

ADDITION/ SUBTRACTION OF 2 DIGIT NUMBERS

1. Below are three addition (subtraction) problems which are classified into Level1, Level 2 and Level 3.

Level1: Find the sums and difference of the numbers indicated on the diagram (vertically/horizontally)

$$\begin{array}{r}
 \boxed{35} \\
 + \\
 \boxed{49} \\
 \hline
 \end{array}
 +
 \begin{array}{r}
 \boxed{49} \\
 + \\
 \boxed{57} \\
 \hline
 \end{array}
 =$$

Level 2: Fill in the empty boxes such that the sums (difference) of the horizontally and vertically arranged numbers satisfy the given outputs

$$\begin{array}{r}
 \boxed{} \\
 + \\
 \boxed{} \\
 \hline
 \end{array}
 +
 \begin{array}{r}
 \boxed{} \\
 + \\
 \boxed{} \\
 \hline
 \end{array}
 = 24$$

$$\begin{array}{r}
 \boxed{} \\
 - \\
 \boxed{} \\
 \hline
 \end{array}
 = 29$$

Level 3: Fill in the empty boxes such that the sums (difference) of the horizontally and vertically arranged numbers satisfy the given outputs.

$$\begin{array}{r}
 \boxed{} \\
 + \\
 \boxed{} \\
 \hline
 \end{array}
 +
 \begin{array}{r}
 \boxed{} \\
 + \\
 \boxed{} \\
 \hline
 \end{array}
 = 8$$

$$\begin{array}{r}
 \boxed{} \\
 - \\
 \boxed{} \\
 \hline
 \end{array}
 = 12$$

HELP

How much of number facts do you know? What two numbers add up to 28? Find out the possibilities?

$$\square + \square = 28?$$

Try this:

$$14 + \square = 28$$

$$\square + 15 = 28$$

$$12 + 16 = \square$$

$$\square + 17 = 28$$

$$10 + \square = 28 \dots$$

Notice that there so many possibilities depending on how developed our **number sense knowledge** is. In the task below, there are unique numbers which can be written in blank boxes and satisfy the conditions given.

Here, you are encouraged to try different options, making meaningful **deductions** from the findings, until you get the required numbers. A '**trial and improve strategy**' seem to be one way of finding the appropriate numbers. Can you think of other ways?

$$\begin{array}{rcl} \square & + & \boxed{19} = 28 \\ & + & \\ & & \square \\ & & = 31 \end{array}$$

What numbers go into the boxes above?

Hint: Try **counting on** from 19 in order to get 28, what number goes into the box:

$$\square + 19 = 28?$$

Can you employ your knowledge of subtraction here? What happens if you subtract 19 from 28? Can this lead us to the number we are looking for?

Look at the number line below. What two numbers can be added to 19 to **finally** land on 31?

$$19 + \square + \square = 31$$



What is the distance between 19 and 29 on this number line, and between 29 and 31? This is certainly a strategy one can use to add numbers. You will notice that in this situation, we had to **bridge to ten** from 19 and added the next unit left, is that not so?

Let us look at the following flashcards in relation to addition of 2 - digit numbers:

1.  +  =

2.  +  =

Without using any pen and paper method nor calculation device, mentally compute the sum of the 2- digit numbers.

In addition to the varied methods you might know, try adding digits from **Left to Right** as quickly as possible and see what you obtain. What answers do you get for questions 1 and 2? What needs to be done if the sum of the unit digits is less than ten; more or equal to ten? Can you come up with a rule which helps you add numbers this way?

3.  +  =

Think of **double numbers**; what two double numbers can quickly help you add the numbers on the flashcards? Find the sum of the numbers on the flashcards.

4.  -  =

Can you see 93 as a 'double $\square + 3$ '? What number must go into the box? Now workout $93 - 45$.

Task 3 and 4 are examples of tasks where one can make use of number facts like **near doubles** to add 2-digit numbers

5.

$$\text{25} = \text{20} + \text{5} \quad + \quad \text{43} = \text{40} + \text{3}$$

Make use of the flashcards above to work out $25 + 43$.

6.

$$\text{54} = \text{50} + \text{4} \quad - \quad \text{32} = \text{30} + \text{2}$$

Use the flashcards with **partitioned** numbers to work out: $54 - 32 =$

7.

$$\text{22} \quad + \quad \text{30} + \text{5} + \text{2}$$

With the help of the flashcards, what is the sum of $22 + 37$? Can you explain what is happening in this method?

8.

$$\text{48} + \text{39} \quad = \quad \text{50} - \text{2} + \text{40} - \text{1}$$

Are the two expressions (sums) on the flashcards of the same value?
Justify your answer.

