HOW MANY SQUARES?

How many squares can you make by joining four points on this grid?
Look for squares of different sizes and also tilted squares

SIMPLE CASES
Start with the 9-pin grid (3 by 3). Can you find six squares?
Then go on to this 16-pin grid.

THE BIG CHALLENGE: Solve the puzzle for the 25-pin grid.

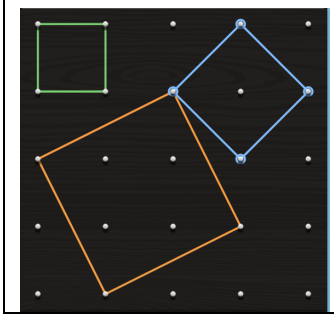
Success depends on a systematic method.

1. Identify all the different types of square
2. Count and record the numbers of squares of each type.
3. Add up to find the total.

What can you say about the areas of the squares?

Try the Square It game on the NRICH site

HELP



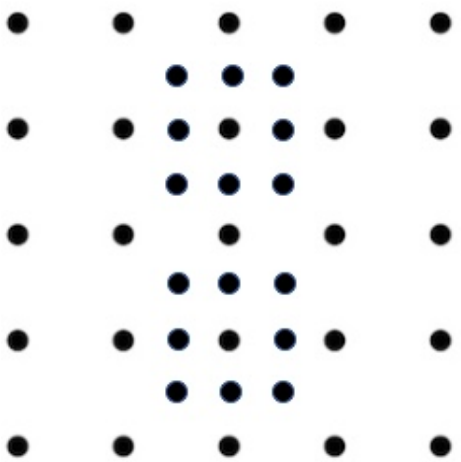
Start with a 4-pin grid (2 by 2) and then 9-pin grid (3 by 3). Next work on a 16-pin grid.

After you have found all the answers for the smaller grids, and only then will you be ready to solve the puzzle for the 25-pin grid.

You should find 1 square on the 4-pin grid and 6 squares on the 9-pin grid.

A geoboard is excellent for exploring all the squares.

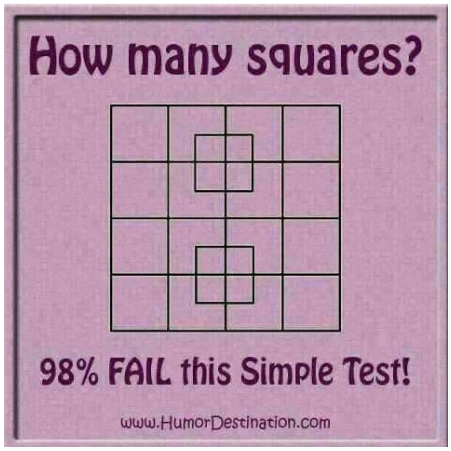
NEXT THE 41 PIN GRID



Find all the squares that can be made by joining 4 dots on this grid.

The view of the same puzzle on the right may help you find all the squares.

There are 4 straight-up-straight-across squares. You must also count tilted squares.



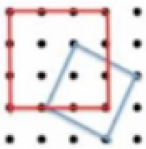
GUIDE FOR HOME-LEARNING

SOLUTION

1. *Identify all the different types of square*
   The squares are classified according to their area. The areas are given in red.
   The chart shows that for the 4-pin grid there is 1 type of square; for the 9-pin grid there are 3 types; for the 16-pin grid there are 5 types and for the 25-pin grid there are 8 types.
   Squares of the same type and area can be transformed to others of the same type by translation, rotation or reflection.

2. *Count and record the numbers of squares of each type.*
   The numbers are shown of each type are given in blue in the chart. Each square counted is made by joining a different set of 4 pins on the geoboard, or 4 dots on a grid drawn on paper.

3. *Add up to find the total.*
   The totals are given in the right hand column.



HOW MANY SQUARES CAN YOU MAKE BY JOINING 4 POINTS ON THIS 25 PIN GRID?

