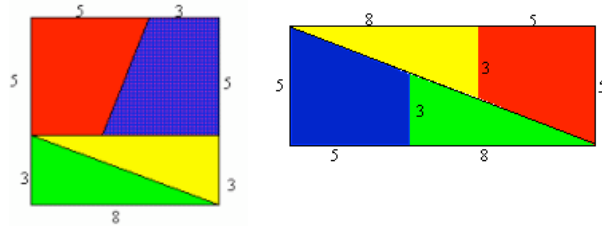


## DOESN'T ADD UP



Look at this square divided into four pieces:  
two identical triangles and two identical trapezia.

The square has edges of length 8 units and area 64 square units.

The edge of length 3 from each triangle matches with the edge of length 3 from a trapezium so that the four pieces from the square now occupy a rectangular space.

The area of the rectangle is 65 square units (13 by 5)

Can the area have changed? Can you explain what has happened?

## HELP

Draw the 4 pieces accurately, measuring the right angles carefully or use squared paper.

Now make both arrangements with your 4 pieces.

Look carefully. Can you explain what happens? Are you sure about it?

## NEXT

Can you make up your own similar problem?

Another set of dimensions are 4, 7, 11 in the corresponding places of 3, 5, 8.

The areas are 121 square units compared to 126 square units.

Draw these pieces and the two arrangements.

# HOME LEARNING AND INCLUSION GUIDE

## THEME: Area

*Young people need to realize that being able to check everything to find a fault and persevering when you don't quickly find what is wrong, is a very important life skill. This activity provides experience that helps young people to develop this skill.*

### Suggestions for Home Learning

#### Young children

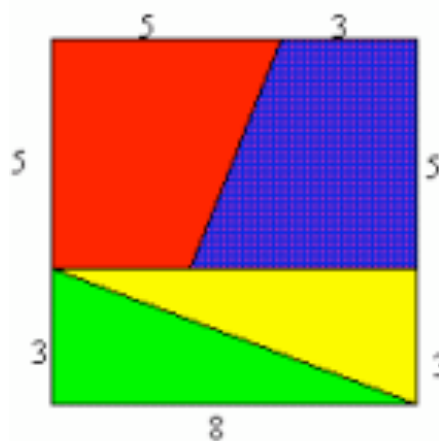
Treat this like a jigsaw with 4 pieces. Make them from scrap card or plastic.

Can the child make a rectangle with 2 pieces?

Can they make a square with 4 pieces?

What does the child notice about the shapes?

Do they know the names of any of the shapes?

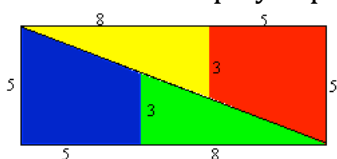


#### Upper Primary

As a preliminary, treat this like a jigsaw with 4 pieces. Make the pieces from scrap card or plastic. Can the learners make a rectangle with 2 pieces? Can they make a square with 4 pieces? What do they notice about the shapes? Do they know the names of any of the shapes?

At this age children will be learning about quadrilaterals and trapezia, they will also be able to recognise the right angles, and this activity may help them to gain a better understanding of area and how to find areas of triangles and quadrilaterals.

After some free play experience, they can explore the area paradox. Everyone assumes that this is a rectangle with area  $5 \times 13 = 65$  square units.



What is wrong? Why is there a difference of 1 square unit of area? Is it mathemagic? No **there must be an**

**explanation. What do people assume when they look at this diagram?** Could it be that their eyes are deceiving them and they are assuming something is true when it is not true?

Encourage the children to be **detectives** and to discover what is going wrong. **The answer is not important**, it is the process of investigation that gives the rich learning experience. Let the children talk about it and try to discover the explanation. If there are no other children at home to talk to about this, then suggest that the child issues a challenge on social media to his or her friends. Who is an expert detective?

**DO NOT tell explain it to them.** Just say that they are developing important skills, and if they persevere, then they will make the all-important discovery.

If this goes on for too long, and the children are losing interest, then you can give clues as below one at a time.

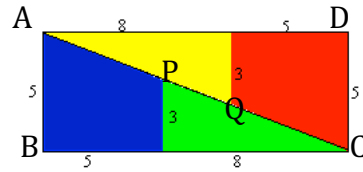
Clue 1: You need to look closely at the line AC.

Clue 2: You need to look closely at points P and Q where three of the shapes meet.

Clue 3: Are you sure that ACB is a triangle? Could it be any other shape?

Clue 4: Is APQC a straight line?