Reinforcement Task (Group/Individual work):

1. In your groups (four/five learners), work out the following. Try using as many different methods as possible:

1.1 $24 + 79 =$

1.2 $37 + 45 =$

1.3 $46 + 39 =$

1.4 $53 + 19 =$

1.5 $18 + 65 + 69 =$

1.6 $37 + 76 + 88 =$

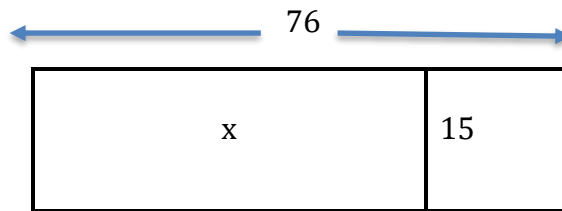
1.7 $86 - 39 - 12 =$

1.8 $57 - 18 - 5 =$

NEXT

Look at the following bars and try to replace letter x by a number which satisfies the given Information:

1.



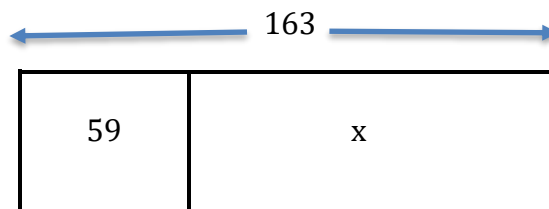
What **fact family** can you deduce from the information given on the bar? What is the value of x?

The first two fact family members are given to you. State the other two **fact family** members.

$$x + 15 = 76$$

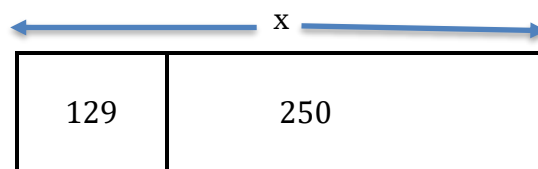
$$15 + x = 76$$

2.



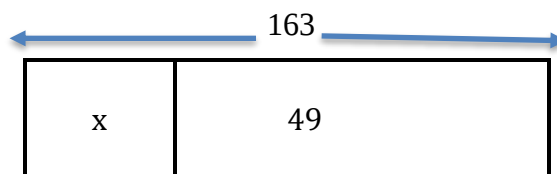
List down four **fact family** members for this bar. Use the fact family to find the value of x.

3.



List down four **fact family** members for this bar. Use the fact family to find the value of x.

4.



List down four **fact family** members for this bar. Use the fact family to find the value of x.

GUIDELINE FOR PARENTS FOR HOMELEARNING

SOLUTION

1. Level 1

$$\begin{array}{ccc} \boxed{35} & + & \boxed{49} = 84 \\ + & & + \end{array}$$

$$\begin{array}{ccc} \boxed{57} & - & \boxed{18} = 39 \\ = 84 & & = 67 \end{array}$$

Level 2

$$\begin{array}{ccc} \boxed{11} & + & \boxed{13} = 24 \\ + & & + \end{array}$$

$$\begin{array}{ccc} \boxed{18} & - & \boxed{11} = 7 \\ = 29 & & = 24 \end{array}$$

Level 3

$$\begin{array}{ccc} \boxed{2.5} & + & \boxed{5.5} = 8 \\ + & & + \end{array}$$

$$\begin{array}{ccc} \boxed{9.5} & - & \boxed{2.5} = 7 \\ = 12 & & = 8 \end{array}$$

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Multi-digit addition and subtraction

$$48 + \square = 71$$

A	B	C	D
29	23	17	27

The correct answer and possible misconceptions:

- A. Wrong answer. Probably guessing
- B. The correct answer: $48 + 23 = 71$
- C. Wrong answer. Probably guessing.
- D. Wrong answer. Probably guessing.

<https://diagnosticquestions.com>

Why do this activity?

This activity allows your child/children to develop skills of addition/subtraction. Counting in principle is a **human need**; we learn a lot of counting skills as we add/subtract numbers, therefore addition/subtraction feeds very well into this **human need** acquisition. In our daily lives, we need to count, quantify, measure objects, etc. This activity is geared towards developing strong flexibility with mental arithmetic. Children develop their own strategies by exploring and discussing in groups, justifying their thinking and the answers they arrive at. Some come up with powerful **models** which help them add/subtract numbers with ease. The activity also feeds in the development of subsequent concepts in Mathematics as a subject: Number theory, Number sense, Measure, Algebra, and virtually many other topics in the Mathematics. This knowledge also translates to other disciplines, be it in Science, Science and Technology, Commerce and indeed many other fields in human activities.

The mental computation handled in this activity encourages thoughtful inspection of a task prior to the application of a step-by-step procedure (which we call an algorithm).

Addition helps children in developing conceptual understanding of multiplication as an operation. They quickly perceive that $7 + 7 + 7 + \dots + 7$ (twelve times) is equal to $7 \times 12 = 84$, so this becomes a handy tool of doing addition of repeated numbers.

Our 21st Century children are surrounded by electronic gadgets such as calculators and indeed smartphones which are embedded with calculating software. If we do not develop these children's mental computation at an early stage, then they are doomed to carrying calculators in their daily walks in life.

