WHICH ONE HAS THE SMALLEST SURFACE AREA?

This picture shows a rectangular prism made from 36 cubes.
How many different rectangular prisms can you make from 36 cubes?
Do they all have the same volume? Why or why not?

Do they all have the same surface area? Why or why not?
Take the area of each face of the cube as one square unit.
Find the surface area of all the rectangular prisms that can be made using 36 small cubes.
You will have to work systematically and take note of your results.
Which one has the smallest surface area?

HELP

Remember each rectangular prism must have a volume of 36 small cubes.
This rectangular prism has a volume of 36 small cubes

What would be the surface area of this rectangular prism?
How many different rectangular prisms can you make with 36 cubes?
Use your multi-links to help you.
For each different rectangular prism that you find work out its surface area.
Record your results in a table.
Ask a friend to check that you have found all the possible rectangular prisms that you can make with 36 cubes.
Which one of these has the smallest surface area?

NEXT

Can you find a rectangular prism (with edges of integer values) that has a surface area of exactly 100 square units?
Is there more than one?
See Rectangular prisms https://aiminghigh.aimssec.ac.za/grade-9-or-10-cuboids/

NOTES FOR TEACHERS

SOLUTION

Rectangular Prism	Volume in cubic units	Surface area in square units
36 × 1 × 1	36	146
18 × 2 × 1	36	112
12 × 3 × 1	36	102
9 × 4 × 1	36	98
9 × 2 × 2	36	80
6 × 6 × 1	36	96
6 × 3 × 2	36	72
4 × 3 × 3	36	66

The rectangular prism with the smallest surface area is 4 x 3 x 3

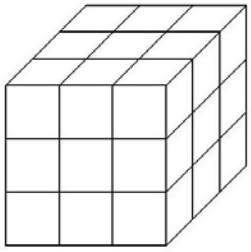
Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The picture is a cube made from smaller cubes. Each small cube has a volume of 1cm³.

i) What is the volume of the large cube?

a) 21cm³
b) 27 cm³
c) 1cm³
d)



ii) What is the surface area of the large cube?

a) 54 cm²
b) 27cm²
c) 6cm²

i) The correct answer:

The correct answer is b) 27 cm³
The large cube has three layers.
Each layer is made from 9 small cubes.
Therefore three layers of 9 small cubes equals 27 small cubes. Each small cube has a volume of 1 cm³.

Possible misconceptions

a) 21 cm³

The learner has only counted the small cubes which are visible. Use multi-link cubes to construct cubes and rectangular prisms to help learners understand that not all of the small cubes will be visible**.**

c) 1 cm³

The learner does not understand the question.

ii) The correct answer:

The correct answer is a) 54cm²
The cube has six faces. Each face is made up of 9 small squares.
Each small square has an area of 1 cm².
Therefore each face has an area of 9 cm².
The cube has six faces.
Therefore the surface area of the cube is 54 cm².

Possible misconceptions

b) 27 cm²

The learner has calculated the surface area of the parts of the cube which are visible.

c) 6 cm²

The learner thinks each side of the large cube has an area of 1 cm².

Why do this activity?

This is a good activity to help learners to develop an understanding of the properties of a rectangular prism and of surface area and volume. It also gives learners practice in working out all the possible factors of a given number.

Learning objectives

In doing this activity students will have an opportunity to:

- To develop the skills of visualization and systematic working.
- To apply knowledge of the factors of 36 and to work systematically to check that they have all of the possible triple factors.
- To develop an understanding of surface area and volume.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems**;
- **work and learn independently** and prepare for lifelong learning;
- **work in a team**:
    - collaborate and work with a partner or group
    - have empathy with others, listen to different points of view
    - develop leadership qualities;
- **communicate** in writing, speaking and listening according to the audience:
    - exchange ideas, criticise, and present information and ideas to others
    - analyze, reason and record ideas effectively;
- **develop life skills and consideration for others** - to show social responsibility – to work for the good of the community.

Suggestions for teaching

You could start by showing the class one of the models and asking them to say how many small cubes have been used to make it. You could give them a few minutes to discuss this question in pairs.

When the class has decided that there are 36 cubes say that each cube has a volume of 1 cubic unit and the area of each face is 1 square unit. Ask the class to find the volume and surface area. Have a class discussion about finding the surface area BY COUNTING SQUARES until all the learners understand how to do this.

Do not tell them to use a formula length × breadth × height – in fact it is not necessary to use this formula.

The class could make a poster to show all the solutions. If they do not find them in the first lesson you could suggest that they keep searching.

Making a list or table is an important and useful way of checking that all possibilities have been included. To round of this activity you could show the group how to do this methodically so that none are included twice, such as 9 x 4 x 1 and 4 x 9 x 1.

One way to do this is to list in order of the size of the numbers so that the above example will always be recorded as 9 x 4 x 1.

Key questions

- Can you split the 36 cubes into shorter equal lengths?
- How many cubes are there in the top layer?
- How many layers?
- How many small cubes all together?
- How many small squares on that face?
- Can you find the areas of all 6 faces?

Follow up

https://aiminghigh.aimssec.ac.za/grades-7-to-10-painted-cube/
https://aiminghigh.aimssec.ac.za/years-8-10-cuboids/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6