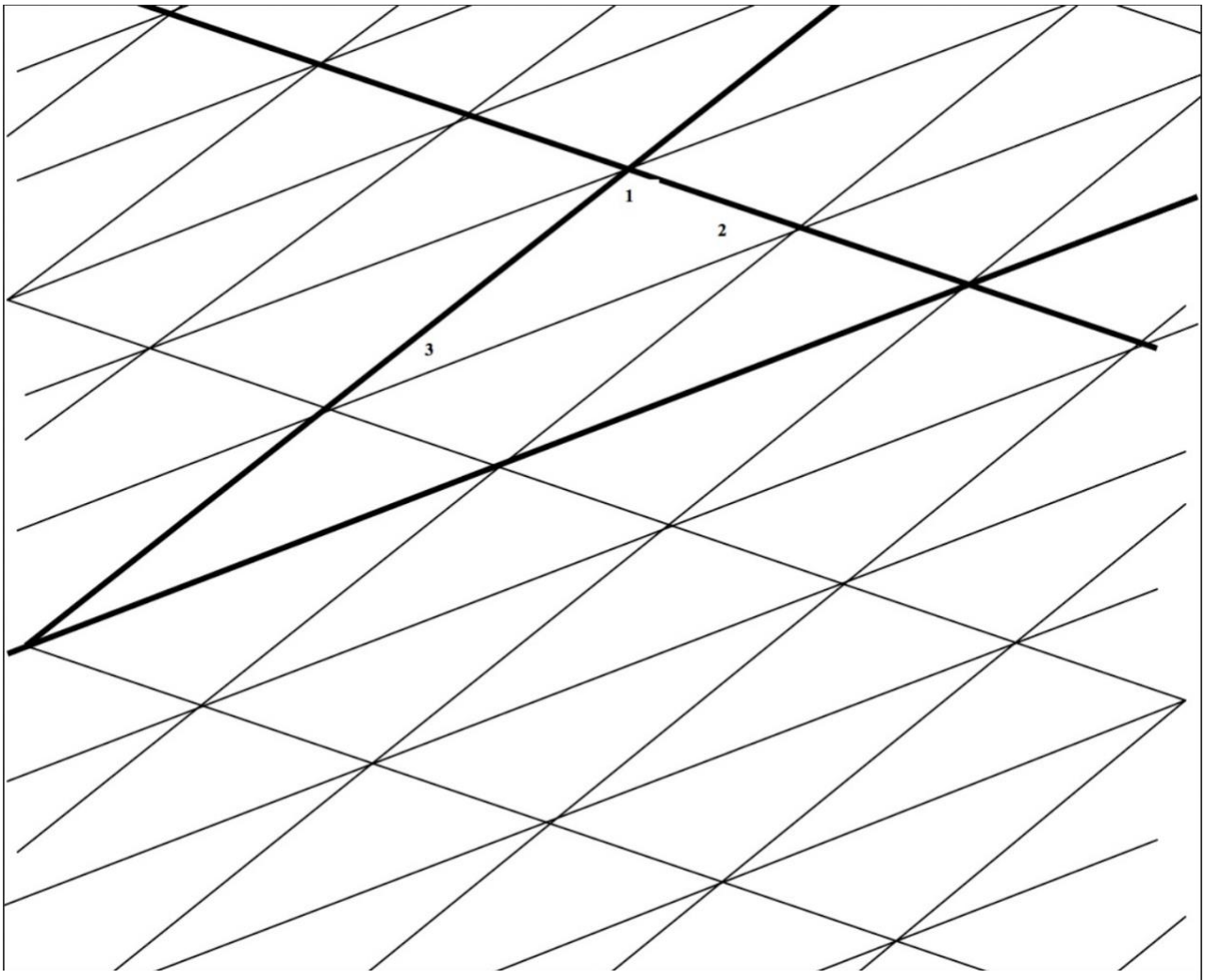
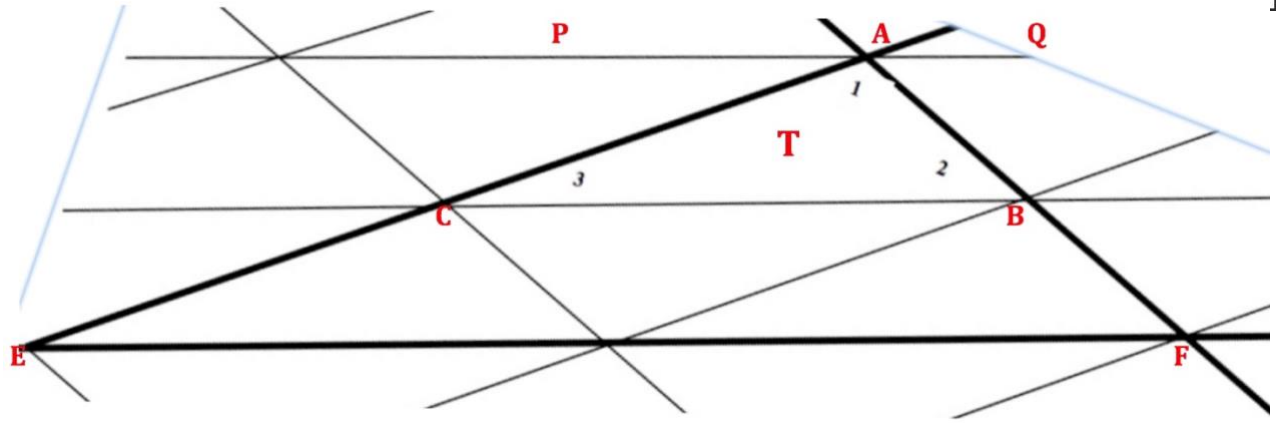
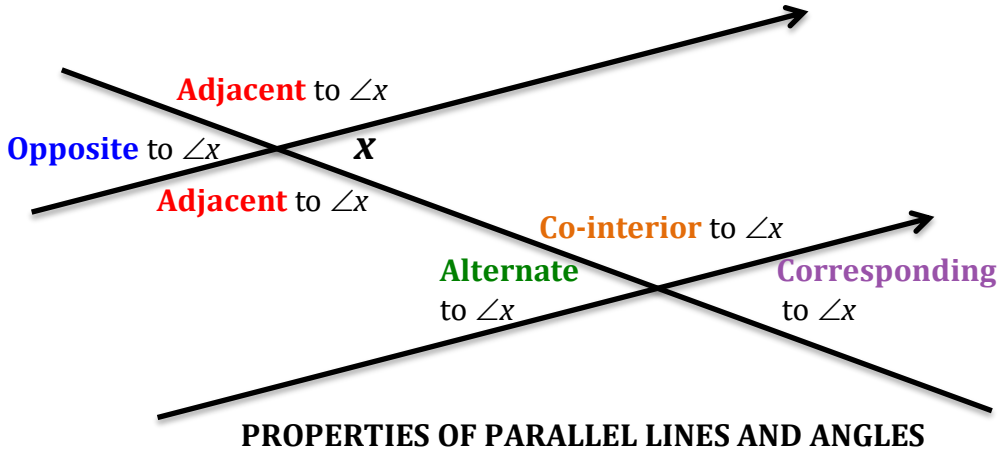
What do you see in this diagram?
***Look at the triangle with angles marked 1, 2 and 3. Call this* T.**
In the enlarged part of the diagram at the bottom of the page label angles equal to these angles.
Why are they equal? What geometrical facts can you see illustrated in this diagram?
Look at the larger heavily outlined triangle.
How many copies of triangle* T *can you see in this triangle?
What else do you notice about this triangle?





