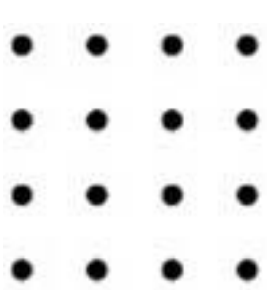
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

THE HOW MANY SQUARES Inclusion and Home Learning Guide is part of a Learning Pack downloadable from the AIMING HIGH website http://aiminghigh.aimssec.za
It provides ideas for activities for inclusion and differentiation in school lessons, guidance for home-learning, and related learning activities for all ages and learning stages from pre-school to school-leaving, all on the Common Theme **SQUARES**
Guidance for school lessons is given in the separate Notes for Teachers documents.
Choose what seems suitable for the attainment levels of your learners.

HOW MANY SQUARES?



How many squares can you make by joining four points on this grid?

Look for squares of different sizes and also tilted squares

SIMPLE CASES

Start with the 9-pin grid (3 by 3). Can you find six squares?

Then go on to this 16-pin grid.

THE BIG CHALLENGE: Solve the puzzle for the 25-pin grid.

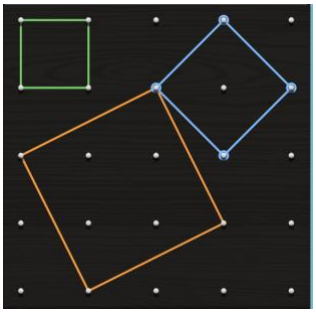
Success depends on a systematic method.

1. Identify all the different types of square
2. Count and record the numbers of squares of each type.
3. Add up to find the total.

What can you say about the areas of the squares?

Try the Square It game on the NRICH site

HELP



Start with a 4-pin grid (2 by 2) and then 9-pin grid (3 by 3).

Next work on a 16-pin grid.

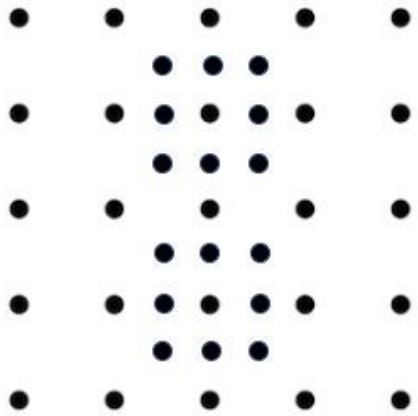
After you have found all the answers for the smaller grids, and only then will you be ready to solve the puzzle for the 25-pin grid.

You should find 1 square on the 4-pin grid and 6 squares on the 9-pin grid.

A geoboard is excellent for exploring all the squares.

NEXT

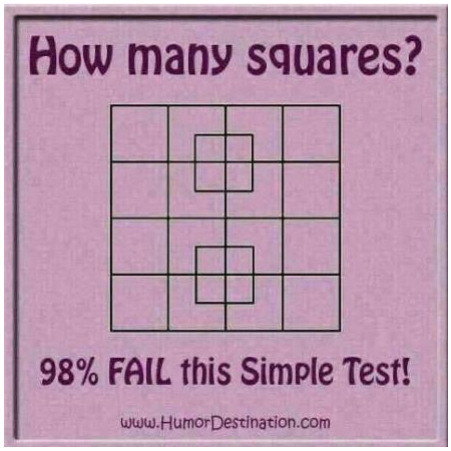
THE 41 PIN GRID



Find all the squares that can be made by joining 4 dots on this grid.

The view of the same puzzle on the right may help you find all the squares.

There are 41 straight up straight across squares. You must also count the tilted squares.



INCLUSION AND HOME LEARNING GUIDE
THEME:

Why do this activity?

This puzzle helps learners to develop problem solving and visualisation skills. The activity can be adapted for different age groups and different attainment levels. Finding different solutions can be treated as a game. Many learners can be given the chance to contribute answers by using a large sheet of paper (flip chart paper) to record each new solution as it is found. Alternatively use 8 sheets, one for each type of square on the 25-pin grid.

This activity can be used to reinforce the concept of area and to develop the understanding that the formula for the area of a triangle is derived from half the area of the enclosing rectangle.

The activity can also be used to reinforce recognition of transformations, and fluency in the associated mathematical language.

