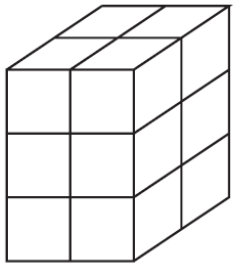
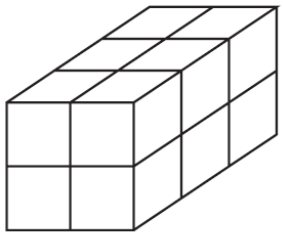
How many can you make?

You have 12 cubes, each with sides 1cm long.
The two rectangular prisms shown below have been made using these 12 cubes.





Are these the same or different?
They are the same.
They can be described as $3 \times 2 \times 2$ rectangular prisms.
There are three other rectangular prisms which can be made from 12 cubes.

1. Describe them.
2. How many different rectangular prisms can you make using:
   a) 16 cubes
   b) 10 cubes
   c) 7 cubes
   d) 5 cubes
   e) 9 cubes?
   Describe each of them in the same way as the example above.

HELP

You will need 16 cubes.
Look at the games you have in the house. Do any of them involve cubes?
For example dice could be placed together to make the different rectangular prisms.
Try and visualise the different possible rectangular prisms without the cubes.
Then use the cubes to check you have found all the possibilities.

NEXT

3. Without drawing them or using cubes, decide how many different rectangular prisms you could make using the following number of cubes: Describe each of them.
   a) 17
   b) 8
   c) 20
   d) 49
   e) 15

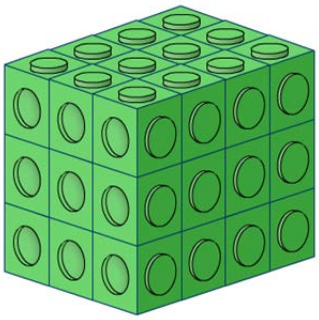
GUIDE FOR PARENTS FOR HOMELEARNING

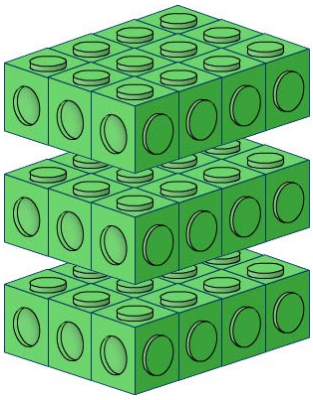
SOLUTION

1. $12 \times 1 \times 1$, $6 \times 2 \times 1$, $4 \times 3 \times 1$
2. a) 4 different rectangular prisms
   $16 \times 1 \times 1$, $8 \times 2 \times 1$, $4 \times 4 \times 1$, $4 \times 2 \times 2$
   b) 2 different rectangular prisms
   $10 \times 1 \times 1$, $5 \times 2 \times 1$
   c) 1 rectangular prism
   $7 \times 1 \times 1$
   d) 1 rectangular prism
   $5 \times 1 \times 1$
   e) 2 rectangular prisms
   $9 \times 1 \times 1$, $3 \times 3 \times 1$
3. a) 1 rectangular prism
   $17 \times 1 \times 1$
   b) 3 rectangular prisms
   $8 \times 1 \times 1$, $4 \times 2 \times 1$, $2 \times 2 \times 2$
   c) 4 rectangular prisms
   $20 \times 1 \times 1$, $10 \times 2 \times 1$, $5 \times 4 \times 1$
   d) 2 rectangular prisms
   $49 \times 1 \times 1$, $7 \times 7 \times 1$
   e) 2 rectangular prisms
   $15 \times 1 \times 1$, $5 \times 3 \times 1$

Diagnostic Assessment This should take about 5–10 minutes.

This should take about 5–10 minutes.
The learner's should have had a lesson on volume of rectangular prisms before trying this activity.
The learners should understand the following;





This rectangular prism is made from 36 small cubes. Each small cube has a side length of 1 cm.
The volume of each small cube is $1cm^3$
You can see that the rectangular prism is made from three layers.
Each layer is made from 12 small cubes. So the volume of each layer is $12cm^3$.
There are 3 layers so the volume of the rectangular prism is $3 \times 12 = 36cm^3$
This rectangular prism can be described as a $3 \times 4 \times 3$ prism. Can you see why?

