MORE MAGIC NUMBERS

This challenge is an extension to MAGIC NUMBERS
https://aiminghigh.aimssec.ac.za/grades-4-to-7-magic-numbers/

LIST 1	LIST 2
$\square \times 8 + 1 = 9$	$(9 - 1) \div 8 =$
$\square \times 8 + 2 = 98$	$(98 - 2) \div 8 =$
$\square \times 8 + 3 = 987$	$(987 - 3) \div 8 =$
$\square \times 8 + 4 = 9876$	$(9876 - 4) \div 8 =$
$\square \times 8 + 5 = 98765$	$(98765 - 5) \div 8 =$
$\square \times 8 + 6 = 987654$	$(987654 - 6) \div 8 =$
$\square \times 8 + 7 = 9876543$	$(9876543 - 7) \div 8 =$
$\square \times 8 + 8 = 98\ 765432$	$(98765432 - 8) \div 8 =$
$\square \times 8 + 9 = 987654321$	$(987654321 - 9) \div 8 =$

Complete the calculations in LIST 2.

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

What do you notice about the connections between the equations:

$$\square \times 8 + 1 = 9 \text{ etc.}$$

and the calculations:

$$(9 - 1) \div 8 = ? \text{ etc.}$$

What do inverse operations have to do with the connections between LIST 1 and LIST 2?

You have been doing algebra!

The equations $\square \times 8 + 1 = 9$ etc. can be written in the form

$$8x + 1 = 9 \text{ etc.}$$

where the letter x represents an unknown number and $8x$ means x multiplied by 8.

When you do algebra you are asked to solve equations. That means to find the number that the letter represents.

How would you use inverse operations to solve equations like $8x + 1 = 9$ etc. ?

HELP

You should do the subtractions first and then divide those answers by 8 each time. You will then have solved 9 equations of the form: $8x + ? = ?$ by finding the value of x each time.

If you are not confident that you know the multiples of 8 it may help you to list them before you start dividing by 8.

NEXT

What happens when you solve $\square \times 9 + 5 = 8888$

and $\square \times 9 + 4 = 88888$

and can you find a pattern and more terms in the pattern?

NOTES FOR TEACHERS

SOLUTION

MORE MAGIC NUMBERS is an extension activity to MAGIC NUMBERS.

LIST [1]

$\square \times 8 + 1 = 9$
$\square \times 8 + 2 = 98$
$\square \times 8 + 3 = 987$
$\square \times 8 + 4 = 9876$
$\square \times 8 + 5 = 98765$
$\square \times 8 + 6 = 987654$
$\square \times 8 + 7 = 9876543$
$\square \times 8 + 8 = 98\,765432$
$\square \times 8 + 9 = 987654321$

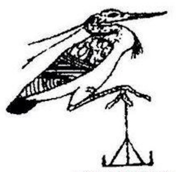
Re-arranging the calculations in LIST 1 to the form
(9 – 1) ÷ 8 = ? and
(98 – 2) ÷ 8 = ? etc in LIST 2
and then re-writing these to the form
$8x + 1 = 9$ and
$8x + 2 = 98$ etc.
should help learners to understand that this activity is all about solving a set of nine *algebraic equations*.

Thinking about the connections between the two lists [1] and [2] and talking about them, with other learners and in class discussions, should help learners to understand how, going from [2] to [1], and also solving linear equations, both involve using inverse operations as follows:

(a) 'undoing' the operation of addition by the inverse operation subtraction
(b) 'undoing' the operation of multiplication by the inverse operation division.

LIST [2]

$(9 - 1) \div 8 = 1$
$(98 - 2) \div 8 = 12$
$(987 - 3) \div 8 = 123$
$(9876 - 4) \div 8 = 1234$
$(98765 - 5) \div 8 = 12345$
$(987654 - 6) \div 8 = 123456$
$(9876543 - 7) \div 8 = 1234567$
$(98765432 - 8) \div 8 = 12345678$
$(987654321 - 9) \div 8 = 123456789$



THE MAGIC NUMBERS

1 x 8 + 1 = 9
12 x 8 + 2 = 98
123 x 8 + 3 = 987
1234 x 8 + 4 = 9876
12345 x 8 + 5 = 98765
123456 x 8 + 6 = 987654
1234567 x 8 + 7 = 9876543
12345678 x 8 + 8 = 98765432
123456789 x 8 + 9 = 987654321

This illustration gives the solutions to the equations.

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:

Which calculation is **not** equal to 39 × 42?

