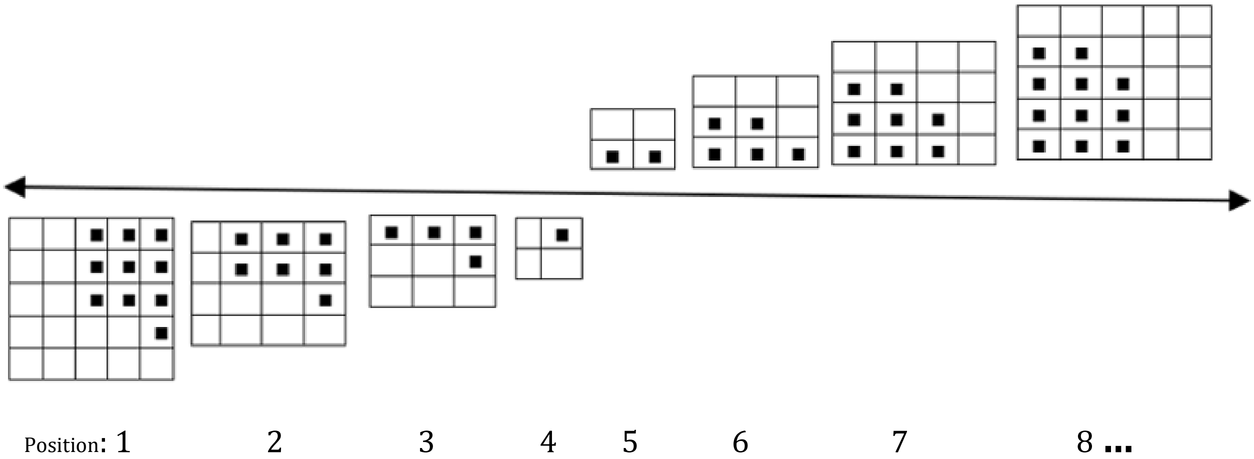
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

LINEAR SEQUENCE



1.1 The square grids below the continuous line depict negative numbers and those above the line positive numbers. Study the square grids shown above with **black squares** forming a pattern. How is the pattern growing? Create square grids 9, 10 and 11; complete them with **black squares** to further develop the pattern.

1.2 How many small **black squares** are in the grids in the 9th, 10th and 11th positions?

1.3 Can you predict the number of small **black squares** in the grids in the 50th and 100th positions?

1.4 What numerical number pattern can be formed from the **black squares** in each grid?

1.5 Obtain the general term, T_n, for this number pattern.

1.6 What is the mathematical name given to this kind of number pattern?

1.7 If there are 602 black squares in a square grid, determine n, the position of the square grid.

HELP

Study the following number pattern: -3, -1, 1, 3, 5, 7, …

Workout the differences between any two consecutive terms of the number pattern.

-3, -1, 1, 3, 5, 7, ...

What do you notice? What can you say about these differences?

2 4 6 8 10 12 …. Multiples of 2 i.e. 2n

-3, -1, 1, 3, 5, 7, ... 2n – 5. **Notice that all these numbers are 5 less than the multiples of 2 (i.e. 2n), hence 2n – 5**.

First differences….

Can you confirm that the n^{th} term of this number pattern is 2n – 5? How can you confirm that?

What is the 10^{th} term, 20^{th} term and the 100^{th} term?

Is 200 a term of this sequence? Justify your answer.

How about 245, can it be a term of this sequence? Justify your answer.

NEXT

Can you create a similar number pattern which develops in such a way that the consecutive terms have a constant difference?

What is the rule of your number pattern?

What are the next two terms of your pattern?

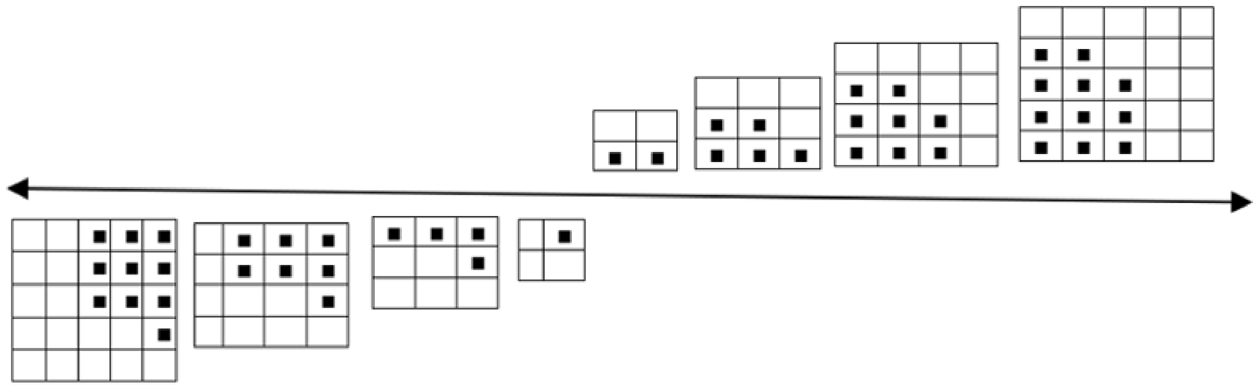
Can you come up with the n^{th} term?