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

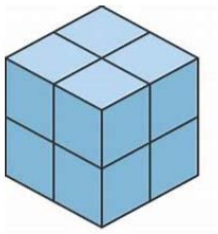
The correct answer and possible misconceptions:

A.
B.
C.
D.

https://diagnosticquestions.com

What is the volume of this rectangular prism?

a) 12 cubic units
b) 7 cubic units
c) 8 cubic units
d) 8 cm²

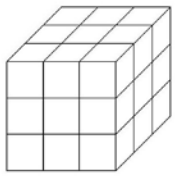


The correct answer is c) 8 cubic units.
Possible misconceptions;

a) 12 cubic units. Possibly trying to use a formula for volume incorrectly.
b) 7 cubic units. The learner is not visualising two layers of 4 small cubes.
c) 8 cm². The learner is confused about the units for area and volume.

How many small cubes have been used to create this larger cube?

a) 36
b) 27
c) 19
d) 9



The correct answer is b) 27
Possible misconceptions;

a) 36. The learner is not visualising 3 layers of cubes. With each layer made up of 9 cubes.
c) 19. The learner has guessed an answer.
d) 9. The learner has only counted one layer of cubes.

Why do this activity?

The learner has to make all the possible rectangular prisms from the number of cubes given.
They have to describe these rectangular prisms using three numbers which relate to the length, height and width of the completed rectangular prism.
Rectangular prisms are another name for cubes and cuboids. In doing this activity learners will have an opportunity to visualise the rectangular prisms and work systematically to find all the possibilities. It will also help them to understand the volume of rectangular prisms. They already know that volume is measured in millilitres. (1ml= 1 cm³)

If each small cube has a height, width and length of 1 cm then its volume is 1cm³.

So for example if their rectangular prism is made from 10 small cubes like these then its volume will be 10 cm^3.
This is the same as 10 ml since $1cm^3 = 1ml$.

Learning objectives

In doing this activity students will have an opportunity to:
Develop the skills of visualisation and systematic working.
Apply knowledge of factors and to work systematically to check that they have all possible triples of factors.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems** in a variety of situations;
- **work and learn independently** and prepare for lifelong learning;

Suggestions for homelearning

Start by making the rectangular prism from 12 dice.
Position the prism so that it looks like the first picture. Reposition it so that it looks like the second picture.
Ask your child if they are the same. Once this has been established, ask your child why it can be described as a $\mathbf{3 \times 2 \times 2}$ prism.
One answer could be the width is 3 units, the length is 2 units and the height is 2 units.
The order of the three numbers used to describe the prism does not matter.
Another answer may be each layer has a width of 3 units and a length of 2 units. There are two identical layers.
Hence it can be described as; $\mathbf{3 \times 2 \times 2}$
Again it does not matter the order of the numbers used.
Then dismantle the prism so that you have 12 single dice.
Ask your child to make a different rectangular prism using the dice.
Once this has been done ask your child to describe it using three numbers like the example above.
Your child should be able to make three different rectangular prisms and name them.
The answers are available for you to check.
Then move onto question 2. Your child will need a maximum of 16 dice.
When you are trying to find the number of different prisms for each number of cubes think about the different sets of three numbers that would multiply together to give that number.
For example for the rectangular prism made from 12 cubes the answers were

$$3 \times 2 \times 2\,, \quad 12 \times 1 \times 1, \quad 6 \times 2 \times 1, \quad 4 \times 3 \times 1$$

If any of the numbers in the triple can be rewritten in a different way then there will be another possible prism that can be made**.**
For example $12 \times 1 \times 1$, ***can be rewritten as*** $6 \times 2 \times 1$ **and** $4 \times 3 \times 1$
$4 \times 3 \times 1$ **can be rewritten as** $3 \times 2 \times 2$
The final answer is the same**.**
It is only when the triple is made from prime numbers that no more can be found.

$3 \times 2 \times 2$ is made from 3 prime numbers as 3 can only be multiplied by 3×1 and 2 can only be multiplied by 2×1 to give the answer.
Once you have completed question 2, ask your child to try question 3 without using the dice.
This is a skill that is important in mathematics called visualisation.
This involves being able to visualise all the possible prisms without making them.
Help your child to think logically by thinking of all the three possible numbers that will multiply to give the number of cubes.

Key questions

1. What three numbers can be multiplied together to describe the prism we have made?
2. Are there other sets of three numbers that can be multiplied together that would also give you this total number of cubes?
3. How many different prisms will you be able to make for this number of cubes?
4. How do you know there are no more possibilities?

Follow up

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6