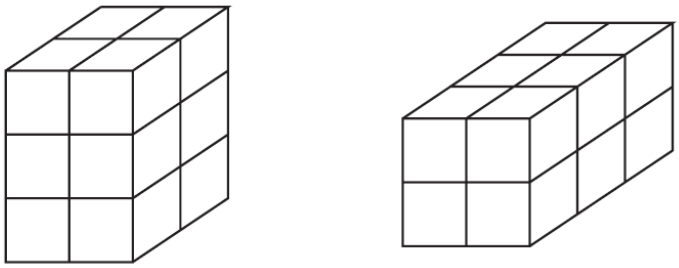
HOW MANY CAN YOU MAKE?

You have 12 cubes, each with sides 1cm long.
The two rectangular prisms shown below have been made using these 12 cubes.



Are these the same or different?
They are the same.
They can be described as $3 \times 2 \times 2$ rectangular prisms.
There are three other rectangular prisms which can be made from 12 cubes.

1. Describe them.
2. How many different rectangular prisms can you make using:
   a) 16 cubes
   b) 10 cubes
   c) 7 cubes
   d) 5 cubes
   e) 9 cubes?
   Describe each of them in the same way as the example above.
3. Without drawing them or using cubes, decide how many different rectangular prisms you could make using the following number of cubes: Describe each of them.
   a) 17
   b) 8
   c) 20
   d) 49
   e) 15

HELP

You will be given 16 cubes.
You can use these to build the different possible rectangular prisms.
Try and visualise the different possible rectangular prisms without the cubes.
Then use the cubes to check you have found all the possibilities.
How many rectangular prisms can be made from 18 cubes?

$(9\times \mathbf{3} \times \mathbf{1})$, $(\mathbf{6} \times \mathbf{3} \times \mathbf{1})$ and $(\mathbf{3} \times \mathbf{3} \times \mathbf{3})$

Three rectangular prisms can be made from 18 cubes.
Which one is made from prime numbers?

$$(\mathbf{3} \times \mathbf{3} \times \mathbf{3})$$

This can be written as $\mathbf{3^3}$
Think about this when you are trying to complete the extension activity.

NEXT

4. Can you determine a general result, which gives you the number of different rectangular prisms it is possible to make, using any number of cubes?
Hint: Write each number of cubes as a product of its prime factors and look at the sum of the powers for each number.

NOTES FOR TEACHERS

SOLUTION

1. $12 \times 1 \times 1, \quad 6 \times 2 \times 1, \quad 4 \times 3 \times 1$
2. a) 4 different rectangular prisms
   $16 \times 1 \times 1, \quad 8 \times 2 \times 1, \quad 4 \times 4 \times 1, \quad 4 \times 2 \times 2$
   b) 2 different rectangular prisms
   $10 \times 1 \times 1, \quad 5 \times 2 \times 1$
   c) 1 rectangular prism
   $7 \times 1 \times 1$
   d) 1 rectangular prism
   $5 \times 1 \times 1$
   e) 2 rectangular prisms
   $9 \times 1 \times 1, \quad 3 \times 3 \times 1$
3. a) 1 rectangular prism
   $17 \times 1 \times 1$
   b) 3 rectangular prisms
   $8 \times 1 \times 1, \quad 4 \times 2 \times 1, \quad 2 \times 2 \times 2$
   c) 4 rectangular prisms
   $20 \times 1 \times 1, \ 10 \times 2 \times 1, \ 5 \times 4 \times 1$
   d) 2 rectangular prisms
   $49 \times 1 \times 1, \quad 7 \times 7 \times 1$
   e) 2 rectangular prisms
   $15 \times 1 \times 1, \ 5 \times 3 \times 1$
4. General results can be deduced from the factorisation into prime numbers
   For example;
   17 cubes $= 17^1$ 1 rectangular prism

8 cubes = 2^3	3 rectangular prisms
20 cubes = $2^2\ 5^1$	3 rectangular prisms
49 cubes = 7^2	2 rectangular prisms
15 cubes = $3^1 5^1$	2 rectangular prisms

Note that this does not always give the exact answer but it gives you the lower boundary.

100 = $2^2 5^2$ 4 is the lower boundary.

You can construct 6 different rectangular prisms.

128 = 2^7 7 is the lower boundary

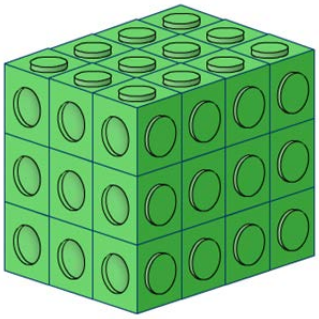
You can construct 8 different rectangular prisms.