Give these clues one by one and encourage the learners to use the clues. This leads to a discussion of the fact that the climb PA is steeper than the climb CP, meaning that ACB is not a triangle. APCB and AQCD are quadrilaterals. There is a VERY thin parallelogram shaped gap along AC which takes up 1 square unit of area.



## Lower Secondary

In lower secondary school this activity can be used to motivate learning about gradients. It is an ideal activity to use before you introduce the definition of gradient because it helps learners to understand why there is a need for this definition.

**Key Questions** *These are questions you ask to help learners to think for themselves. STOP yourself telling children what to do next. Instead ask a KEY QUESTION.*

- What do we know about areas of squares?
- What do we know about areas of rectangles?
- What do we know about areas of triangles?
- What do we know about areas of trapezia?
- Do you believe that a shape can change its area when it is moved around?
- Have you tried drawing the diagrams on squared paper?
- What makes a line straight?

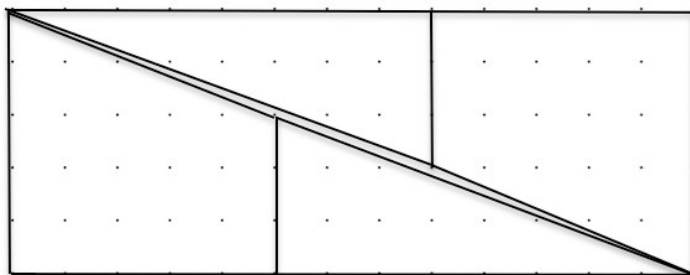
If learners have already met the definition of gradient than you can ask the Key Questions:

- What do you know about gradients?
- Can you use gradients knowledge in this problem?

## Years 9 to 12

At this stage learners should already have met gradients. If they have not seen it before then Doesn't Add Up is a good activity, both to help learners to develop life skills, and also to help them to review concepts of area and gradient. They should be able to solve the problem and give a good explanation of this paradox.

## SOLUTION



The discrepancy is due to the slightly different slopes (gradients) of the edges making quadrilaterals look like triangles.

The areas do not change but the pieces do not fit closely together

to make the rectangle as it might appear.

The sloping edges of the trapezium and the triangle do not form a straight line across the diagonal of the rectangle.

The gradient of the sloping edges of each triangle is  $\frac{3}{8} = 0,375$

The gradient of the sloping edge of each trapezium is  $\frac{2}{5} = 0,4$  (slightly 'steeper').

There is a very thin parallelogram shaped space across the diagonal of the rectangle making a gap of area 1 square unit inside the rectangle.

## Why do this activity?

This problem gives learners of all ages the opportunity to practice mathematical thinking and visualization and it gives them an experience that helps to develop the important skill of checking every detail of a system to find out what has gone wrong.

The geometrical ideas of area, conservation of area and slopes (gradients) of lines are fundamental.

Learners should at first be surprised by the almost magical change in area. They should know that the pieces don't change in area simply by being moved around (**conservation of area**) and they should be motivated to find an explanation.

This problem provides an experience to show us that we must be careful what we assume and there is a need to check things out. This problem is also motivating because it is a trick that seems to be magic and it could be used to impress parents and friends.

## Learning objectives

In doing this activity students will have an opportunity to:

- review areas of simple 2D shapes;
- review gradients of line segments.

## Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop perseverance**;
- **develop the skill of checking details** to find where an error lies;
- **develop observational and visualization skills**.

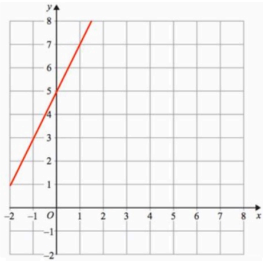
### Diagnostic Assessment

This can be used before or after the lesson.

Show this question and say:

**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**

What is the gradient of the line?



A      B      C      D

-2      2      1      3

1. Notice how the learners respond. Ask them to explain why they gave their answer and **DO NOT** say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer so as to clarify their own thinking by putting it into words, and to develop their communication skills.

3. If you have a group, try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the learners again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers

**The correct answer is B. The gradient is 2.**

A. Student does not understand and is using the intercept on the x-axis.

C. No understanding of gradient. With slope or gradient 1 the line would be at 45 degrees.

D. Probably a guess

<https://diagnosticquestions.com>

## Follow up

Muggles Magic: <https://aiminghigh.aimssec.ac.za/years-7-10-muggles-magic/>



Subscribe to MATHS TOYS if you have not already done so.

<https://www.youtube.com/c/mathstoys> Or go to YouTube, search for Maths Toys.

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links: <https://aiminghigh.aimssec.ac.za>

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