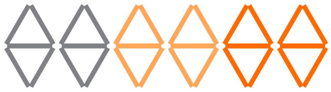
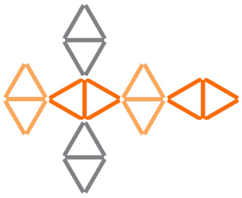
ICOSAHEDRON PUZZLE



This practical activity involves making a skeletal model of an icosahedron by tying together sticks made from paper rolled tightly around pieces of string.

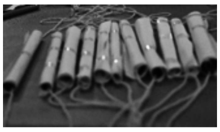
Make 6 rhombuses as shown with 5 sticks for each one.



Tie the rhombuses together to make this pattern noting how the different coloured rhombuses fit into the pattern.

Then make it into an icosahedron. Note that there are exactly 5 edges meeting at all vertices.

What properties of the icosahedron do you notice?



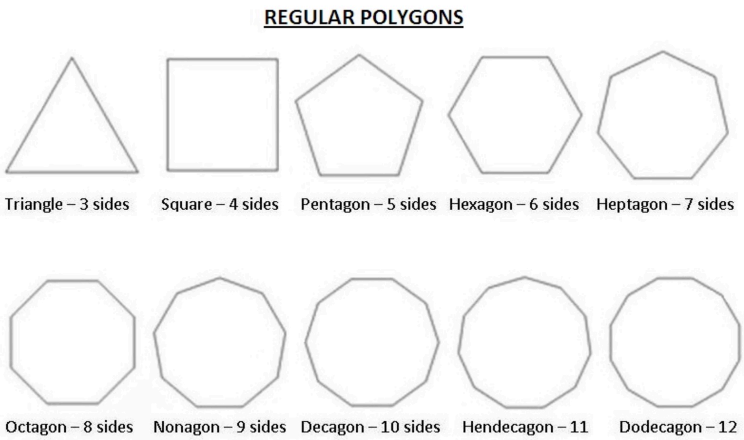
When making your paper sticks it helps to fix the string onto the paper with sticky tape before starting to roll it so the string stays in place. Roll the paper tightly around the string and secure it with sticky tape. Four 10.5 cm sticks can be made from one A4 sheet of paper cut into 4.

HELP

All angles are equal in a regular polygon and all the edges are equal.

The faces of regular polyhedra are all regular polygons and there are the same number of faces meeting at each vertex.

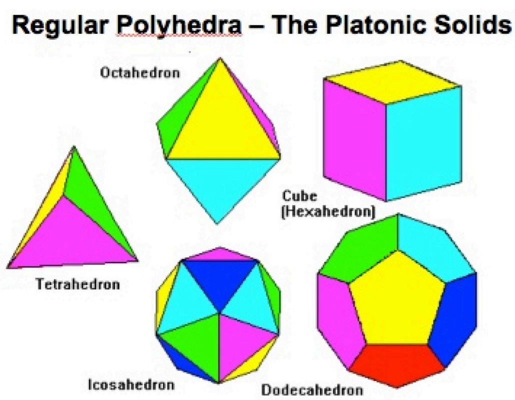
The faces of the icosahedron are all equilateral triangles.



NEXT

Study as many polyhedra as you can and fill in the following table recording the number of faces, edges and vertices of these polyhedra. What do you notice?

The soccer ball is an inflated buckyball or Buckminster fullerine made with 20 hexagons and 12 pentagons. The buckeyball bonding of carbon atoms is found in soot. The structure is used in building geodesic domes.



Name of polyhedron	Number of faces **F**	Number of edges **E**	Number of vertices **V**
Cube			
Tetrahedron			
Octahedron			
Icosahedron			
Dodecahedron			
Buckyball			
Square based pyramid			
Triangular prism			
Cuboid			

Play with the numbers F, V and E for each solid. Add and subtract these numbers. What do you notice? Can you discover a relationship between F, V and E?

See this article on Euler's Formula and Topology about the relationship between F, V and E that you should have discovered, and the importance of this result in mathematics:
https://nrich.maths.org/1384

NOTES FOR TEACHERS

SOLUTION

The icosahedron has **20 triangular faces, 30 edges** and **12 vertices**.

All the faces of the icosahedron are congruent equilateral triangles (regular polygons). This makes the icosahedron a **regular polyhedron** (solid).

It is **one of the five Platonic or regular polyhedra**. The other regular polyhedra are the tetrahedron, octahedron, cube and dodecahedron.

All the **edges are the same length**.

At each vertex 5 faces meet and 5 edges meet.

The lines through **opposite vertices** are axes of **rotational symmetry of order 5** (by angle 72°).

The lines through the **centres of opposite f**aces are axes of rotational symmetry of order 3 (by angle 120°).

The lines through the **midpoints of opposite edges** are axes of **rotational symmetry of order 2** (by angle 180°).

In all cases F+V-E=2 where F is the number of faces, V the number of vertices and E the number of edges. This is Euler's formula. Let the learners discover this for themselves. DO NOT tell them the formula and if a few learners discover it, ask them to keep their discovery quiet so that other learners can make their own discoveries.

Why do this activity?

Learners enjoy making models from paper sticks and you can use scrap paper so there is very little cost. You can make models of different 3D objects and hang them from the ceiling in your classroom. Handling the models will make it much easier for the learners to visualise the shapes and remember their properties. Symmetry is a very important property studied in higher mathematics and widely applicable in other sciences.

Learning objectives

In doing this activity students will have an opportunity to:

- become more familiar with properties of the icosahedron;
- gain a better understanding of the concept of regularity in geometrical shapes;
- improve their ability to visualise the icosahedron and its symmetries by handling the model;
- gain a better understanding of mirror symmetry and rotational symmetry.

