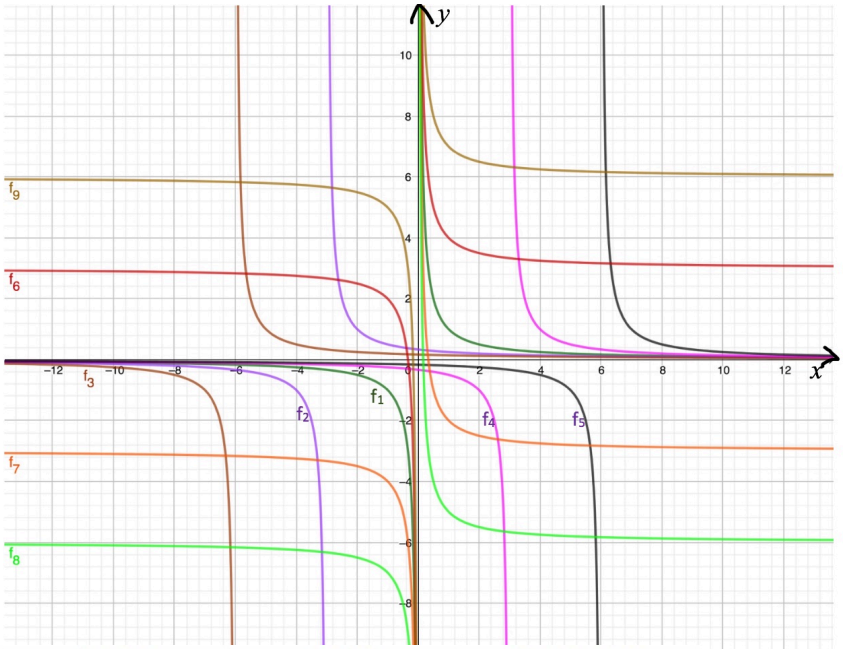
MATCHING HYPERBOLAS



Which one of the graphs represents the function $f(x) = \frac{1}{x}$?

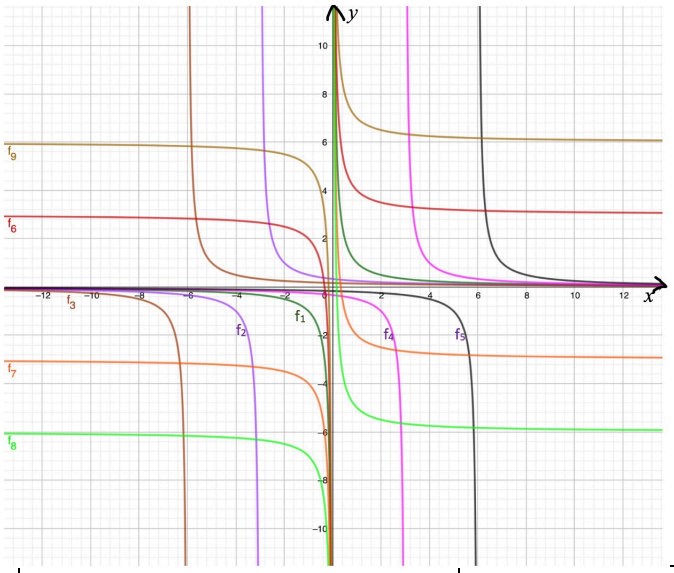
The table below has been jumbled up. Match the graphs labelled f_1 to f_9 to their equations in the table below and to the descriptions of the transformations of $f(x) = \frac{1}{x}$.

Cut up a copy of the table on page 4 and rearrange the pieces into 9 matching sets.

Equations	Graphs	Transformations
1. $y = \frac{1}{x} + 3$	f_1	A. $f(x)$ translated 6 units to left
2. $y = \frac{1}{x-3}$	f_2	B. $f(x)$
3. $y = \frac{1}{x} - 6$	f_3	C. $f(x)$ translated 6 units down
4. $y = \frac{1}{x+3}$	f_4	D. $f(x)$ translated 3 units up
5. $y = \frac{1}{x} - 3$	f_5	E. $f(x)$ translated 3 units to left
6. $y = \frac{1}{x}$	f_6	F. $f(x)$ translated 6 units to right
7. $y = \frac{1}{x+6}$	f_7	G. $f(x)$ translated 6 units up
8. $y = \frac{1}{x} + 6$	f_8	H. $f(x)$ translated 3 units down
9. $y = \frac{1}{x-6}$	f_9	I. $f(x)$ translated 3 units to right

2. Given: $g(x) = \frac{3}{x+1} - 2$

2.1 Write down the coordinates of the y-intercept (or intercepts) of $g(x)$.
2.2 Write down the coordinates of the x-intercept (or intercepts) of $g(x)$.
2.3 Write down the equation of the vertical asymptote of $g(x)$.
2.4 Write down the equation of the horizontal asymptote of $g(x)$.
2.5 Sketch the graph of $g(x)$.