A (39 × 40) + (39 × 2)

B (30 × 40) + (9 × 2)

C 39 × 2 × 3 × 7

D 42 × 39

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is C. 39 × 42 = (30 × 40) + (30 × 2) + (9 × 40) + (9 × 2).
Learners giving this answer have omitted two parts of this calculation.
https://diagnosticquestions.com

Why do this activity?
The activity engages learners in step by step calculations that form part of a pattern but only involve numbers. The next step into algebra is very natural and easy because the learners have already been solving equations without using letters. Engaging with a pattern helps learners to keep a check on their answers and also to appreciate the beauty of number.

Learning objectives
In doing this activity students will have an opportunity to:
- practise subtraction and division calculations;
- deepen understanding that addition and subtraction are inverse operations and multiplication and division are inverse operations;
- investigate and discover for themselves the methods for solving linear equations;
- gain an understanding of the process of solving linear equations.

Generic competences (some suggestions, select from list or write your own)
In doing this activity students will have an opportunity to:
- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills.
- **visualize** and develop the skill of interpreting visual images to represent concepts and situations;
- **persevere and work systematically** to investigate different cases.

Suggestions for teaching
Start with the Diagnostic Quiz as a warm up. Tell the learners that the More Magic Numbers question involves rearranging calculations to give equivalent calculations and in mathematics there are often many different ways to write down the same statement. Very often solving a problem depends on finding a simple way to write down a statement.

Note that by using this number pattern learners are introduced to methods of solving linear equations that depend on understanding the nature of addition, multiplication and their inverses. This introduction to algebra focusses on understanding and not on rules that learners just follow blindly.

If the class has recently done the Magic Numbers activity then they are ready to follow the steps as given on page 1 without any introduction from the teacher. Simply copy and give out the worksheet or copy it on the board. Otherwise either allow the use of calculators or, if you want the whole focus of the lesson to be on methods of solving linear equations, first revise the method of division by chunking. Do not use the division algorithm.

Use the example of sharing a fortune between 9 lucky prize winners by sharing first the millions, then the hundred thousands, then the ten thousands, then the thousands, then the hundreds, then the tens, then the units. So, to do the sharing 888 885 ÷ 9 first split 888 885 into chunks that are all multiples of 9:
(810 000 + 78 885) then (810 000 + 72 000 + 6 885) then (810 000 + 72 000 + 6 300 + 585) then (810 000 + 72 000 + 6 300 + 540 + 45) which is
= (9 × 90 000) + (9 × 8 000) + (9 × 700) + (9 × 60) + (9 × 5). **So 888 885 ÷ 9 = 98 765**

Then ask the question: If you want to find the number to put in the box in $\square \times 8 + 2 = 98$ why do you subtract 2 from 98? Here the answer should be because +2 and -2 are inverse operations one undoes the other, like doing up shoelaces and undoing them. At the top of LIST 1 this will seem the obvious thing to do and it is natural to follow the same steps as you work down the list.

Once the learners have found the numbers to put in the boxes in LIST 1 ask learners to come to the board to explain exactly what they did for $\square \times 8 + 7 = 9876543$
Do **not** accept the answer just because it fits the pattern. Insist that the learner goes through the **whole process** of subtracting 7 then dividing by 8. If the first learner you ask to do this cannot go beyond the pattern ask for another learner to help out. When everyone understands that
STEP 1 is subtraction
STEP 2 is division
then go on to discuss how the number relationship can be written as
AN EQUATION $8x + 7 = 9876543$ and that finding the value of x is the same as filling the box in LIST 1.
Then discuss inverse operations + and – and also × and ÷.
Then congratulate the class that now they have found out how to solve equations in algebra.

Key questions

- When you divide by 8 can you see how it might help you to write down the multiples of 8: 8, 16. 24, 32 … etc.?
- Can you chunk that big number into numbers that are divisible exactly by 8?
- Can you imagine sharing first the millions, then the hundred thousands, then the ten thousands, then the thousands, then the hundreds, then the tens, then the units?
- If you want to find the number to put in the box in $\square \times 8 + 3 = 987$ why do you subtract 3 from 8987? (N.B. Here the answer should be because +3 and -3 are inverse operations one undoes the other, like doing up shoelaces and undoing them. At the top of LIST 1 this seems the obvious thing to do and it is natural to follow the same steps as you work down the list.)
- If you want to find the number to put in the box in $\square \times 8 = 987648$ why do you divide by 8? (N.B. Here the answer should be because ×8 and ÷8 are inverse operations again because one operation undoes the other.)

Follow up

Beautiful Numbers https://aiminghigh.aimssec.ac.za/years-6-8-beautiful-numbers/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6