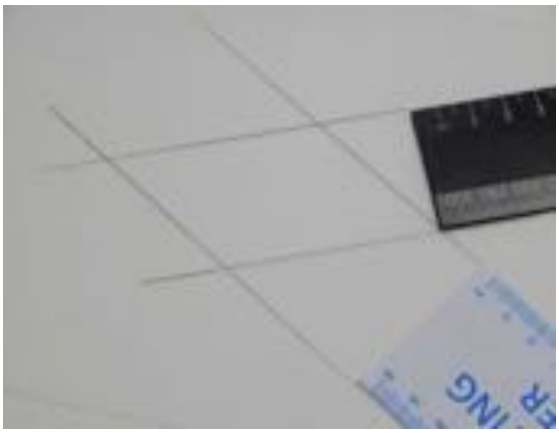
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE
suggests related learning activities for all ages from 4 to 18
on the theme of PARALLELS

Just choose whatever seems suitable for your group of learners

The original **PROPERTIES OF PARALLELOGRAMS** activity was designed for Years 8 - 10

PROPERTIES OF PARALLELOGRAMS



We start from this definition:

A parallelogram is a quadrilateral with both pairs of opposite edges parallel.

In this investigation you will discover some other properties of parallelograms.

Step 1: Use two rulers with different widths to draw a parallelogram on tracing paper or baking paper. Make sure that it is not a rhombus which has the adjacent edges equal in length.

Step 2: Place a second piece of tracing paper over the first and copy the parallelogram.

Step 3: Slide the second tracing paper over the first to compare the lengths of the opposite edges of the parallelogram.

How do the lengths of the opposite edges compare?

What did you discover? Call this

Conjecture 1: Both pairs of opposite edges of a parallelogram are ...

Step 4: Rotate or flip the second piece of paper and compare the sizes of the opposite angles of the parallelogram.

How do the opposite angles compare?

What did you discover? Call this

Conjecture 2: Both pairs of opposite angles of a parallelogram are ...

Step 5: What can you discover about the point of intersection of the diagonals? Draw or fold the two diagonals of the parallelogram. Place a dot on their intersection.

Fold to compare the lengths of the two segments on each diagonal.

What did you discover? Call this

Conjecture 3. It could be: The point of intersection of the diagonals of a parallelogram...

Step 6: Now fold the diagonals of one of the sheets and compare angles between the diagonals. Are the angles equal in size?

Do the diagonals bisect the angles? Justify your answer.

Step 8: Use the paper with the folded diagonal, rotate it and see if it the area of the triangle formed by the diagonal fits on to the triangles of the second paper.

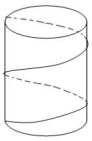
What did you discover? Call this

Conjecture 4. It could be: The diagonal of a parallelogram …

(Idea taken from: Michael Serra, "Patty Paper Geometry", Key Curriculum Press, 1994, pp 88 – 89)

HELP

This is a parallelogram made from the core of a toilet roll, pulled apart very carefully, making the curved surface of the cylinder flat, so that the spiral (a helix) became two parallel edges of the parallelogram. To make it stay flat it was stuck down on the top of a desk. Try this for yourself.



If the opposite edges of a parallelogram are parallel, which angles are equal?

If the opposite edges are parallel, which angles add up to 180?

You have the original parallelogram that you drew and a copy on tracing paper.

By matching lengths and angles in the copy to lengths and angles in the original, try to discover which lengths and angles in a parallelogram are equal.

What can you discover this way about the diagonals of a parallelogram and the angles they form?

What can you discover about the opposite edges or opposite angles of a parallelogram?

NEXT

You started with this definition:

A parallelogram is a quadrilateral with both pairs of opposite edges parallel.

Draw one of the diagonals so that the parallelogram is split into two triangles.

Knowing that the opposite edges are parallel, what can you say about the angles?

Look for congruent triangles.

Can you prove your conjectures?

INCLUSION AND HOME LEARNING GUIDE

Theme: PARALLELS

SOLUTION

These are only conjectures. Once convinced of these facts learners should go on to find proofs that they are in fact true.

Conjecture 1: Both pairs of opposite edges of a parallelogram are **equal.**

Conjecture 2: Both pairs of opposite angles of a parallelogram are **equal.**

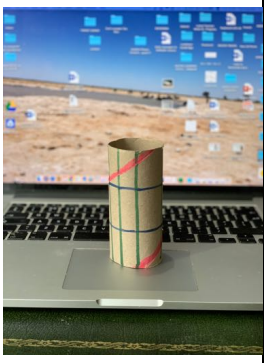
Conjecture 3: The point of intersection of the diagonals of a parallelogram **bisects each diagonal**. Note that the two diagonals are not equal in length.