HELP

It helps to work with a partner if you can. You might use a cardboard or plastic triangle cut from scrap material as a template. Draw around it to make your own tessellation like the one shown. You can refer to the diagram below to help you to know what words to use.



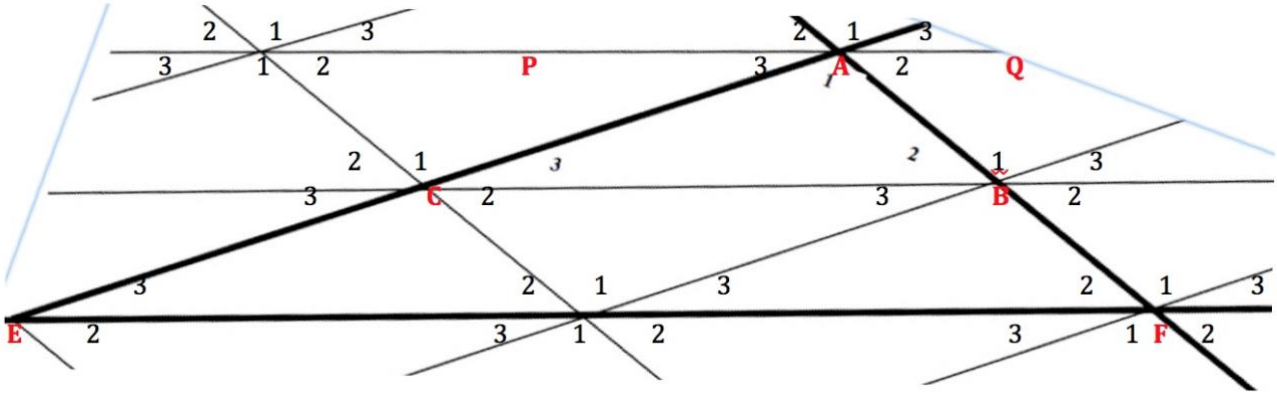
PROPERTIES OF PARALLEL LINES AND ANGLES

NEXT

Draw your own tessellations using ruler, protractor and compasses.

INCLUSION AND HOME LEARNING GUIDE

SOLUTION



All the angles marked 1 in the diagram are equal to ∠CAB as vertically opposite angles are equal and corresponding angles between parallel lines are equal.

Similarly all the angles marked 2 are equal and all the angles marked 3 are equal for the same reasons. This labelling can be continued indefinitely in all directions across the whole tessellation.

The diagram illustrates the theorem that the angles of a triangle add up to 180º. The line PAQ is parallel to CB so the alternate angles ∠PAC = ∠ACB (angle 3) and ∠QAB = ∠ABC (angle 2). As the angles on a straight line add up to 180º so

∠PAC + ∠CAB + ∠BAQ = ∠3 + ∠1 + ∠2 = ∠ACB + ∠CAB + ∠ABC = 180º

The ΔABC (also labelled T) is similar to ΔAFE as the three angles in each triangle are equal. The lengths of the edges of ΔAFE are twice the lengths of the edges of ΔABC so ΔAFE is an enlargement of ΔABC by a scale factor 2 with the centre of the enlargement at A. There are 4 copies of ΔABC in ΔAFE so the area scale factor of the enlargement is 4.

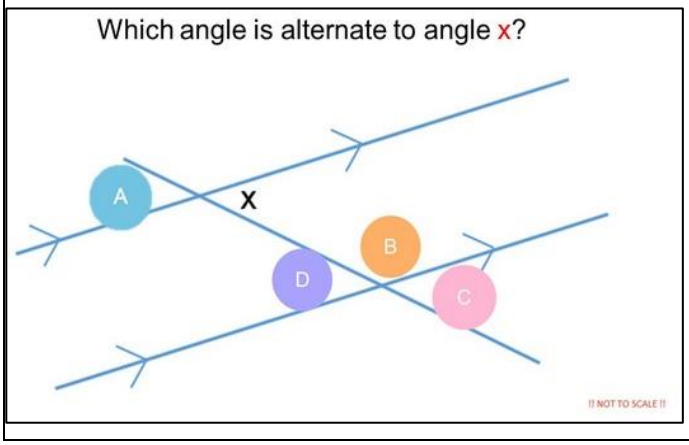
Many more geometrical facts can be seen from this diagram.

Diagnostic Assessment

This should take about 5–10 minutes. It can be used before or after the lesson.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask them all to explain why they gave their answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer. Try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.

3. **Ask the learners again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**

The correct answer is: D

Common Misconceptions

Wrong answers or guesses here mean that the learner does not know the word 'alternate'. Learning and remembering the mathematical words is not easy particularly if the English is an additional language. This example gives the teacher a basis for reviewing the angle properties of parallels and, most importantly, helping the learners to remember the words.

A wall chart would be helpful (See page 4) https://diagnosticquestions.com

Why do this activity?

This activity can be used with different age groups to engage learners in looking carefully at different geometrical properties illustrated in the diagram and talking about these properties. Learners can be asked to make a list of everything that they notice and then all the observations can be shared. This is a very open ended activity giving plenty of scope for learners to think mathematically, to develop their visualisation skills and to contribute to the class discussion. It can be revision or lead into introducing some new geometrical concepts.

Learning objectives

In doing this activity students will have an opportunity for a review of the geometry of parallel lines, angles, triangles and polygons.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts and situations;

Suggestions for teaching

Resources: *Tracing paper would be a useful resource here.*
Worksheets downloaded and copied.

Start the session with the Diagnostic Quiz. Then give out the worksheet on page 1 and ask the learners to read it individually and to start to answer the questions, writing their answers in their workbooks. Working individually to read the question for themselves gives learners practice in reading information and answering mathematical questions. These are skills that they need to develop so that they can do this in tests and in later life.