The activity can lead to work on analytic geometry, and to understanding that gradient and perpendicularity depend on 'distance across' and 'distance up'. It can also lead to a proof of Pythagoras Theorem (see https://aiminghigh.aimssec.ac.za/pythagoras-jigsaw/)

Learning objectives

In doing this activity students will have an opportunity to:

- to develop problem solving and visualisation skills and experience of working systematically;
- to develop concepts of area;
- to develop concepts of translation, reflection and rotation;
- to introduce the ideas of distance, gradient and perpendicularity in analytic geometry;
- to lead up to work on Pythagoras Theorem.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts and situations;
- interpret and **solve problems**;
- **work in a team to** collaborate and work with a partner or group.

Starter Activity for All

Play the **Square It** game as a group split in 2 teams.

Teams take it in turns to mark a dot on the grid with their colour and claim that dot.

The winner is the first to claim four dots in their own colour that can be joined by straight lines to form a square.

See https://aiminghigh.aimssec.ac.za/square-it-game/

If you have access to a computer then your group can try to beat the computer following this link (see http://nrich.maths.org/2526).

Early Years and Lower Primary

Play the Square It game on a 9-pin grid.

Count all the squares that can be made on a 4-pin and 9-pin geoboard or grid.

Upper Primary

Play the Square It game on a 16-pin grid as a class in 2 teams.

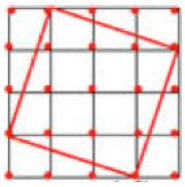
Count all the squares that can be made on a 4-pin, 9-pin and 16-pin geoboard or grid.

Lower Secondary

Play the Square It game on a 25-pin grid as a class in 2 teams. Learners can then play in pairs. Then point out that they will have a better chance of winning if they can find and be aware of all possible squares on the grid.

Then ask the class how many different squares they can find on the grid, either 16-pin (4 by 4) or 25- pin (5 by 5). Have a class discussion about what makes squares *different*: either joining 4 different points or different in area (size). It works well to say that squares OF THE SAME TYPE have the same size and same area. Then the group should look for ALL squares of each type that can be made by joining different sets of 4 dots (or pins on a geoboard). Squares can be any size and can be tilted.

A good way to make it easier for learners to find and record all possible squares is for them to classify the squares by area and work systematically to find them all and record their results in the table below. They should write the areas, and the numbers of each type, beside the sketches in the table and fill in the totals in the right-hand column. The easiest way to calculate the areas of tilted squares is to use the subtraction method.

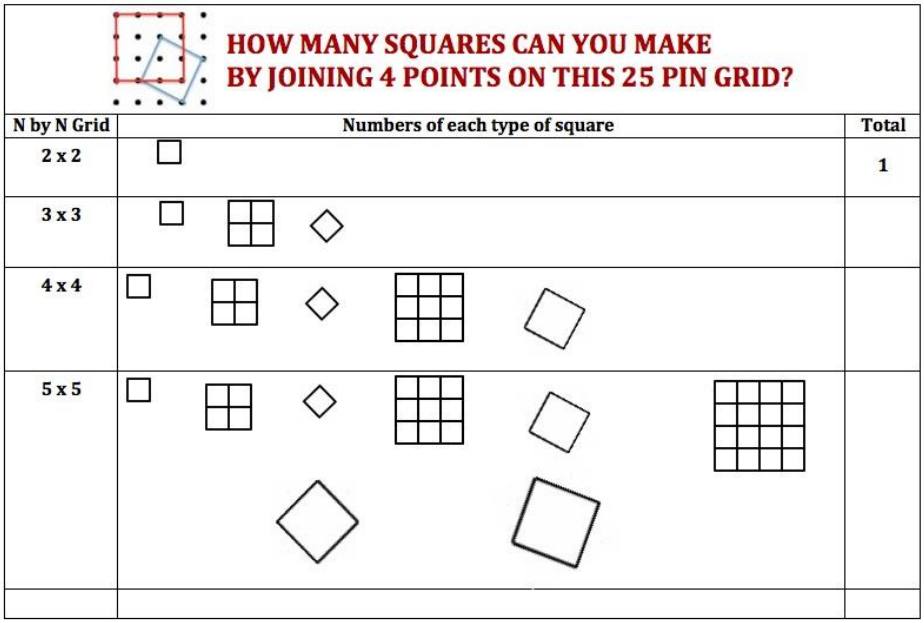


Subtraction Method for Area

Area of large square = 16 square units.

Areas of 4 outer triangles (each ½ area of rectangle) = 1½ square units.

Area of tilted square = 16 – (4 × 1½) = 10 square units.



