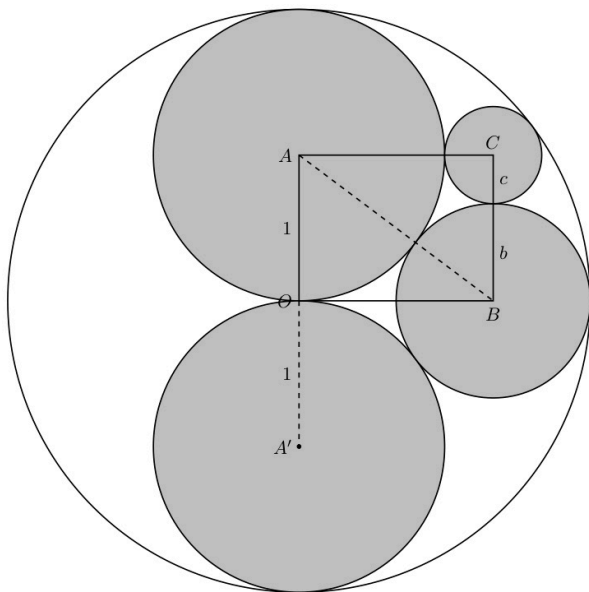# KISSING CIRCLES

The outer circle, centre O, has radius 2 units. The points O, A, A$^{/}$, B and C are centres of circles that are tangent to each other where they touch.

It can be shown that OACB is a rectangle.

1. Prove that OA⊥OB

2. Find the radii b and c and the ratio of the radii of the circles with centres A, B and C.

3. What do you notice about triangle OAB?

HELP

Where the circles touch they have a common tangent and each radius is perpendicular to the tangent. What does this tell you about the lines joining the centres of the circles?

NEXT – *for students who enjoy challenges and are not daunted by having to do a lot of algebra.*

The proof that OACB is a rectangle can be done using coordinate geometry, the formula for the distance between two points and the fact that the circles all touch each other.
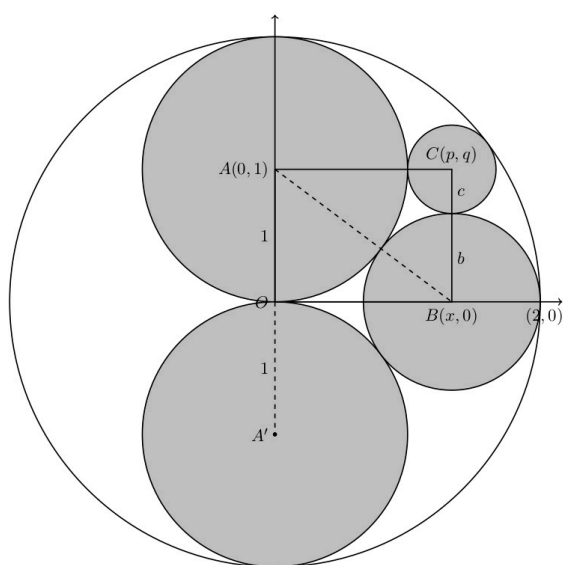
1. Prove that OA⊥O. Then use Pythagoras Theorem for Δ AOB and the fact that the circle centre B touches the outer circle to show that $b = \frac{2}{3}$.

2. The circle centre C touches 3 other circles, use this:

a) Write down 3 equations involving p, q and c and, from them, 3 expressions equal to $p^2 + q^2 - c^2$.

b) Find 3 linear expressions involving p, q and c.

c) Find an expression giving p in terms of c.

d) Find an expression giving q in terms of c.

e) Eliminate p and q to produce a quadratic equation in c and solve this equation to find c.

3. For each value of c, find p and q and identify the two circles to which these values correspond.

4. Explain why the values found show that OACB is a rectangle.