1. Which one of the graphs represents the function $f(x) = \frac{1}{x}$?
Match the graphs f_1 to f_9 to their equations numbered 1 to 9 and to the descriptions of the transformations of $f(x) = \frac{1}{x}$ labelled A to I.

Equation 1 has been done for you.

Equation	Graph	Description/Transformation
1. $y = \frac{1}{x} + 3$	f_6	$f(x)$ translated 3 units up

2. Given $g(x) = \frac{3}{x+1} - 2$ write in the table below:

2.1 the coordinates of the y-intercept of $g(x)$	
2.2 the coordinates of the x-intercept of $g(x)$	
2.3 the equation of the vertical asymptote of $g(x)$	
2.4 the equation of the horizontal asymptote of $g(x)$.	
2.5 Sketch the graph of $g(x)$.	

HELP

Start with one of the equations. Find the value of x that makes y infinite which gives you the equation of the vertical asymptote.

Find what happens to y as x gets bigger and bigger. This will give you the horizontal asymptote.

These two facts should help you to identify the graph.

NEXT

Make up a similar activity of your own, perhaps with just 4 graphs and 12 cards altogether. Then exchange your work with another learner and each try to complete the matching activity created by the other learner, but also to spot any errors that they think have been made in creating the problem. The learners should try to resolve issues about errors and come up with a set of 24 cards between them that provide a similar activity to the original one.

Cut the table into 27 cards and rearrange them into 9 matching sets.

Equations	Graphs	Transformations
1. $y = \frac{1}{x} + 3$	f_1	**A.** $f(x)$ translated 6 units to left
2. $y = \frac{1}{x-3}$	f_2	**B.** $f(x)$
3. $y = \frac{1}{x} - 6$	f_3	**C.** $f(x)$ translated 6 units down
4. $y = \frac{1}{x+3}$	f_4	**D.** $f(x)$ translated 3 units up
5. $y = \frac{1}{x} - 3$	f_5	**E.** $f(x)$ translated 3 units to left
6. $y = \frac{1}{x}$	f_6	**F.** $f(x)$ translated 6 units to right
7. $y = \frac{1}{x+6}$	f_7	**G.** $f(x)$ translated 6 units up
8. $y = \frac{1}{x} + 6$	f_8	**H.** $f(x)$ translated 3 units down
9. $y = \frac{1}{x-6}$	f_9	**I.** $f(x)$ translated 3 units to right

NOTES FOR TEACHERS

SOLUTION

1.Which one of the graphs represents the function $f(x) = \frac{1}{x}$?

Match the graphs labelled f_1 to f_9 to their equations and to the descriptions of the transformations of the graph of $f(x)$

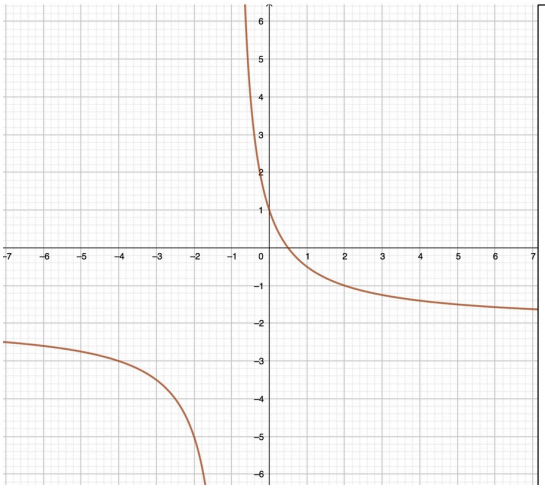
1: f_6 D - translation 3 units up
2: f_4 E – translation 3 units to left
3: f_8 C – translation 6 units down
4: f_2 L - translation 3 units to the right
5: f_7 H – translation 3 units down
6: f_1 B - the parent function in this family
7: f_3 F – translation 6 units to the right
8: f_9 G – translation 6 units up
9. f_5 A – translation 6 units to the left

