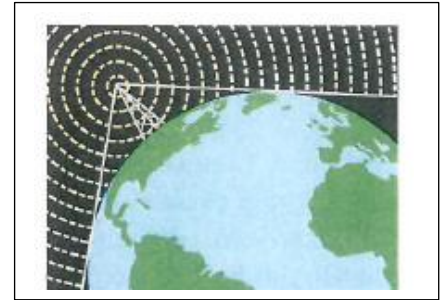


APPLYING THE TANGENT THEOREM

Radio and TV stations broadcast from high towers.

Their signals are picked up by radios and TV's in houses within a certain radius. Because the Earth is spherical, these signals do not get picked up beyond the point of contact of the near horizontal tangent line as illustrated in the sketch.



What is the maximum broadcasting radius from a radio tower 70m tall?

Assume that the circumference of the Earth is 42 650km.
 Give your answer to the nearest km.

HELP

Make a drawing and let the distance from Tower (T) to point of contact (A) be d and draw in the line from (A) to centre of earth (C). Then complete triangle TAC.

What angle is formed between the tangent and the radius?

If it is a right angle, what other theorem can you use in triangle TAC?

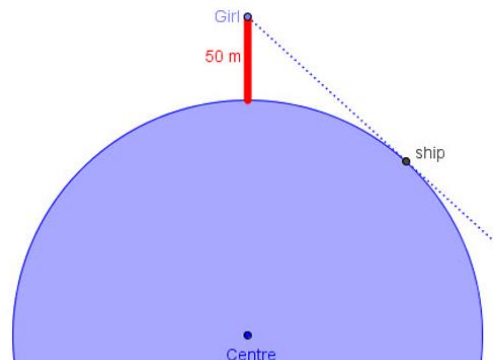
Can you find the radius of the earth?

Remember units must be the same.

NEXT

Can you apply your method to various heights of towers or similar situations?

1. A girl sits in a building, 50 m high, and stares at the sea looking for ships. How far can she see? (Assume that the circumference of the Earth is 42 650km. Give your answer to the nearest km).



2. You are standing 12 meters from a cylindrical storage tank. The distance from you to the point of tangency on the tank is 35 m.
 What is the radius of the tank?

NOTES FOR TEACHERS

SOLUTION

Circumference of earth = $2\pi r = 42\,650$

$$r = \frac{42\,650}{2\pi} = 6\,787,958 \approx 6\,788 \text{ km}$$

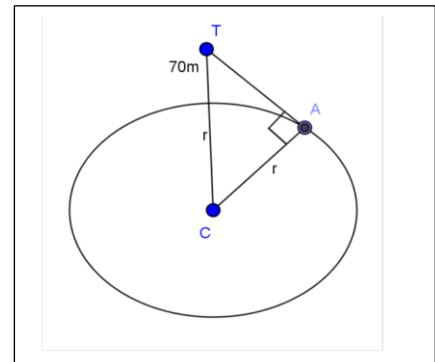
Let the distance from Tower (T) to point of contact (A)

be d and draw in line from (A) to centre of earth (C)

TA is perpendicular to AC (tangent, radius)

$$\begin{aligned} d^2 &= (r + .07)^2 - r^2 &= 6788,07^2 - 6788^2 \\ &= 950.3249 \end{aligned}$$

$$d = 30,827 \approx 31 \text{ km}$$




This example was taken from Michael Serra's book: "Discovering Geometry: An Investigative Approach", Fourth Edition, Key Curriculum Press, 2008, p514.

DIAGNOSTIC ASSESSMENT

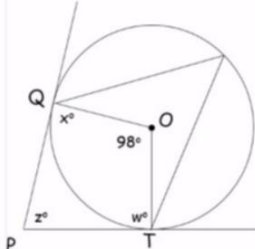
This should take about 5–10 minutes.

1. Write or show the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



PQ and PT are tangents to a circle with centre O. Find the angle z .



A

 $z = 49^\circ$

C

 $z = 98^\circ$

B

 $z = 82^\circ$

D

 $z = 196^\circ$

A
B
C
D

✓

- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

B. is the correct answer. $360^\circ - 98^\circ - 90^\circ - 90^\circ = 82^\circ$ as the angles between a radius and tangent = 90°

Common Misconceptions

A z is not equal to 49° , the angle on the opposite circumference which is half of the angle at the centre.

C z is not equal to 98° , the angle at the centre O

D z is not equal to 196° , which is double 98.

<https://diagnosticquestions.com>

Why do this activity?

This activity applies the tangent theorem to real life problems. This usually motivates learners to want to learn more about geometry and its applications.

Learning objectives

In doing this activity students will have an opportunity to:

See the value of the theorem and make a drawing as one of the first steps in solving a problem.

Generic competences

In doing this activity students will have an opportunity to:

Make connections and see the interrelatedness of different concepts in mathematics (circumference, Pythagoras Theorem and tangents).

Suggestions for teaching

Do this activity after discovering that the angle between the tangent and the radius is 90° .

Key questions

How are you going to tackle this?

What information do you have? What do you need to find out?

Would a diagram help?

Which facts have you learned today that you can use?

Refer to the help section on page 1.

Follow up

The solution to the question in the “Next” section on the worksheet on page 1 is:

Circumference of earth = $2\pi r = 42\,650$

$$r = \frac{42\,650}{2\pi} = 6\,787,958 \approx 6\,788 \text{ km}$$

Let the distance from girl (G) to ship be d and draw in line from ship (S) to centre of earth (C)

SC is perpendicular to GS (tangent, radius)

$$d^2 = (r + .05)^2 - r^2 = 6788,05^2 - 6788^2 = 678,8025$$

$$d = 26,05 \approx 26 \text{ km}$$

This example was taken from Gerrit Stols's PDF book: "Circle Geometry for High School students", Gerrit Stols, 2014 available from his website accessed 19 February 2020

<https://schoolmaths.com/documents/Circle%20geometry.pdf> p 37.

For more ideas follow the links:

<https://aiminghigh.aimssec.ac.za/years-10-11-investigating-circle-theorems/>

<https://aiminghigh.aimssec.ac.za/years-10-12-circle-inscribed-in-quadrilateral/>

<p>Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339</p>				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6