Find the 50^{th} term of your sequence?

Is 172 (or any number of your choice picked at random) a term of the sequence?

NOTES FOR TEACHERS

SOLUTION



Position: 1 2 3 4 5 6 7 8 ...

1.1 The following number pattern can be deduced from square grids given above.

-10, -7, -4, -1, 2, 5, 8, 11,…

+3 +3 +3 +3 +3 +3 +3

Notice that the pattern develops by the **rule 'add 3'** to each successive term. In other words, there is a constant difference of +3 between any two consecutive terms in the sequence.

Take note:

3 x 1	3 x 2	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7……3 x n		
3	6	9	12	15	18	21	3n	Multiples of 3.

-10, -7, -4, -1, 2, 5, 8, 11,…

+3 +3 +3 +3 +3 +3 +3

Notice that ALL the numbers in the pattern are 13 less than the multiples of 3, that is, **3n – 13**, which gives us the n^{th} term.

Check this out:

3n – 13:

n = 1: 3 (1) – 13 = 3 – 13 = -10

n = 2: 3 (2) – 13 = 6 – 13 = - 7

n = 3: 3 (3) – 13 = 9 – 13 = - 4

n = 4: 3 (4) – 13 = 12 – 13 = -1

n = 5: 3 (5) – 13 = 15 – 13 = 2

n = 6: 3 (6) – 13 = 18 – 13 = 5

n = 7: 3 (7) – 13 = 21 – 13 = 8

n = 8: 3 (8) – 13 = 24 – 13 = 11

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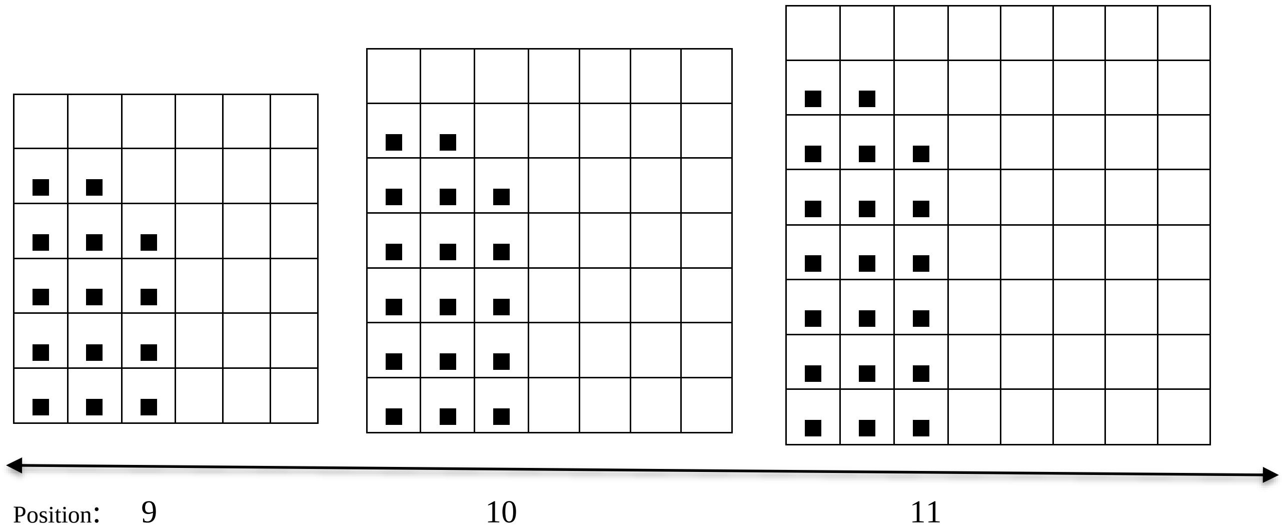
n: 3 (n) – 13 = **3n – 13**



1.2 9th position: 14 **black squares**

10th position: 17 **black squares**

11th position: 20 **black squares**

1.3 It seems **'3 times the position number minus 13'** gives us the number of **black squares.**

∴ 3 x 50 – 13 = 150 – 13 = 137

3 x 100 – 13 = 300 – 13 = 287

∴ There are **137** black squares in the grid in **50th** position and **287** in the grid in **100th** position.