According to your learning objective for the lesson, set the challenge to the class to work in pairs or small groups.

Share the answers in a plenary session. If learners have not found all the answers in the lesson the teacher should NOT tell them the remaining answers. Better, challenge the learners to find more answers in the next few days. Put a large sheet of paper (flip chart paper) on the wall to record each new solution as it is found.

Key Questions

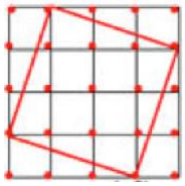
- How can you be sure that those 4 dots form a square?
- What is different about those two squares?
- What is the same about those two squares?
- Are there any more squares like that (of that size) joining different dots on the grid?

Upper Secondary

Play the Square It game on a 5 by 5 (25-pin) grid or geoboard.

You will have a better chance of winning if you can find and be aware of all possible squares on the grid.

A good way to make it easier for yourself to find and record all possible squares, classify the squares by area and work systematically to find them all. Record your results in the table below. Write the areas, and the numbers of each type, beside the sketches in the table and fill in the totals in the right-hand column.
The easiest way to calculate the areas of tilted squares is to use the subtraction method.



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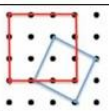
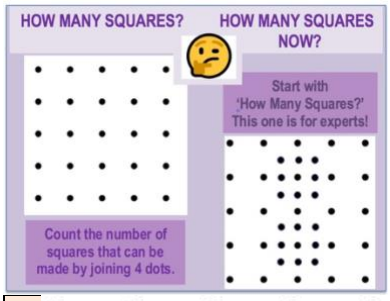


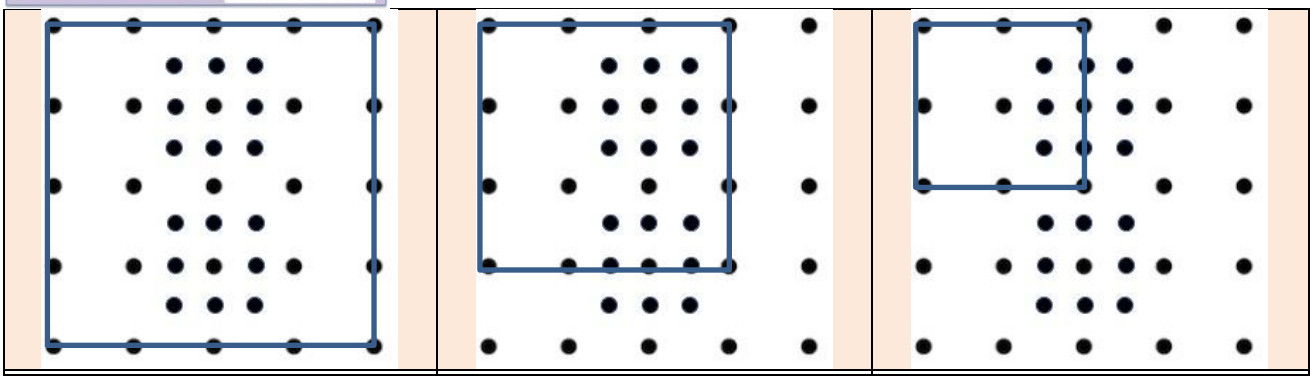
HOW MANY SQUARES CAN YOU MAKE BY JOINING 4 POINTS ON THIS 25 PIN GRID?

