AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

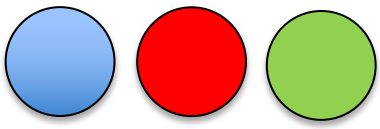
PATTERN is the theme for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 5 to 17+

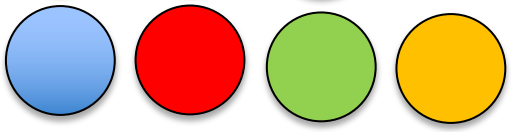
Just choose whatever seems suitable for your group of learners.

The PERMUTATION PUZZLE activity was designed for Lower and Upper Secondary

PERMUTATION PUZZLE



How many different ways can you arrange 3 counters in a row. Record the arrangements.



Now try 4 counters.

What about 5 counters or more?

Can you see a pattern?

You can use counters or cut out pieces of paper or card.

HELP

How many ways can you place the first counter?
How many ways can you place the second counter?
How many arrangements is that for 2 counters?
You have 2 counters there, how many ways can you insert a 3rd counter in the line?
How many arrangements is that for 3 counters?
Can you carry this process on?

NEXT

Can you discover a formula for the number of arrangements, otherwise called *permutations*, for n counters in a row?

INCLUSION AND HOME LEARNING GUIDE

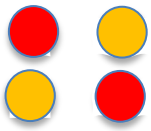
THEME: Pattern

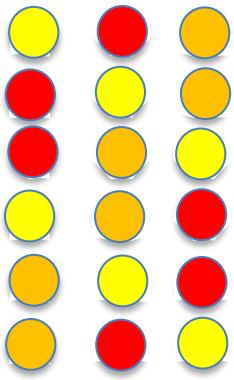
Young children

Non mathematicians may not realize how important the concept of pattern is in mathematics. Explore different ways of arranging a set of objects in different orders is a valuable learning experience.

Simply ask the child to arrange two or three soft toys, or toy cars, or any other objects, in different orders. For the youngest children there is no need for counting.

Playing with colours is a good way to explore patterns. What they have just done with toys, young children can do with two or three colours, either rearranging coloured counters (which can be made from paper) or using colouring pens.





Sometimes we have a fixed pattern, like the colours in a rainbow.

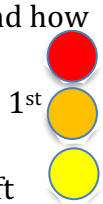
Sometimes it's interesting to explore different patterns made by re-arranging the order of things.

Upper Primary

Introduce the task as given on page 1. Learners can work individually or in pairs to find the number of arrangements of 3 objects of different colours.

Help learners to discover the number of arrangements for themselves and avoid telling them how to do it. When most of your group have found that the answer with 3 counters is 6 arrangements ask them to explain what answer they found and how they found it. Key Questions: Answer:

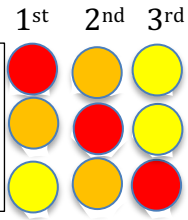
1. How many ways can the 1st place be filled? 3 ways:
2. How many ways can the 2nd place be filled? 2 ways for each 1st choice:
3. How many ways can the 3rd place be filled? 1 way, there is 1 counter left

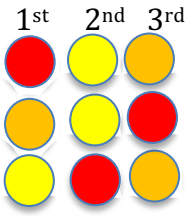


After red you can put orange or yellow

After orange you can put red or yellow

After yellow you can put orange or red





PEOPLE MATHS

If you have 3 people you can enact this by arranging them in line in 6 different ways.

Secondary Years 7 to 11

Introduce the task as given on page 1. Learners can work individually or in pairs to find the number of arrangements of 3 objects of different colours as described above for primary. Ask the **Key Questions**

- How many ways can you place the first counter? (or first person in line) (3 choices)
- What choices have you now for second counter? (2 choices)
- How many arrangements is that for 3 counters? (3 × 2 = 6 arrangements)

Let learners discover what to do for themselves and avoid telling them how to do it, rather keep asking questions. When the child or most of the group have found the answer with 3 counters ask them to explain what answer they found and how they found it. Ask "how do you know that you have found all the arrangements?"
There is **more than one method** so LISTEN and share methods.

Suppose we write R for red, O for orange and Y for yellow.

The alternative method is to imagine 2 counters (or people) in order R O or O R

There are 3 places to put the 3rd counter: in first, middle or last place giving the arrangements Y R O ; R Y O ; R O Y and Y O R ; O Y R ; O R Y making 6 arrangements.

Explain that the mathematical word used for 'arrangement' is '*permutation*'.

Then ask the learners to find the number of permutations for 4 objects and 5 objects. Can they see a pattern?
There are:
permutations for 2 objects,
6 permutations for 3 objects,
24 for permutations for 4 objects.