1.4 The number pattern extracted from the geometric pattern:

-10, -7, -4, -1, 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, …

1.5 The general term will be $\mathbf{T_n = 3n - 13}$ as seen from the predictions above: 1.3

1.6 From the general term $\mathbf{T_n = 3n - 13}$ which is in the form $y = mx + c$, one can deduce that this sequence is a **linear sequence** or a **linear number pattern.**

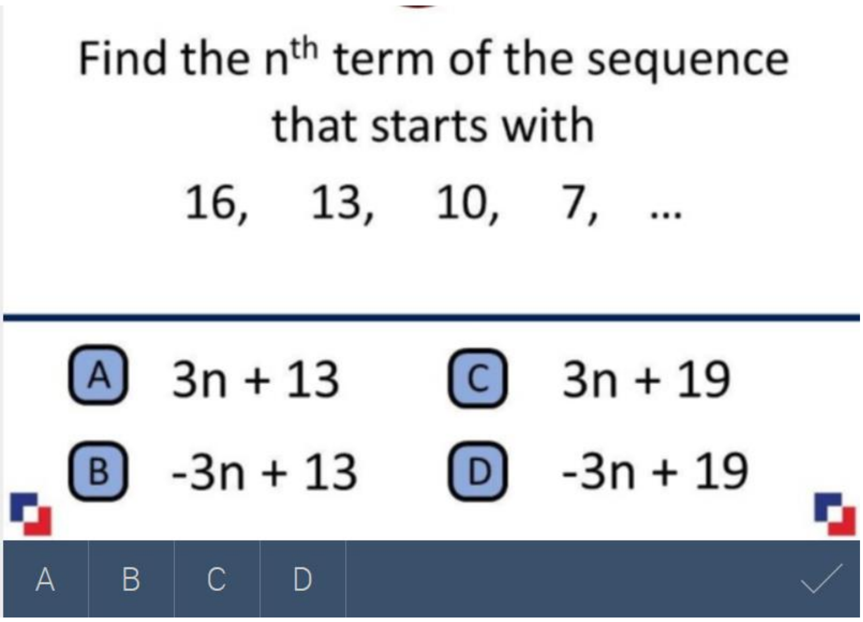
1.7 If there are 602 black squares, n, the position of the square grid containing these black squares will be obtained by solving the following algebraic equation:

$$3n - 13 = 602$$
$$3n = 615$$
$$n = 205$$

$\therefore$ The grid in position 205 will contain 602 black squares.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer and possible misconceptions:

A. Guessing
B. 13 might imply some guessing
C. Guessing
D. Is the correct answer: -3n + 19

https://diagnosticquestions.com

Why do this activity?

This activity help learners develop their observation skills; as they come up with a number pattern extracted from a geometric pattern and can develop it further, by so doing they develop skills of predicting, evaluating and monitoring the outcome of a given situation. When they are in full control of the sequences, they can make informed decisions based on the rules or facts about the sequences. Algebraic skills are further sharpened in this kind of activity, especially when they come up with the rule for the n^{th} term and further predicting terms of the sequence using the rule.

Learning objectives

In doing this activity students will have an opportunity to:

- **investigate geometric and numeric patterns** and find rules for the patterns in sequences;
- generalize number patterns and rules describing sequences, moving naturally from geometric to arithmetic to algebra, to **find formulae** for the n^{th} terms of the sequences;
- **think mathematically**, reason logically and give explanations.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly,** be creative and innovative and apply knowledge and skills;
- **visualise** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems**;
- **work and learning dependently** and prepare for lifelong learning;
- **work in a team**:
  - collaborate and work with a partner or group
  - have empathy with others, listen to different points of view
  - develop leadership qualities;
- **communicate** in writing, speaking and listening;
  - exchange ideas, criticise, and present information and ideas to others
  - analyse, reason and record ideas effectively.
- **develop life skills and consideration for others-** to show social responsibility- to work for the good of the community.

Suggestions for teaching

Organise your learners into small groups to promote group discussions for this kind of activity. Start with the diagnostic question as a warm-up. Ask the learners to describe how the pattern on the diagnostic question develops. Pamphlets of the diagnostic question can be availed to each group of learners for discussions.

Is there any rule which seems to be applicable in developing the pattern? Let learners prove the validity of the rule through substitution. This task will eliminate guessing, for example, some learners might substitute $n = 1$ in $3n + 13$ to get 16, then rush to conclude that $3n + 13$ is the rule, without realizing that the pattern is actually a decreasing one, and this rule will in fact not generate the subsequent terms. Through some critical thinking, intuition or otherwise, learners should pick up that **$-3n + 19$** is the rule for this sequence in the diagnostic question.