N by N Grid	Numbers of each type of square	Total
2 x 2	Area1 : 1	1
3 x 3	Area1 : 4 Area 4 : 1 Area 2 : 1	6
4 x 4	Area1 : 9 Area4 : 4 Area 2 : 4 Area 9 : 1 Area 5 : 2	20
5 x 5	Area 1 : 16 Area 4 : 9 Area 2 : 9 Area 9 : 4 Area 5 : 8 Area 16 : 1 Area 8 : 1 Area 10 : 2	50

Why do this activity?

This puzzle helps learners to develop problem solving and visualisation skills. The activity can be adapted for different age groups and different attainment levels. Finding different solutions can be treated as a game. Many learners can be given the chance to contribute answers by using a large sheet of paper (flip chart paper) to record each new solution as it is found. Alternatively use 8 sheets, one for each type of square on the 25 pin grid,

This activity can be used to reinforce the concept of area and to develop the understanding that the formula for the area of a triangle is derived from half the area of the enclosing rectangle.

The activity can also be used to reinforce recognition of transformations, and fluency in the associated mathematical language.

The activity can lead to work on analytic geometry, and to understanding that gradient and perpendicularity depend on 'distance across' and 'distance up'. It can also lead to a proof of Pythagoras Theorem (see https://aiminghigh.aimssec.ac.za/years-8-12-pythagoras-jigsaw/)

Learning objectives

Lessons can be planned to use this lesson to work towards one or more of the following:

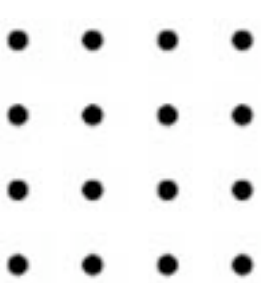
- to develop problem solving and visualisation skills and experience of working systematically;
- to develop concepts of area;
- to develop concepts of translation, reflection and rotation;
- to introduce the ideas of distance, gradient and perpendicularity in analytic geometry;
- to lead up to work on Pythagoras Theorem.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts and situations;
- interpret and **solve problems**;
- **work in a team to** collaborate and work with a partner or group.

Suggestions for home-learning



Play the **Square It** game as a group or in pairs. See https://aiminghigh.aimssec.ac.za/years-4-10-square-it-game/
If you have access to a computer then your group can try to beat the computer following this link (see http://nrich.maths.org/2526).

Players take it in turns to mark a dot on the grid with their colour and claim that dot.

The winner is the first to claim four dots in their own colour that can be joined by straight lines to form a square.

Squares can be any size and can be tilted.

Learners can then play in pairs against each other.

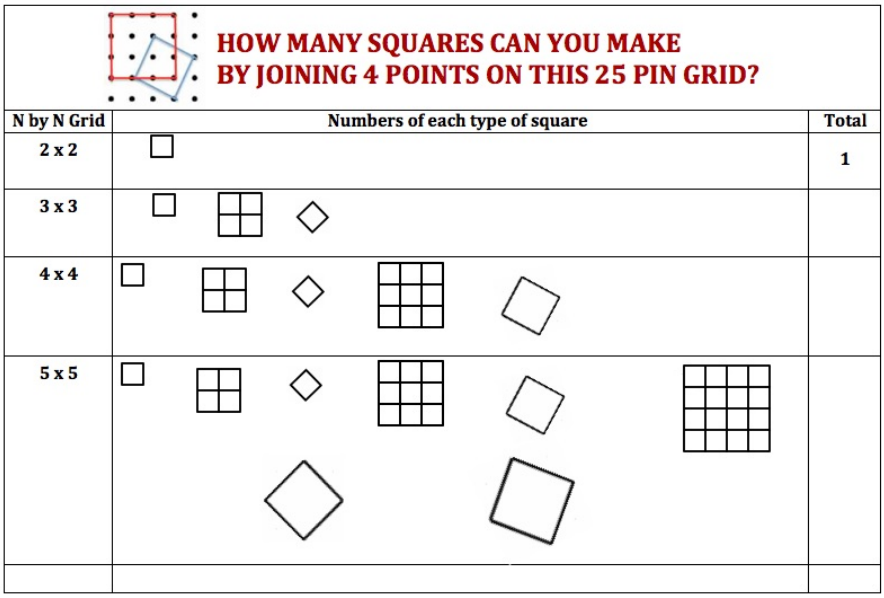
Then ask the class how many different squares they can find on the grid, either 16-pin (4 by 4) or 25- pin (5 by 5).

