SANDWICHES

Suppose you have two 1's, two 2's and two 3's.
Arrange these six digits in a list so that:

between the two 1's there is one digit giving 1?1,
between the two 2's there are two digits giving 2??2,
between the two 3's there are three digits giving 3???3.

Can you do the same if you only have 1's and 2's? Explain your answer.

Can you do the same if you include two fours, and between the two 4's there are four digits?

The illustration shows a 7-sandwich using the pairs of all the whole numbers from 1 to 7.

Can you make another 7 sandwich?

For some values of n it is possible to make n-sandwiches using pairs of every positive integer from 1 to n, and arranging the list according to the rules given above, and for other values of n it is not possible. The big challenge is to find when it is and when it is not possible.

HELP

You may want to stop with n=4.

NEXT

The process of working systematically through all possible cases can be tedious and time consuming. If you can, discuss with other people the best systematic approach and how you can share the work of searching for solutions and checking. Such work offers a valuable learning experience and prepares you for using computers in problem solving, not just in mathematics but also in other fields.

For example, one person could look for sandwiches that start with 17 then with 16, 15 etc. and another person could look for sandwiches that start with 71 then with 61, 51 etc.

In order to construct a computer program you have first to plan a systematic approach to the problem. Another extension could be to write a computer program to find solutions and to test which values of n yield n-sandwiches.

The alternate colours are the key to the proof that for some values of n you can make n-sandwiches and for some values of n it is impossible.

7 1 3 1 6 4 3 5 7 2 4 6 2 5

Finally, there is a method of proving that for certain values of n it is impossible to make n-sandwiches. The argument is simple and requires only very elementary mathematics. The proof falls into the 'Aha' category - once you see it, it seems obvious and amazingly simple, but the choice of method called for real inspiration in the first place. The picture gives a clue. Can you think of the proof?