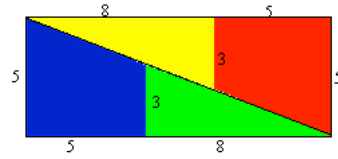
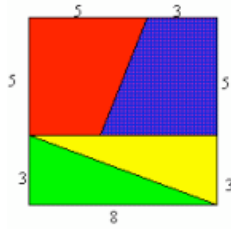


DOESN'T ADD UP



Look at this square divided into four pieces:
two identical triangles and two identical trapezia.

The square has edges of length 8 units and area 64 square units.

The edge of length 3 from each triangle matches with the edge of length 3 from a trapezium so that the four pieces from the square now occupy a rectangular space.

The area of the rectangle is 65 square units (13 by 5)

Can the area have changed? Can you explain what has happened?

HELP

Draw the 4 pieces accurately, measuring the right angles carefully or use squared paper.

Now make both arrangements with your 4 pieces.

Look carefully. Can you explain what happens? Are you sure about it?

NEXT

Can you make up your own similar problem?

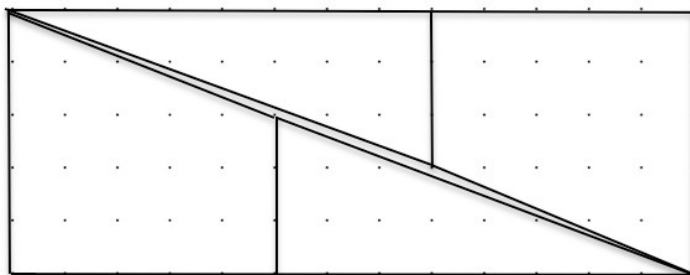
Another set of dimensions are 4, 7, 11 in the corresponding places of 3, 5, 8.

The areas are 121 square units compared to 126 square units.

Draw these pieces and the two arrangements.

NOTES FOR TEACHERS

SOLUTION



The discrepancy is due to the slightly different slopes (gradients) of the edges making quadrilaterals look like triangles.

The areas do not change but the pieces do not fit closely together

to make the rectangle as it might appear.

The sloping edges of the trapezium and the triangle do not form a straight line across the diagonal of the rectangle.

The gradient of the sloping edges of each triangle is $3/8 = 0,375$

The gradient of the sloping edge of each trapezium is $2/5 = 0,4$ (slightly 'steeper').

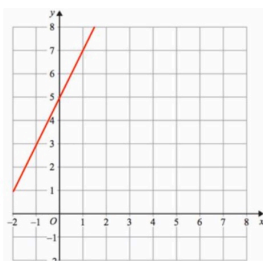
There is a very thin parallelogram shaped space across the diagonal of the rectangle making a gap of area 1 square unit inside the rectangle.

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

What is the gradient of the line?



A

-2

B

2

C

1

D

3

2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. **DO NOT** say whether it is right or wrong just thank the learner for giving the answer.

3. It is important for learners to explain the reason for their answer because putting their thoughts into words improves their understanding and helps to develop communication skills.

4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

5. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is B. The gradient is 2.

A. Student does not understand and is using the intercept on the x-axis.

C. No understanding of gradient. With slope or gradient 1 the line would be at 45 degrees.

D. Probably a guess.

<https://diagnosticquestions.com>

Why do this activity?

This problem gives learners of the opportunity to practice mathematical thinking and visualization and it gives them an experience that helps to develop the important skill of checking every detail of a system to find out what has gone wrong.

The geometrical ideas of area, conservation of area and slopes (gradients) of lines are fundamental.

Learners should at first be surprised by the almost magical change in area. They should know that the pieces don't change in area simply by being moved around (**conservation of area**) and they should be motivated to find an explanation.

This problem provides an experience to show us that we must be careful what we assume and there is a need to check things out. This problem is also motivating because it is a trick that seems to be magic and it could be used to impress parents and friends.

Learning objectives

In doing this activity students will have an opportunity to:

- review areas of simple 2D shapes;
- review gradients of line segments.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop observational and visualization skills**.

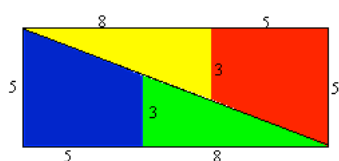
Suggestions for teaching

WARM-UP OR LESSON STARTER

For older learners this could be a warm-up task or lesson starter that is ready on the board so learners get busy as soon as they sit down in class. It provides a challenge and an opportunity for revision of basic knowledge and understanding of area and gradients. It need not take up much lesson time, especially for upper secondary school students.

For younger learners, suggest that they draw the diagrams carefully on squared paper or dotty squared paper or construct accurate diagrams if squared paper is not available and even if they work in pairs, learners should draw their own diagrams.

You could ask the learners to work out the areas of the separate pieces. Everyone



assumes that this is a rectangle with area $5 \times 13 = 65$ square units. What is wrong? Why is there a difference of 1 square unit of area?

Is it magic? No **there must be an explanation. What do people assume when they look at this diagram?** Could it be that their eyes are deceiving them and they are assuming something is true when it is not true?

Encourage the learners to be **detectives** and to discover what is going wrong. **The answer is not important**, it is the process of investigation that gives the rich learning experience. **DO NOT tell explain it to them.** Just say that they are developing important skills, and if they persevere, then they will make the all-important discovery.

Let the learners talk about it and try to discover the explanation. Who is an expert detective? It's best not to feel that you need to finish this in one lesson.

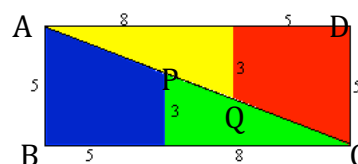
Come back to it later. If nobody finds an explanation then you can give clues as below one at a time.

Clue 1: You need to look closely at the line AC.

Clue 2: You need to look closely at points P and Q where three of the shapes meet.

Clue 3: Are you sure that ACB is a triangle? Could it be any other shape?

Clue 4: Is APQC a straight line?



Give these clues one by one and encourage the learners to use the clues. This leads to a discussion of the fact that that PA is steeper than CP, meaning that ACB is not a triangle. APCB and AQCD are quadrilaterals. There is a VERY thin parallelogram shaped gap along AC which takes up 1 square unit of area.

Doesn't Add Up is an ideal activity to use before you introduce the definition of gradient because it helps learners to understand why there is a need for this definition. If learners have already met the definition of gradient than you can ask "What do you know about gradients?" and "Can you use gradients knowledge in this problem?"

You may want to use the Diagnostic Quiz to assess whether the learners understand gradients at the end of the lesson.

Key questions

- What do we know about areas of squares?
- What do we know about areas of rectangles?
- What do we know about areas of triangles?
- What do we know about areas of trapezia?
- Do you believe that a shape can change its area when it is moved around?
- Have you tried drawing the diagrams on squared paper?
- What makes a line straight?
- What do you know about gradients? Can you use that knowledge in this problem?

Follow up

Also see the problem Muggles Magic

<https://aiminghigh.aimssec.ac.za/years-7-10-muggles-magic/>



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