Have a class discussion about what you take to be ***different***: either joining 4 different points or different in area (size). It works well to say that squares OF THE SAME TYPE have the same size and area. Then the class should look for ALL squares of each type that can be made by joining different sets of 4 dots (or pins on a geoboard).

According to your learning objective for the lesson, set the challenge to the class to work in pairs or small groups.

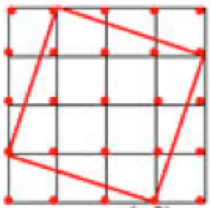
Share the answers in a plenary session. If they have not found all the answers in the lesson the teacher should NOT tell them the remaining answers. Better, challenge the learners to find more answers in the next few days. Put a large sheet of paper (flip chart paper) on the wall to record each new solution as it is found.

Example of one possible lesson – (Learning objective: To develop problem solving and visualisation skills and experience of working systematically.)



Give learners copies of this diagram and dotty grids and ask them to work out how many squares of each type there are if squares are different when they join different dots.

In another lesson you can ask the learners to work out the areas of each different type of square.



Subtraction Method for Area

Area of large square = 16 square units.

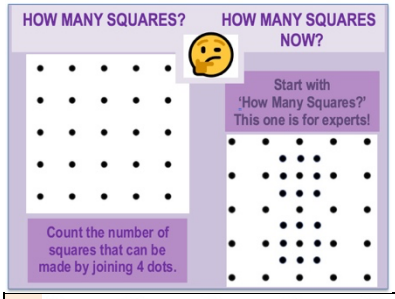
Areas of 4 outer triangles (each ½ area of rectangle) = 1½ square units.

Area of tilted square = 16 – (4 × 1½) = 10 square units.