You may wish to give the HELP strip to learners who you expect to struggle, especially to those who got the answers wrong twice in the Diagnostic Quiz. Or if you have a mixed age group in your household, give the HELP strip to the younger learners (age 10 and upwards).

The NEXT strip is for learners who finish the activity ahead of the rest of the class and who would benefit from an extra challenge.

You could give out tracing paper. Greaseproof paper as used in the kitchen is good for this. Suggest that learners should trace the diagram onto the tracing paper, then move the tracing paper around to identify copies of triangle T, equal angles etc. and they should make lists of what they discover.

If you have a group then learners can share ideas. Some children will have noticed properties that others did not see and older learners can explain to younger learners what they see in the diagram. Finally summarize what has been learned.

The group could make a poster listing all the geometrical properties that the learners have noticed. You could challenge the learners to find more properties not on the list.

Key Questions

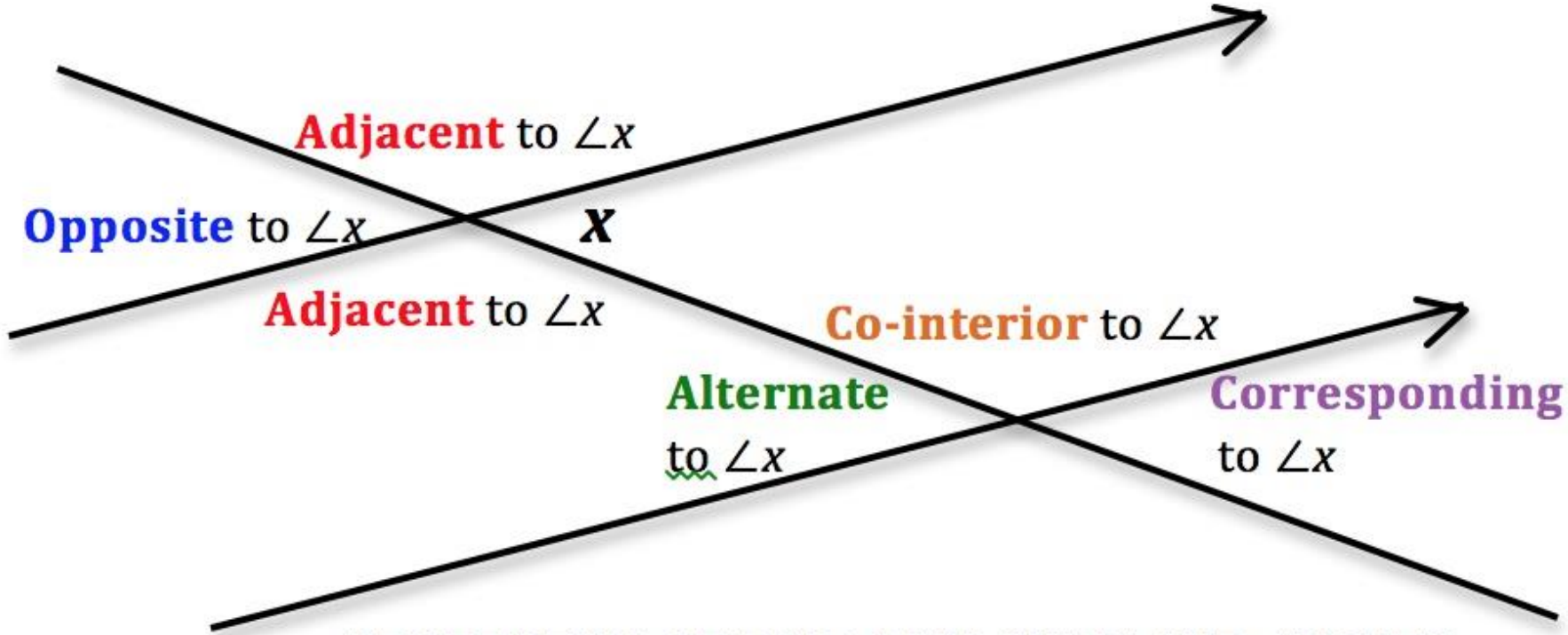
- What shapes do you see?
- Which angles are equal? Why?
- What properties of the parallel lines do you see?
- Can you see similar triangles? How do you know they are similar?
- Can you see similar parallelograms? How do you know they are similar?

Follow up

Tessellating Quadrilaterals https://aiminghigh.aimssec.ac.za/years-7-12-tessellating-quadrilaterals/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6



PROPERTIES OF PARALLEL LINES AND ANGLES