Learning objectives

In doing this activity your children will have an opportunity to:

- Develop their own strategies for addition and subtraction of 2-digit numbers.
- Develop mental computation skills with regards to addition/subtraction of numbers.
- Develop an understanding of number sense.
- Develop flexibility with arithmetical operations.
- Develop problem solving skills

Generic competences

In doing this activity your children will have an opportunity to:

- **think mathematically**, reason logically and give explanations and justifications;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems** in a variety of situations;
- **work and learn independently** and prepare for lifelong learning;
- **work in a team**:
 - collaborate and work with a partner or group
 - have empathy with others, listen to different points of view
 - develop leadership qualities;
- **communicate** in writing, speaking and listening according to the audience:
 - exchange ideas, criticise, and present information and ideas to others
 - analyze, reason and record ideas effectively;
- **develop life skills and consideration for others** - to show social responsibility – to work for the good of the community.

Suggestions for Homelearning

Start the lesson with the diagnostic assessment. This should take about 5 to 8 minutes.

Steps for diagnostic assessment:

1. Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Multi-digit addition and subtraction

$$48 + \square = 71$$

A	B	C	D
29	23	17	27

The correct answer and possible misconceptions:

- A. Wrong answer. Probably guessing.
- B. The correct answer: 23, since $48 + 23 = 71$.
- C. Wrong answer. Probably guessing.
- D. Wrong answer. Probably guessing.

After discussing the solution to the diagnostic question, find out how much of number facts does your child know, through presenting this question:

$$\square + \square = 28? \quad \text{What numbers can go into the boxes to make the calculation correct?}$$

Let the children explore possibilities. There are many. To assist them, take them through the following task.

$$14 + \square = 28$$

$$\square + 15 = 28$$

$$12 + 16 = \square$$

$$\square + 17 = 28$$

$$10 + \square = 28$$

Look at the number pattern generated in this task.

$$14 + 14 = 28$$

$$13 + 15 = 28$$

$$12 + 16 = 28$$

$$11 + 17 = 28$$

$$10 + 18 = 28$$

Mathematics is full of patterns. There are so many possibilities which satisfy the condition.

Encourage children to explore more possibilities.

Take the children through the **flashcards** for question 1 – 8. Allow them to engage any strategy to answer the questions. It is interesting to take note of the following strategies, amongst other possible strategies children can come up with:

1. Adding by '**bridging to ten**': $84 + 8 = 84 + 6 + 2 = 90 + 2 = 92$

2. '**Bridging to ten**' on an empty number line: $19 + 12 = 19 + 10 + 2 = 29 + 2 = 31$



3. **Sequence/Stringing** strategy: $27 + 35 = 27 + 30 + 5 = 57 + 5 = 62$

$$34 + 29 = 34 + 20 + 5 + 4 = 54 + 5 + 4 = 59 + 4 = 63$$

4. **Split ten** strategy: $37 + 45 = 30 + 40 + 7 + 5 = 70 + 12 = 70 + 10 + 2 = 80 + 2 = 82$

5. **Compensation**: Compensation is a strategy that involves adding or subtracting a number larger than the number specified in the calculation- usually the next high multiple of ten, and then modifying the answer by 'compensating' for the extra bit added or subtracted.

$$\begin{aligned} \text{(i)} \quad 46 + 39 &= 46 + 40 - 1 \\ &= 86 - 1 \\ &= 85 \end{aligned}$$

$$\text{(ii)} \quad 46 + 39 = 46 + 40 - 1 + 86 - 1 = 85$$

$$\text{(iii)} \quad 19 - 14 = 5 \xrightarrow{\text{yields}} 18 - 14 = ? \text{ It becomes immediately clear to some learners that the answer is 1 less than 5, i.e. 4.}$$

6. **Partitioning**: $67 + 56 = 60 + 50 + 7 + 6 = 110 + 13 = 120 + 3 = 123$

7. **Near doubles:** $17 + 19 = 17 + 17 + 2 = 34 + 2 = 36$
 $93 - 45 = 2(45) + 3 - 45 = 45 + 3 - 45 = 45 + 3 = 48$
 Seeing **93** as a 'double **45** + **3**' is a skill a learner needs to develop.
8. **Bridging to ten & compensation:** $28 + 35 = 30 + 35 - 2 = 65 - 2 = 63$

It will be interesting to see children coming up with some strategies different from these ones mentioned above.

As a consolidation exercise wrap up this activity with your child by engaging him/her in a question and answer session where they do calculations, explaining how they found the answers, justifying their solutions.

Key questions

1. Can you explain how to add/subtract the following numbers:

$$35 + 49 =$$

$$57 - 18 = ?$$

2. What do you notice about the numbers which are supposed to be in boxes connected by the diagonal indicated by the arrow on the diagram below? Justify your answer.

$$\begin{array}{rcl} \boxed{} & + & \boxed{} = 24 \\ & + & \\ \boxed{} & - & \boxed{} = 7 \\ 29 & & 24 \end{array}$$

3. Can you give an example of a strategy you employed in solving these addition/subtraction problems?

Follow up

<https://aiminghigh.aimssec.ac.za/years-3-7-handspan-pace/>
<https://aiminghigh.aimssec.ac.za/years-4-7-checkit/>
<https://aiminghigh.aimssec.ac.za/years-7-10-target-game/>
<https://aiminghigh.aimssec.ac.za/years-3-10-i-am-eight/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6