You might then pose the problem:
Suppose 10 children line up in single file in a queue for lunch.
How many ways could they be arranged in line?
What happens when another person joins the line?

Having worked out the number of permutations for 2, 3 , 4 and 5 objects, think about 10. Ask how many permutations for 10 people in a queue?

If that seems difficult think about 6 in line? Can anyone see a pattern? Can anyone explain the rule for calculating the number of permutations for 10 objects?

Nobody needs to do the calculation but, if you have calculators with a $\boxed{n!}$ key (called *factorial n*) then you'll find that the answer is more than 3 million. Factorial 10, written 10! = 10 × 9 × 8 × … × 3 × 2 ×1 = 3,628,800.

Finish with the Diagnostic Quiz and follow steps 1 to 5. This relates to learners' everyday experience. Discuss similar problems. For example if you have 2 coloured T-shirts and one pair of jeans how many ways can you dress (2). What about the answer for 2 coloured T-shirts with either one pair of jeans or one pair of shorts (4).

Years 12 and 13

The application of counting principles is required in Grade 12 in South Africa and very widely elsewhere. It follows directly from the methods of counting permutations from first principles rather than applying formula.

Discuss the following example from the South African curriculum:
How many 3 character codes can be formed if the first character must be a letter and second and third characters must be digits (if 26 letters and 10 digits are allowed)?
Answer 26 × 10 × 10 = 2600

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

You are choosing what clothes to wear.
You can put on a blue shirt or a red shirt or a green shirt.
You can put on either jeans or shorts.
How many different ways can you dress?

A. 5 **B. 6** **C. 2** **D. 4**

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer to give them an opportunity to put their thoughts into words, to develop communication skills and perhaps gain a better understanding.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**

The correct answer is B. 3 × 2 =6
outfits: blueT/jeans, redT/jeans, greenT/jeans,
blueT/shorts, redT/shorts, greenT/shorts.

A. If learners add 3 +2 it means they are not thinking logically, just combining the two numbers. This is typical of learners who don't expect to understand the maths and see doing maths as just following rules that make no sense. https://diagnosticquestions.com

SOLUTION

For 3 counters there are 3 choices of colour for the first counter, 2 choices for the second counter and only one choice for the third counter.

So there are 3 × 2 × 1 = 6 arrangements or permutations.

They can be recorded by drawing or by colour, for example, using R for red and O for orange and Y for yellow, the arrangements could be recorded as listed in the box.

ROY
RYO
ORY
OYR
YOR
YRO

For 4 counters, there are 4 choices of colour for the first counter, 3 for the second, 2 for the third and 1 for the fourth.

So there are 4 × 3 × 2 × 1 = 24 permutations.

For 5 counters it is $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations.

And this pattern continues…

For n counters it is $n \times (n - 1) \times (n - 2) \times \ldots \times 3 \times 2 \times 1$ permutations. This number is written using an exclamation mark n! and you will find it on your calculator.

Why do this activity?

Solving puzzles can be enjoyable and the Permutation Puzzle is not difficult to solve practically using 3 bottle tops or any 3 objects.

This puzzle is suitable for quite young children and even for 17 and 18 year olds.

Whereas with 8 year olds it is sufficient to find the answer for 3 objects, for 18 year olds they should learn the word 'permutation', understand how to find and explain the general formula, and be able to apply the same ideas in solving the sort of probability problems they will meet in school leaving examinations.

Being able to generalise a problem is important in mathematics. Learners can easily solve this puzzle and they can be encouraged to look for a pattern so they can find the rule that gives the answer for ANY NUMBER. In that way even in primary school, learners can generalise patterns for themselves by finding the answer for 2 counters, 3 or 4 or 5 counters and so on and here many learners can discover the rule for themselves. When they do that, tell them that they are thinking like real mathematicians and praise their efforts.

Learning objectives

In doing this activity students will have an opportunity to:

- experiment with manipulatives (concrete objects that can be handled) to investigate a general number pattern;
- understand and learn the mathematical word *permutation* and its meaning;
- discover a general rule for permutations and explain the rule in words or using a formula.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **visualize** and develop the skill of interpreting and creating visual images;
- independently **solve a problem** that has many important applications**.**

Follow up

Nines and Tens https://aiminghigh.aimssec.ac.za/years-7-10-nines-and-tens/
Red Evens https://aiminghigh.aimssec.ac.za/years-10-red-even/

Using the concept of permutations in solving problems involving probability:
Same sweets: https://aiminghigh.aimssec.ac.za/years-4-7-same-sweets/
In a Box https://aiminghigh.aimssec.ac.za/years-6-12-in-a-box/

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the MATHS TOYS YouTube Channel
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13