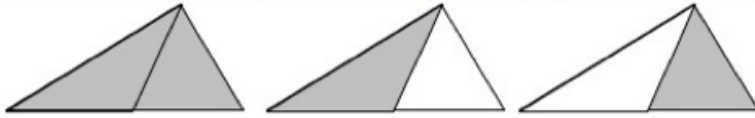
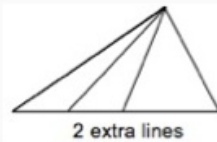


COUNTING TRIANGLES

A triangle with one extra line gives 3 triangles altogether



How many triangles can you find when you draw in 2 extra lines? What about 3 extra lines?



We can go on drawing in extra lines.



How many triangles will there be with...
...4 extra lines? ...5 extra lines? ...8 extra lines? ...20 extra lines?

Help

For 2 extra lines, draw the diagram 6 times and shade in 6 different triangles to show why the answer is 6 for 2 extra lines.

For 3 extra lines, it might help you to draw lots of copies of the diagram with 3 extra lines and shade the different triangles.

Try to find a number pattern and use it to find a rule for the number of triangles for 20 extra lines. Check to see if your rule works for 4 extra lines and 5 extra lines.

Extension

Explain the pattern or rule that you use to work out the number of triangles for 20 extra lines. Why does the rule work?

Can you use the same rule for 100 extra lines?

What about n extra lines where n can take any whole number value? What is the formula for the number of triangles for n extra lines?

NOTES FOR TEACHERS

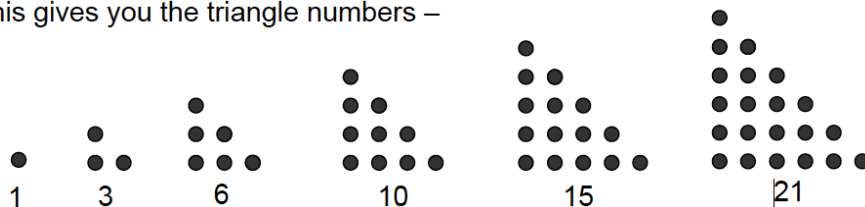
SOLUTION

Number of extra lines	1	2	3	4	5	6	7	8
Number of triangles	3	6	10	15	21	28	36	45

Each time you add a new line you have all the triangles in the diagram already **plus extra triangles**. The number of triangles goes up by one more than it did last time.

For n extra lines there will be $(n + 1)$ extra triangles making $1 + 2 + 3 + \dots + n + (n + 1)$ triangles in total.

This gives you the triangle numbers –



Each extra line adds one extra small triangle and the triangles are counted singly or in twos, or threes etc up to the number of small triangles in the large one.

Number of extra lines	Single Δ s	2- Δ s	3- Δ s	4- Δ s	5- Δ s	6- Δ s	Total
0	1						1
1	2	1					3
2	3	2	1				6
3	4	3	2	1			10
4	5	4	3	2	1		15
5	6	5	4	3	2	1	21

With n extra lines there will be

$$1 + 2 + 3 + \dots + (n + 1) = \frac{1}{2}(n + 1)(n + 2) \text{ triangles.}$$

With 20 extra lines there will be $\frac{1}{2} \times 21 \times 22 = 231$ triangles.

Diagnostic Assessment This should take about 5–10 minutes.

- Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Sequences

How many red balls would be in the fifth picture?



A	B	C	D
16	19	10	15

The correct answer is D as $1 + 2 + 3 + 4 + 5 = 15$

Possible misconceptions:

A. Students have mistakenly added 6 or just guessed.

B. Probably just a guess.

C. This is the 4th term in the sequence.

<https://diagnosticquestions.com>

Why do this activity?

This activity involves learners in an investigation. They need to count the triangles in each new diagram as they add an extra line to the diagram and look for a pattern in the sequence. They must work systematically, and record their results so that they can see the pattern.

Learning objectives

In doing this activity students will have an opportunity to:

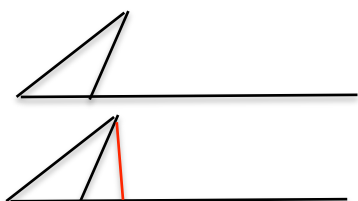
- investigate and extend geometric and numeric patterns looking for relationships between numbers;
- represent the patterns in a table of results;
- describe and justify the general rules for the observed relationships between numbers in learners' own words or in algebraic language.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all. In doing this activity students will have an opportunity to:

- solve and interpret problems in a variety of situations;
- develop the skill of interpreting and creating visual images.

Suggestions for teaching



Draw this diagram on the board. Ask the class “How many triangles?”

Now draw an extra line and ask “How many triangles?”
Make sure that the learners understand the question.

Ask the learners to work in pairs to find the number of triangles for 2 extra lines. Discuss this as a class. Then ask them to find the number of triangles for 3, 4, and 5 ... extra lines and see if they can spot a pattern.

Ask learners to explain what they have discovered.

Key questions

- How many **extra** triangles do you add?
- How can you be sure that you have found all the triangles?
- How can you record your results so you can see clearly if there is a pattern?
- How does the number of triangles go up each time you add a new line?
- Can you see a pattern in the increase in the number of triangles?
- Where have you seen this number pattern before?

Follow up

Triangle Number Picture <https://aiminghigh.aimssec.ac.za/years-9-to-12-triangle-number-picture/>

Further activities involving number patterns and sequences:

Equipattern <https://aiminghigh.aimssec.ac.za/years-5-to-9-equipattern/>

Mystic Rose <https://aiminghigh.aimssec.ac.za/years-7-to-12-mystic-rose/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6