Conjecture 4: The diagonal of a parallelogram bisects it's area

Suggestions for Home Learning

Early Years and Lower Primary

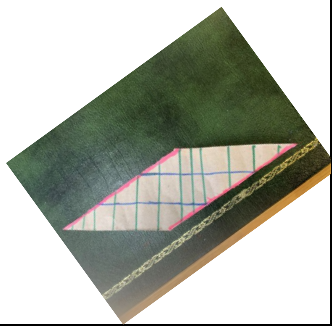
Look for parallel lines in your home, edges of a book or a table, edges in packets and in toys, lines in tiling on floors, on the walls of the room etc. etc. 'These lines are parallel, those lines are not.' is good enough as an explanation of the word for younger children. You want to aim, after a while, for the children to be able to point out parallel lines to you as well as you pointing them out, and for the children to use the word 'parallel' as part of their everyday language.



The picture of a toilet roll core sitting on a computer shows some lines that you can call parallel. The vertical lines are parallel. The circular rings around the cylinder are parallel. Also, the pink spiral (called a helix) around the cylinder comes full circle around in a line parallel to itself above and below. It is also the line where the toilet role core comes apart.

Compare the helix with a spiral staircase.

If you have a mixed-age group, then an older child can do this toilet roll demonstration, marking parallel lines like this on a toilet roll core (preferably using 3 colours). The younger ones can watch and talk about what they think it will look like when it is taken apart and the cylinder is made flat.



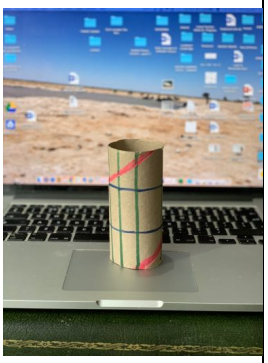
The second picture shows the same toilet roll core opened out and stuck down on a flat surface.

Upper Primary

Resources needed: One toilet roll core for each pair of children. Extras because of discards if the experiment goes wrong. Rulers. Coloured marking pens.

The activity here is exactly the same as for young children, but older primary age children should be able to do the 'toilet core' experiment for themselves, working in pairs and talking about what it will look like when they split open the toilet roll core and flatten it out. Obviously 'between-agers' should be encouraged to do it themselves but may need some help.

Look for parallel lines in your home, edges of a book or a table, edges in packets and in toys, lines in tiling on floors, on the walls of the room etc. etc. 'These lines are parallel, those lines are not.' is good enough as an explanation of the word for younger children. You want to aim, after a while, for the children to be able to point out parallel lines to you as well as you pointing them out, and for the children to use the word 'parallel' as part of their everyday language.



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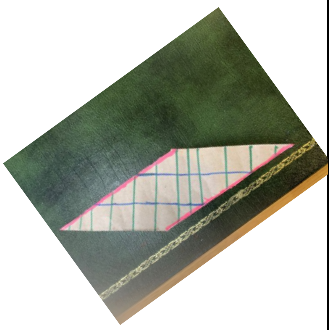

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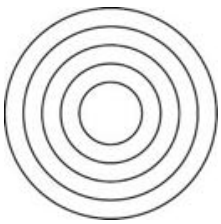


For older children we may say 'these lines are always the same distance apart so they never meet' which includes curved lines like concentric circles. But we should talk about 'parallel lines' as something different because, by definition, parallel lines are always straight lines in geometry.

The distinction between concentric circles and the helix on the toilet roll is quite subtle and interesting. A surface that can be spread out flat onto a plane without breaking has zero curvature like a flat surface. The helix is essentially a straight line on a flat surface.

Lower Secondary

Resources needed: Pencils. Rulers of different widths. Tracing paper or waxed baking paper.

The **Properties of Parallelograms** activity on page 1 is designed for this age group. Ideally the children should have their own copies of the question on pages 1 and 2. Children who find it difficult to understand the question, and to get started, should be given the HELP strip. There is a lot of reading, but this is excellent practice for learners. Think of the life skills that will give them a better chance of a good future. Maths facts in their heads alone are useless without skills like reading comprehension and the ability to use and apply what they have learnt.

After about 10 minutes, if you have several children, get them to work in pairs, to compare what they understand and what they notice, to explain their ideas to each other, and perhaps to produce a joint solution.

To help them, if they seem to need it, you can ask Key Questions, but stop yourself from telling them what to do.

After they finish the Steps in the work that lead them to make each conjecture, you can ask them to explain the conjecture and why they believe in it.

Make sure that they understand that these are only conjectures and still need to be proved.

Key questions

- Did you answer the question?
- What did you discover about the opposite edges of a parallelogram?
- You know that the opposite edges are parallel, what does that tell you about the angles?
- You know that the opposite edges are parallel, so which angles add up to 180^{0}.
- What did you discover about the opposite angles of a parallelogram?
- What did you discover about the diagonals of a parallelogram and the angles they form?

Years 9 and 10

These learners should do the **Properties of Parallelograms** activity and then they should prove the conjectures using what they know about parallel lines and about congruent triangles.
Starting from this definition:

A parallelogram is a quadrilateral with both pairs of opposite edges parallel.

Draw one of the diagonals so that the parallelogram is split into two triangles.
Knowing that the opposite edges are parallel, what can you say about the angles?
Look for congruent triangles.
Can you prove your conjectures?