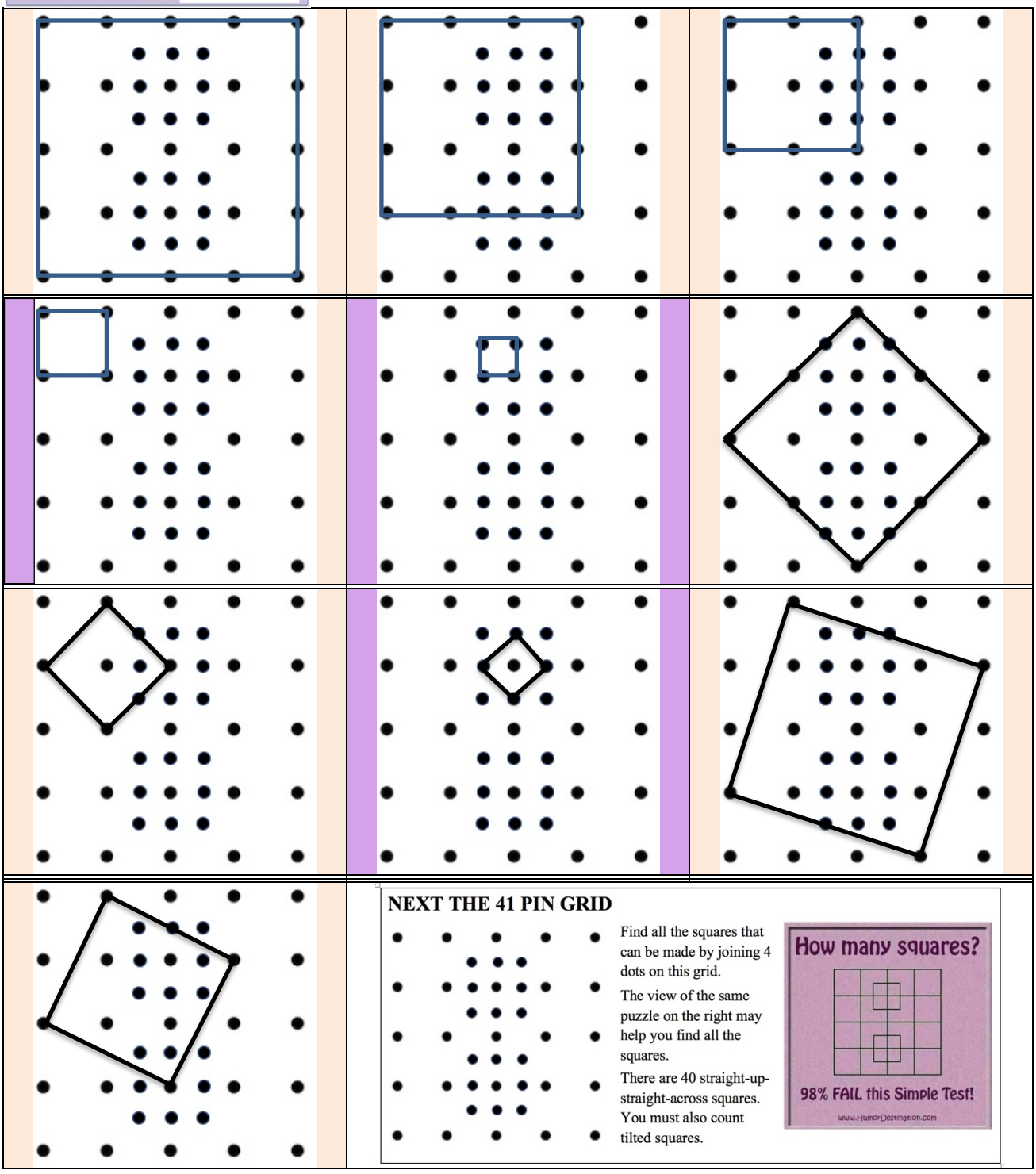
Key questions (remember to ask learners to explain their answers)

- How can you be sure that those 4 dots form a square?
- What is different about those two squares?
- What is the same about those two squares?
- Are there any more squares like that (of that size) joining different dots on the grid?
- How would you find the area of that square?

On the left you see the ‘How Many Squares’ puzzle for a 25-pin grid (5 by 5).



Start by solving the puzzle for a 9-pin grid (3 by 3).
Only move on to the16-pin and 25-pin puzzles when you understand why there are 6 solutions to the 9-pin puzzle.
When you have solved the 25-pin puzzle, then do the 41-pin puzzle?
This help-sheet gives you all the types of square.
The pink cells in this table apply to the 25-pin puzzle.
You need the lilac cells for the 41-pin puzzle.



NEXT THE 41 PIN GRID

Find all the squares that can be made by joining 4 dots on this grid.

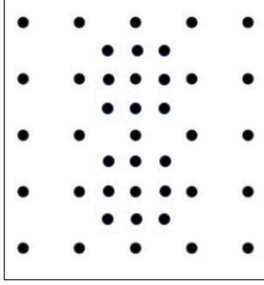
The view of the same puzzle on the right may help you find all the squares.

There are 40 straight-up-straight-across squares. You must also count tilted squares.

Follow up

The numbers of squares of each type and their areas are given below.

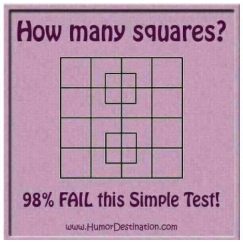
NEXT THE 41 PIN GRID



Find all the squares that can be made by joining 4 dots on this grid.

The view of the same puzzle on the right may help you find all the squares.

There are 41 straight-up-straight-across squares. You must also count tilted squares.



Make a wall poster. Praise each learner as they find a new solution to add to the poster that nobody has discovered before.