Give out posters to each group. Allow them to reproduce the geometric pattern of the sequence under study, the one of square grids. “Can anyone tell how the **black squares** in the grids are forming a pattern? How is the pattern developing? Help learners realise that the numbers portrayed by the grids underneath the continuous line are negative and those above are positive.

"Can you write down the number pattern extracted from the geometric pattern?" Try to extend the geometric pattern on your posters to include positions 9, 10, 11, 12, clearly inserting the small **black squares** within the grids in such a way that there is continuity of the pattern.

Let the learners list the number pattern up to position 20 in their posters, allowing them to discuss the development of the number pattern within their groups. They should come up with a number sequence like this:

-10, -7, -4, -1, 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, …

"Can you predict the 21st, 22nd, 50th and 100th terms in the sequence?" "Can you come up with the nth term of the sequence?" Realising that the common difference is 3, and that each position is generated by a multiple of 3 plus a certain constant c, that is 3n + c. Substituting the value of n for each position, say n = 1: 3 (1) + c = -10 $\xrightarrow{yields}$ c = -10 – 3 = -13

∴ **3n – 13** is the required nth term, **T_n = 3n – 13** of the sequence.

Ask learners to pick on any three of more of the terms in the sequence, substitute in T_n the value of n (position in the sequence) and see if the nth term expression generates the correct terms in the sequence.

In the HELP section, learners can confirm 2n – 5 is the generator of the sequence i.e. the nth term, by substituting any position of the terms in the sequence and see if they can obtain the corresponding terms to the said positions.
e.g. n = 3: 2 (3) – 5 = 6 – 5 = 1 and for sure this is the 3rd term in this sequence.

10th term: 2 (10) – 5 = 20 – 5 = 15
20th term: 2 (20) – 5 = 40 – 5 = 35
100th term: 2 (100) – 5 = 200 – 5 = 195

To check whether 200 is a term of this sequence:
Solve 2n – 5 = 200
2n = 205
n = 102.5 since n should be an integer value (position), 200 is not a term of this sequence.
Do the same to check for 245:
Solve 2n – 5 = 245
2n = 250
n = $\frac{250}{2} = 125$, yes 245 is a term in the sequence as n = 125 is an integer value.

NEXT this section allows learners to be more creative and come up with their own sequences of their choice, apply the knowledge learnt. Allow them to pick on any number of their choice, 172 was just a number picked at random.

Key questions

- Can you see a pattern in the image presented to you?
- How is the image developing?
- Can you draw grids in position 9, 10, 11 and 12, showing the small **black squares**?
- Can you predict 100th term of the number pattern?

- Can you come up with an **algebraic expression, the n^{th} term,** which generates the terms of this sequence?
- How can you prove that the n^{th} term you obtained is correct?

Follow up

- https://aiminghigh.aimssec.ac.za/years-7-9-shifting-times-tables/
- https://aiminghigh.aimssec.ac.za/years-7-9-steps/
- https://aiminghigh.aimssec.ac.za/wp-content/uploads/2018/01/LS-A2-Sequences-and-Patterns.pdf
- https://aiminghigh.aimssec.ac.za/?s=squares+pattern

A going through the follow up activities on the Aiming High website, ask ALL groups to summarise what they have learnt in this lesson, draw up this summarise on the given posters for presentation in front of the whole class on a mentioned suitable date.

<table>
<tr><td colspan="5">Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.
For resources for teaching A level mathematics see https://nrich.maths.org/12339</td></tr>
<tr><td></td><td>Lower Primary
or Foundation Phase
Age 5 to 9</td><td>Upper Primary

Age 9 to 11</td><td>Lower Secondary

Age 11 to 14</td><td>Upper Secondary

Age 15+</td></tr>
<tr><td>South Africa</td><td>Grades R and 1 to 3</td><td>Grades 4 to 6</td><td>Grades 7 to 9</td><td>Grades 10 to 12</td></tr>
<tr><td>USA</td><td>Kindergarten and G1 to 3</td><td>Grades 4 to 6</td><td>Grades 7 to 9</td><td>Grades 10 to 12</td></tr>
<tr><td>UK</td><td>Reception and Years 1 to 3</td><td>Years 4 to 6</td><td>Years 7 to 9</td><td>Years 10 to 13</td></tr>
<tr><td>East Africa</td><td>Nursery and Primary 1 to 3</td><td>Primary 4 to 6</td><td>Secondary 1 to 3</td><td>Secondary 4 to 6</td></tr>
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