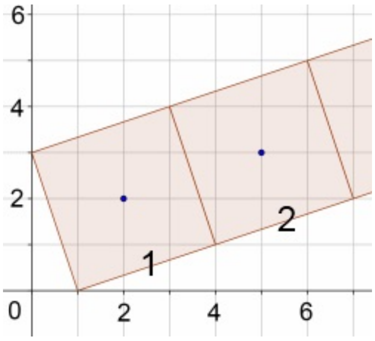
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
TEACHER NETWORK

COORDINATE PATTERNS



Imagine this pattern of squares continuing on and on ...
How do you find the coordinates of the centre of square 1 if you know that (0, 3), (3, 4) and (4, 1) are vertices?
What are the coordinates of the centre of the 20th square?

Imagine the sequence of squares extending to the left ... -2, -1, 0, 1, 2, 3, ...

Can you explain why the centre of square -1 is (-4 ; 0)?
What are the coordinates of the centre of square -10?

Explain how you can find the coordinates of the vertices of square 2?
Now the big challenge: explain how you can find the coordinates of the vertices of square n?

Help

You will be able to solve this puzzle if you use what you can find out from the diagram about gradients.

It's all about distances across and up.

How many steps across and up is it between the vertices at the tops of the squares?

Can you see why that tells you that the gradient of the top edges of the squares is ⅓ ?

What other line segments in the diagram have the same gradient? Why?

Use these ideas to answer the question.

Extension

Explain how you could work out the coordinates of the vertices of the *n*th square.
Find a formula for the coordinates of the vertices of the *n*th square.

NOTES FOR TEACHERS

SOLUTION

The 4th vertex of square 1 is 3 units across and 1 unit up from (0, 3) that is (3, 4).
The centre of the square is the midpoint of (0, 3) and (4, 1) that is (½(0+4), ½(3+1)) = (2, 2).
Similarly it is the midpoint of (1, 0) and (3, 4) that is (½(1+3), ½(0+4)) = (2, 2).

To get the coordinates of the centre of the next square in the sequence you go across 3 units and up one unit.
So square 20 has centre (2 + 19×3, 2+19×1) = (59, 21)

The centre of square -1 is (2 + 2 ×(-3), 2 + 2 ×(-1)) = (-4, 0)
The coordinates of the centre of square number -10 is (2 + 11(-3), 2+11×(-1)) = (-31, -9)

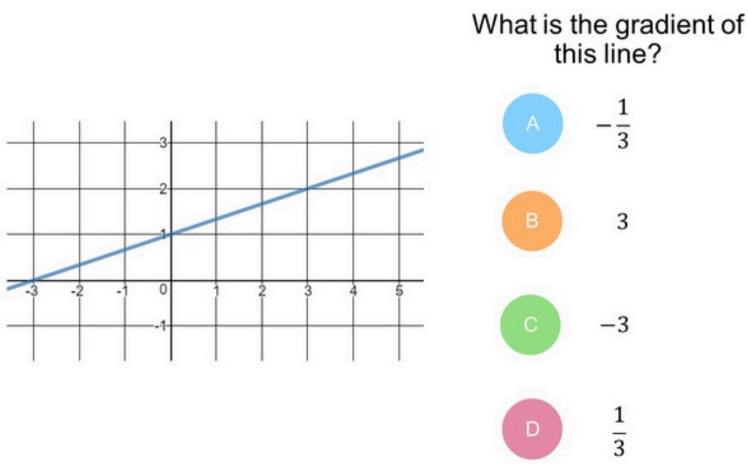
To go from the vertices of square 1 to the corresponding vertices of square n you go ***across 3 units (n-1) times and up one unit (n-1) times.***

The coordinates of the vertices of the square 2 are:
(0 + 3(2-1) , 3 + (2-1)) = (3, 4)
(1 + 3(2-1) , 0 + (2-1)) = (4, 1)
(4 + 3(2-1) , 1 + (2-1)) = (7, 2)
(3 + 3(2-1) , 4 + (2-1)) = (6, 5)

The coordinates of the vertices of the nth square are:
(0 + 3(n-1) , 3 + (n-1)) = (3n - 3, 2 + n)
(1 + 3(n-1) , 0 + (n-1)) = (3n - 2, n - 1)
(4 + 3(n-1) , 1 + (n-1)) = (3n + 1, n)
(3 + 3(n-1) , 4 + (n-1)) = (3n, 3 + n)

Diagnostic Assessment
This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



D. is the correct answer.
Common Misconceptions
A. The learner has used the wrong sign.
B. The learner has flipped the fraction.
C. The learner has flipped the fraction and used the wrong sign.

https://diagnosticquestions.com

Why do this activity?