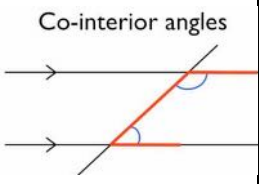
Years 11, 12 and 13

This is written for you to read for yourself. If you need help perhaps you can discuss it with other students or ask a teacher.

Work through the activity on page 1 and then prove the 4 conjectures.

Discuss the meaning of the word **PARALLEL.**

These following assumptions are the basis of **Euclidean Geometry** but they can be expressed in other ways:



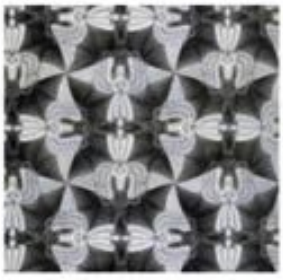
straight lines and points exist and don't have to be explained;
points are the intersections of lines;
a straight line is the shortest distance between two points;
all right angles are equal;
two lines are parallel if and only if the co-interior angles add up to two right angles;
two straight lines in the same infinite plane, that do not intersect at any point, are said to be parallel;
THE PARALLEL POSTULATE: ***through a point, not on a given line, there is one and only one line through that point parallel to the given line.***

In **Spherical Geometry** (also called Elliptic Geometry) there are **no parallel lines**. Think about the North Pole. What are the lines of shortest distance from the North Pole to the South Pole? Yes, lines of longitude. There are infinitely many of them. So it is impossible to find a line through the North Pole parallel to a line through the South Pole.

In **Hyperbolic Geometry** there are infinitely many parallel lines through a point parallel to a given line. This is the geometry used by Einstein in his special theory of relativity.

The artist M.C. Escher used the different geometries of the sphere the flat plane and the hyperbolic plane to give wonderful pictures. These are from The Magic of M.C. Escher, Thames and Hudson, 2000.

Sphere with Angles and Devils (P.92)

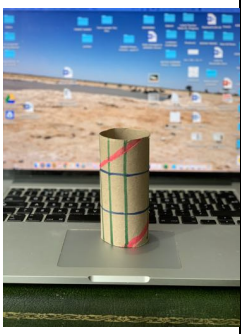


Regular Division of the Plane # 45 (P.93)

Circle limit IV (Heaven and Hell) (P.180)

Moving from 2 to 3 dimensions there is more to think about.

Does this spiral staircase remind you of the toilet roll core experiment for young children described on page 3? Did you observe that the curved lines on the surface of the cylinder became straight when it was flattened? The cylinder is what is called a 'ruled surface'.



If you are interested, you might also talk about lines in parallel planes in 3 dimensional space.

Why do this activity?

This activity helps learners to notice the properties of parallelograms that seem apparent and to make conjectures about what appears to be true about parallelograms. It should then be used as an incentive to the learners to use and apply what they know about congruent triangles in order to prove the conjectures that they have made.

Learning objectives

In doing this activity students will have an opportunity to discover for themselves the properties of parallelograms and parallels.

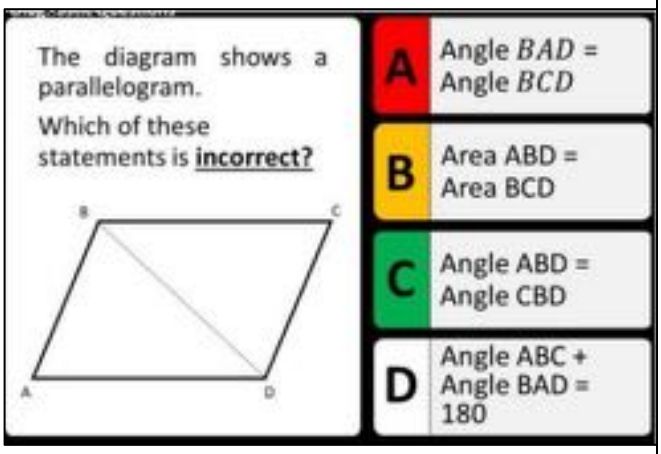
Generic competences

In doing this activity students will have an opportunity to learn to use the correct vocabulary and to get into the habit of checking and asking themselves whether the answer makes sense.

DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes. It can be used before or after the lesson.

Show the question to the learners and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D".



1. Notice how the learners respond. Ask the learners to explain why they or gave their answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer because it helps them to clarify their own thinking and to develop communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote again for the right answer by putting up 1, 2, 3 or 4 fingers.
5. Notice if there is a change and who gave right and wrong answers.

The correct answer is: C
Possible misconceptions:
A. Possibly the learner did not select the intended angles or does not know that the opposite angles are equal.
B. Possibly the learner does not recognise these as congruent triangles.
D. Learners should recognise the two angles add up to 180 as they are co-interior with BC parallel to AD. https://diagnosticquestions.com

Follow up

Properties of quadrilaterals
https://aiminghigh.aimssec.ac.za/years-8-10-properties-of-quadrilaterals/
Tessellating quadrilaterals
https://aiminghigh.aimssec.ac.za/years-7-12-tessellating-quadrilaterals/