2. Given: $g(x) = \frac{3}{x+1} - 2$

2.1 the coordinates of the y-intercept of $g(x)$	(0 ; 1)
2.2 the coordinates of the x-intercept of $g(x)$	(0.5 ; 0)
2.3 the equation of the vertical asymptote of $g(x)$	x = -1
2.4 the equation of the horizontal asymptote of $g(x)$.	y = -2
2.5 Sketch the graph of $g(x)$. *See graph in Diagnostic Quiz below.*	

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



The equation of this hyperbola is

$A. y = \frac{3}{x-1} - 2$

$B. y = \frac{3}{x+1} - 2$

$C. y = \frac{3}{x+1} + 2$

$D. y = \frac{3}{x-1} + 2$

2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is B

A. The mistake here is failure to recognise that the vertical asymptote for graph A is y= -1
C. The mistake here could be failure to understand the use of the sign in translating graphs up or down.
D. This combines the mistakes in A and C. https://diagnosticquestions.com

Why do this activity?

This activity provides practice for learners in recognizing transformations of graphs of functions and identifying the effects of changes in parameters in the equations that give transformations of the graphs.

Learning objectives

In doing this activity students will have an opportunity to:

- gain a deeper understanding of asymptotes and reciprocal functions;
- gain a deeper understanding of transformations of graphs.

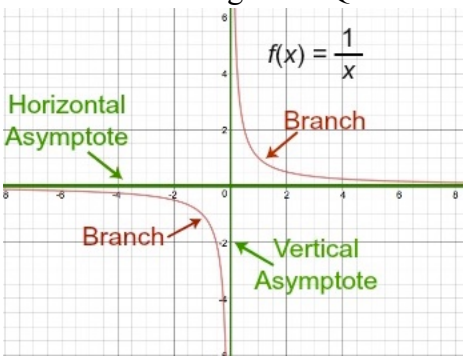
Generic competences (some suggestions, select from list or write your own)

In doing this activity students will have an opportunity to work with a team to create a poster.

Suggestions for teaching

Resources: Flip chart paper and dry markers. Graphing software is useful but not essential.

Start with the Diagnostic Quiz to review the simplest reciprocal function with the graph of a rectangular hyperbola.



Explain the general form of a reciprocal function is $r(x) = \frac{a}{x-h} + k$ and that the graphs of reciprocal functions are made up of two branches and asymptotes, which are horizontal and vertical lines that the graph approaches but doesn't touch. Explain that the lesson will be about the effects on the graph of changing the parameters *a, h* and *k* and the class will work in groups to make posters to explain these changes.

Make copies of the table and give to the learners to cut out into 27 separate cards. Then they can work in groups of 4 to sort and match the cards into sets.

Alternatively copy the cards in very large writing onto 27 separate sheets of A4 paper and stick them on the board in a random order. Ask the learners to work in pairs to match each equation with its graph and with the description of the transformation. Those who finish first can carry on to answer question 2 or this could be a follow-up or homework task.

Then ask the learners to come up to the board one at a time to rearrange the sheets that were stuck on the board into 9 sets.

Either in the same lesson if there is time, or in the next lesson, give each group a sheet of flip chart paper and a dry marker. If you don't have space in the classroom for learners to make posters then take them into the school hall or similar space where they can sit on the floor around their piece of flip chart paper.

Ask the learners to design and make a poster that could be used in teaching about the effects on the graph of $r(x) = \frac{1}{x}$ of changing the parameters *a, h* and *k* in the equation $r(x) = \frac{a}{x-h} + k$. Tell them that the poster should explain why and how the different transformations arise from changing each of the parameters. Explain to the class that they are doing this not just to deepen their own understanding of the maths but also to help them develop skills that employers are looking for, that will be useful in the world of work (for example in advertising).

Key questions

- What is the y-intercept for that graph?
- How do you find the y-intercept from the equation?
- What is the y coordinate when x = 0 for that function?
- What transformation has changed that graph to that one?
- Does the graph show an increasing or decreasing function?
- What transformation of the function corresponds to that change in the equation?
- What change in the equation corresponds to that transformation of the function?

Follow up

See the similar questions on changing parameters in families of functions:
Matching Exponentials https://aiminghigh.aimssec.ac.za/years-11-12-matching-exponentials/
Parabolic Patterns https://nrich.maths.org/parabolicpatterns
More Parabolic Patterns https://nrich.maths.org/777
Parabolas Again https://nrich.maths.org/791
Sine Problem https://nrich.maths.org/436
Tangled Trig Graphs https://aiminghigh.aimssec.ac.za/years-10-12-tangled-trig-graphs/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.
For resources for teaching A level mathematics see https://nrich.maths.org/12339

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6