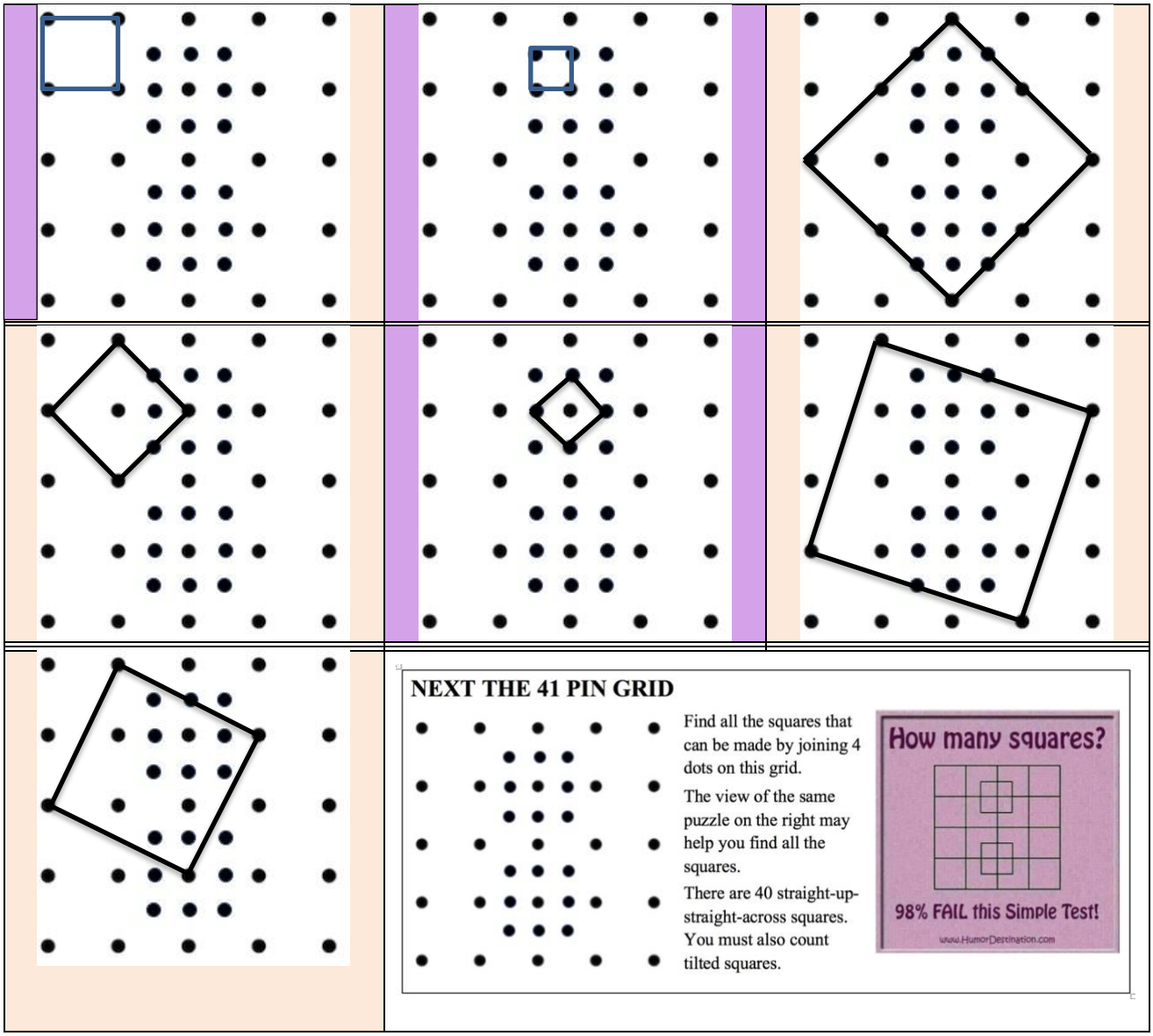
N by N Grid	Numbers of each type of square	Total
2 x 2		1
3 x 3		
4 x 4		
5 x 5		

THE BIG CHALLENGE – THE 41 PIN PUZZZLE



When you have solved the 25-pin puzzle, then do the 41-pin puzzle which has an extra 16 pins and a total of 41 axis aligned (straight across and straight up and down) squares. Can you find them all? The numbers of squares of each type and their areas are given below. How many squares are there altogether? Can you fill in the numbers and areas of each type in the table below identifying them all? Pink strips indicate grids for the squares on the 25-pin board and lilac strips for the squares that use the extra pins on the 41-pin grid.