Generic competences

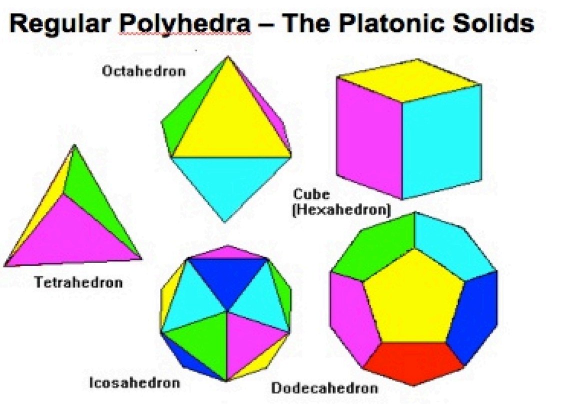
In doing this activity students will have an opportunity to **visualize** and develop the skill of creating and interpreting visual images to represent concepts and situations.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

How many of these solids have triangular faces?

A. All of them **B.** The tetrahedron
C. Three of the **D.** The icosahedron



2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4.Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

C. is the correct answer.
3 of these solids have triangular faces, the tetrahedron, the octahedron and the icosahedron.

Common Misconceptions

A. Learners giving this answer don't understand the concept of faces of a solid.

B. Learners may think the tetrahedron is somehow triangular and not understand that the question was about triangular faces. A question ***"how many …?"*** requires an answer that is a number.

D. This solid has triangular faces but these learners are not answering the question about "***how many …?"***

https://diagnosticquestions.com

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Suggestions for teaching

You might like to start by making tetrahedra and octahedra as these are easier and quicker to make. Perhaps have the learners work in groups of four or six because it helps to have someone to hold the shape while another person ties the knots. It is important to tell everyone to tie the knots firmly so that they can undo them if they make a mistake. **If you don't want to display all the models then untie the knots and keep the sticks for other activities.**

Avoid using the word 'side' as it is ambiguous, instead use the term 'edge' for a line and 'face' for a flat surface. Side is often used for the edge of a flat shape but also, in 3 dimensions, for faces, walls and areas in one direction or another.

Before they start to make the model of the icosahedron they first need to make 30 paper sticks. Groups of 6 work well because each learner can make their own rhombus and then be responsible for fitting it into the model. It is important to tell the learners to tie the knots firmly but so that they can undo them if they make a mistake. When they finish their models the learners should make lists of the properties that they notice.

The whole class should then be involved in suggesting the properties which the teacher can summarize and write on the board. The learners should check their own list of properties and copy the complete list into their workbooks.

Demonstrate rotational symmetry by holding the model at two points and turning it through the same angle a few times into a corresponding position and finally back to its initial position.

For opposite vertices you get rotational symmetry of order 5, turning through angles of 72^o,
for midpoints of opposite edges you get rotational symmetry of order 2 turning by angles of 180^o,
and for centres of opposite faces you get rotational symmetry of order 3 turning by angles of 120^o.

Hang one example of each model from the ceiling of your classroom.

APPLICATIONS OF SYMMETRY

In addition to applications to art and design, symmetry is fundamentally important in mathematics and science. In architecture and engineering, it is symmetry that gives structures strength and rigidity. In physics, chemistry and crystallography it is important to understand the symmetry groups of substances at microscopic and macroscopic levels.

Key questions *These are questions you ask to help learners to **think for themselves**.*

***STOP yourself** telling children what to do next. Instead ask a **KEY QUESTION**.*

- Have you checked that all your sticks are the same length?
- What is that shape called?
- How many edges meet at that vertex? Is that right?
- Have you got 5 edges meeting at that vertex?
- How many faces are there altogether?
- How many edges are there altogether?
- How many vertices are there altogether?

- Can you find any rotational symmetry?
- Hold your model at two opposite vertices, turn it round slowly and stop when it gets to fill the same position in space.
- How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

- Show me the midpoint of an edge. Now where is the midpoint of the opposite edge? Hold those 2 points.
- Now turn the icosahedron round slowly. Is there any rotational symmetry?
- How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

- Show me the centre of a face. Where is the centre of the opposite face? Hold those 2 points.
- Now turn the icosahedron round slowly. Is there any rotational symmetry?
- How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

Follow up

Make more polyhedra from paper sticks and hang them from the ceiling of your classroom in two distinct groups: the regular polyhedra and the irregular polyhedra such as the cuboid and other prisms and pyramids other than regular tetrahedra.

Metre Measures https://aiminghigh.aimssec.ac.za/years-3-7-metre-measures/
Tets and Octs Puzzle https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/
Wholesome Rectangles https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/

<table>
<tr><td colspan="5">Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.</td></tr>
<tr><td></td><td>Lower Primary
or Foundation Phase
Age 5 to 9</td><td>Upper Primary
Age 9 to 11</td><td>Lower Secondary
Age 11 to 14</td><td>Upper Secondary
Age 15+</td></tr>
<tr><td>South Africa</td><td>Grades R and 1 to 3</td><td>Grades 4 to 6</td><td>Grades 7 to 9</td><td>Grades 10 to 12</td></tr>
<tr><td>USA</td><td>Kindergarten and G1 to 3</td><td>Grades 4 to 6</td><td>Grades 7 to 9</td><td>Grades 10 to 12</td></tr>
<tr><td>UK</td><td>Reception and Years 1 to 3</td><td>Years 4 to 6</td><td>Years 7 to 9</td><td>Years 10 to 13</td></tr>
<tr><td>East Africa</td><td>Nursery and Primary 1 to 3</td><td>Primary 4 to 6</td><td>Secondary 1 to 3</td><td>Secondary 4 to 6</td></tr>
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