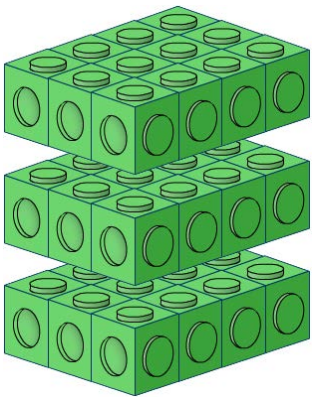
Diagnostic Assessment

This should take about 5–10 minutes.

The learner's should have had a lesson on volume of rectangular prisms before trying this activity.

The learners should understand the following;





This rectangular prism is made from 36 small cubes. Each small cube has a side length of 1 cm.

The volume of each small cube is 1cm³

You can see that the rectangular prism is made from three layers.

Each layer is made from 12 small cubes. So the volume of each layer is 12cm³.

There are 3 layers so the volume of the rectangular prism is $3 \times 12 = 36cm^3$

This rectangular prism can be described as a $3 \times 4 \times 3$ prism. Can you see why?

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

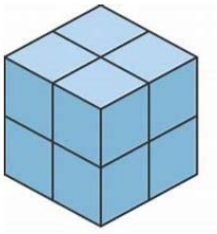
The correct answer and possible misconceptions:

A.
B.
C.
D.

https://diagnosticquestions.com

What is the volume of this rectangular prism?

a) 12 cubic units
b) 7 cubic units
c) 8 cubic units
d) 8 cm^2

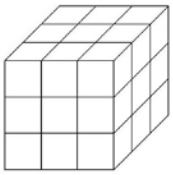


The correct answer is c) 8 cubic units.
Possible misconceptions;

a) 12 cubic units. Possibly trying to use a formula for volume incorrectly.
b) 7 cubic units. The learner is not visualising two layers of 4 small cubes.
c) 8 cm^2. The learner is confused about the units for area and volume.

How many small cubes have been used to create this larger cube?

a) 36
b) 27
c) 19
d) 9



The correct answer is b) 27
Possible misconceptions;

a) 36. The learner is not visualising 3 layers of cubes. With each layer made up of 9 cubes.
c) 19. The learner has guessed an answer.
d) 9. The learner has only counted one layer of cubes.

Why do this activity?

This is a good activity to help learners to develop an understanding of the properties of a rectangular prism and relate these to its volume.
It also gives learners practice in working out all the possible factors of a given number.

Learning objectives

In doing this activity students will have an opportunity to:
Develop the skills of visualization and systematic working.
Apply knowledge of factors and to work systematically to check that they have all possible triples of factors.
Recognise triples that are made from prime numbers.
Develop a deeper understanding of calculating volume.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems** in a variety of situations;
- **work and learn independently** and prepare for lifelong learning;
- **work in a team**:
    - collaborate and work with a partner or group
    - have empathy with others, listen to different points of view
    - develop leadership qualities;
- **communicate** in writing, speaking and listening according to the audience:
    - exchange ideas, criticise, and present information and ideas to others
    - analyze, reason and record ideas effectively;
- **develop life skills and consideration for others -** to show social responsibility – to work for the good of the community.

Suggestions for teaching

You could start by showing the class a model of a rectangular prism made from cubic centimetres.
You could hold up one cubic centimeter and ask the learners how many cubic centimetres were used to make the model.
You could ask them to explain how they got their answer.
Encourage the learners to think of the layers of cubic centimetres making up the model. Ask them to use the concept of layers to get the answer for the total number of cubic centimetres.
Then ask the learners if they could make different rectangular prisms with the same volume as this rectangular prism.
You could then ask them how you would describe the different rectangular prisms thinking about how the layers were composed and the number of layers.

Key questions

1. What is the volume of this small cube?
2. Why?
3. How many small cubes were used to make this rectangular prism?
4. How did you get your answer?
5. How many layers make up this rectangular prism?
6. How many small cubes are in each layer?
7. How did you calculate this answer?
8. How could you calculate the volume using the layers?
9. How many different rectangular prisms could you make using the same number of small cubes?
10. How would you describe these rectangular prisms using three numbers?
11. Will all of these rectangular prisms have the same volume?
12. Why?
13. What is a prime number?
14. Which of the rectangular prisms that you have made can be described using at least one prime number?

Follow up

https://aiminghigh.aimssec.ac.za/which-one-has-the-smallest-surface-area/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see https://nrich.maths.org/12339 Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6