NEXT THE 41 PIN GRID

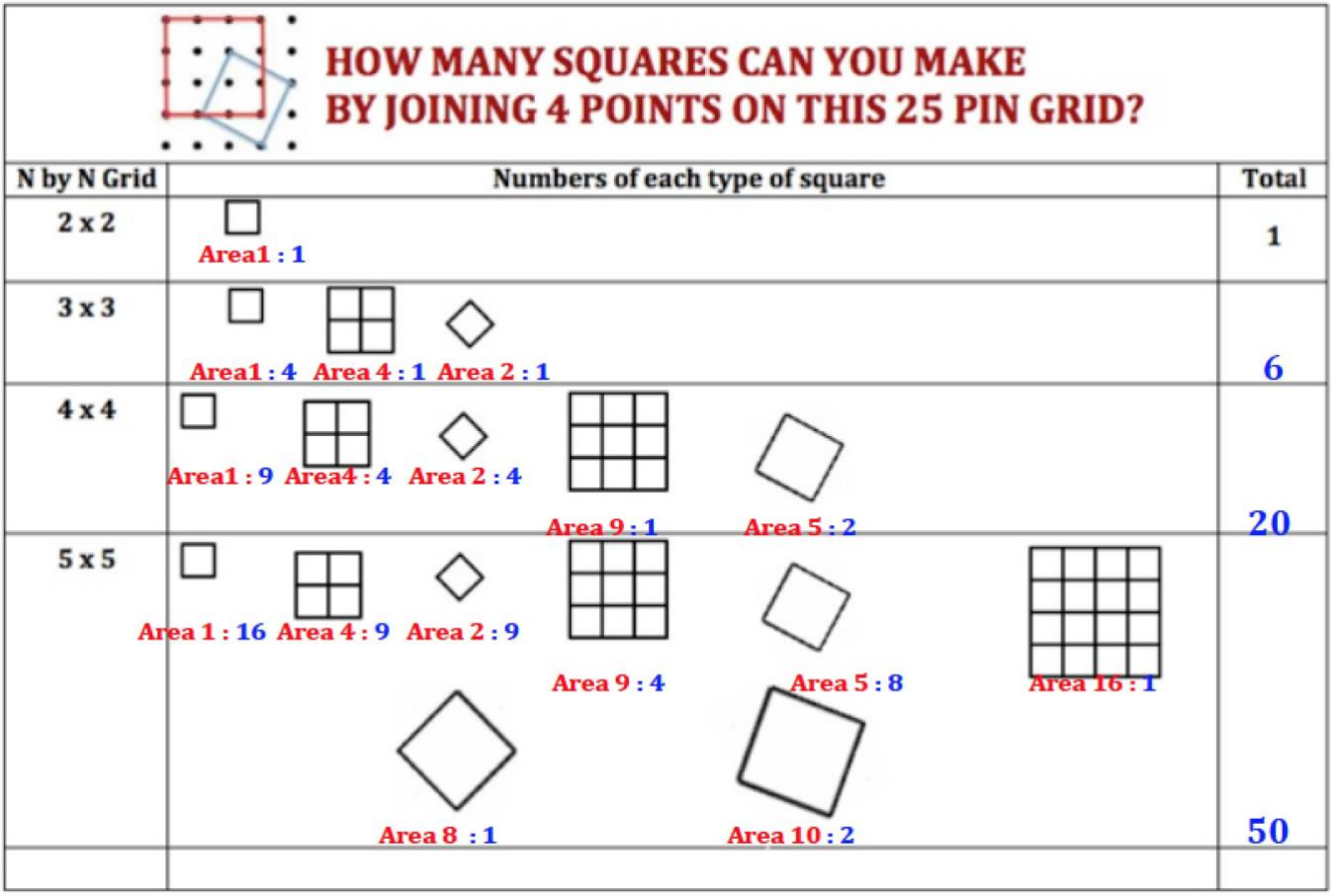
Find all the squares that can be made by joining 4 dots on this grid.

The view of the same puzzle on the right may help you find all the squares.

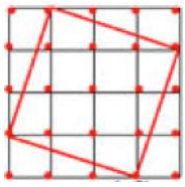
There are 40 straight-up-straight-across squares. You must also count tilted squares.

SOLUTION

1. *Identify all the different types of square.*
   The squares are classified according to their area. Squares of the same type and area can be transformed to others of the same type by translation, rotation or reflection. The chart shows that for the 4-pin grid there is 1 type of square; for the 9-pin grid there are 3 types; for the 16-pin grid there are 5 types and for the 25-pin grid there are 8 types.
2. *Count and record the numbers of squares of each type.*
   The numbers of each type are given in blue in the chart. Each square counted is made by joining a different set of 4 pins on the geoboard, or 4 dots on a grid drawn on paper.
3. *Add up to find the total*.
   The totals are given in the right hand column.



It can be proved that the total number of squares on an n by n grid is $n^2(n^2-1)/12$.



All squares can be enclosed in an axis aligned square bounding box as illustrated in the diagram. To prove the result we calculate the number of bounding boxes and the number of squares in each one and multiply the results.

This involves using the well-known formulas for the sums of series of k terms, k^2 terms and k^3 terms from k =1 to n-1 and then simplifying the result.

THE 41 PIN PUZZLE

The total is 73