This activity offers a good opportunity for learners to discuss patterns and find convincing arguments for their solutions. They are led to generalising from the particular cases. The activity is accessible to all students and the students who struggle will gain a better understanding of coordinates (gradients, midpoints etc.) and of sequences even if they are not able to formulate the algebraic generalisation for themselves. The higher achieving learners will benefit from the opportunity to make conjectures, to justify them and to formulate their own algebraic expressions for the nth terms of the sequences.

Intended learning outcomes

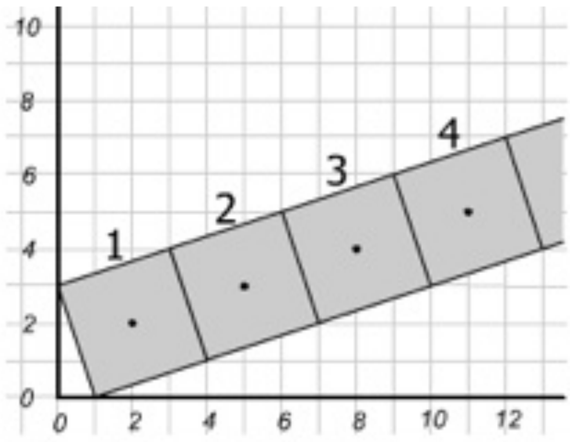
- Deeper understanding of gradients, midpoints and repeating patterns leading to sequences.
- For abler and older learners: practice in making conjectures about patterns, in proving conjectures and in formulating their own algebraic expressions for the nth terms of the sequences.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- think mathematically and to reason logically;
- solve problems;
- develop the skill of interpreting and creating visual images to represent concepts and situations;
- to communicate in writing and speaking:
    - o have empathy with others, listen to different points of view
    - o communicate, exchange ideas, criticise, and present information and ideas to others
    - o analyze, reason and record ideas effectively;
- co-operate - to collaborate/work in a team.



Possible approach

Start with the diagnostic question. This will make it more likely that learners will notice that from one centre to the next you go 1 up and 3 across.

Show this image and say:

1. Have a look at this image. What do you notice about it?

2. Can you work out the coordinates of the centre of square number 3?

3. I wonder if you can now work out the coordinates of the centre of square number 20 from the image, without working out the centres of the squares in between.

4. 'Spend a short while thinking about it on your own, then discuss it with your partner, and together develop a convincing explanation for your answer to share with the class.'

As learners are working, if they get stuck you could ask the following key questions to focus their attention on ideas they might use:

5. How do you move from one square to the next?
6. Can you explain why the centre of square 1 is (2 ; 2)?
7. What do you notice about the x coordinates of the centres?
8. What do you notice about the y coordinates of the centres?

While pairs are talking, circulate and eavesdrop on discussions, drawing attention to mistakes and making a mental note of any students with clear explanations.

Bring the class together and invite those learners with interesting or elegant strategies to present their ideas to the rest of the class.

“In a while I'm going to choose a square and ask you to work out the coordinates of one of the vertices. Can you find a quick and elegant method?"

"Again, you may want to start by working on your own before discussing it with your partner."

Finally bring the class together and challenge them with a few examples. Learners could be asked to display their solutions on their mini-whiteboards. Allow some time for discussion of their strategies.

Extend the discussion by suggesting “Imagine the sequence of squares extending to the left ... -2, -1, 0, 1, 2, 3, ... Can you explain why the centre of square -1 is (-4 ; 0)? What are the coordinates of the centre of square number -10?” Again give the learners time to work on this then bring the class together to dicuss their findings.

Key Questions

- How do you move from one square to the next?
- Can you explain why the centre of square 1 is (2 ; 2)?
- What do you notice about the x coordinates of the centres?
- What do you notice about the y coordinates of the centres?

Follow up

Before working on this problem students could develop fluency with linear sequences by taking a look at Shifting Times Tables https://aiminghigh.aimssec.ac.za/years-7-9-shifting-times-tables/
For an introduction to coordinates: https://aiminghigh.aimssec.ac.za/years-6-to-9-people-coordinates/
A learning activity about gradient: https://aiminghigh.aimssec.ac.za/years-10-12-lines/
Another learning activity about gradient: https://aiminghigh.aimssec.ac.za/years-10-12-line-match/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**