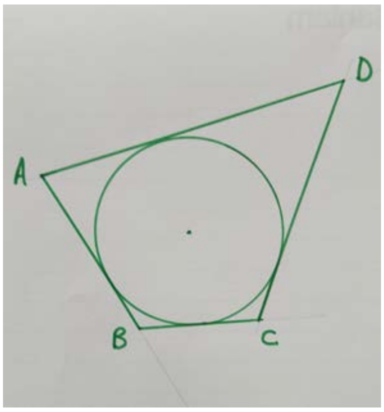
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Investigating by adding pairs of opposite sides of a quadrilateral with a circle inscribed.

Activity
Let the learners in the class draw their own circles with any radius they choose and then draw 4 tangents making a quadrilateral with the circle inside touching the quadrilateral at 4 points.



Then ask them to measure the lengths of the edges of the quadrilateral ABCD.

What do they notice?

Make a table on the board with 8 columns
and let each learner record the details of their own special quad.

Lengths

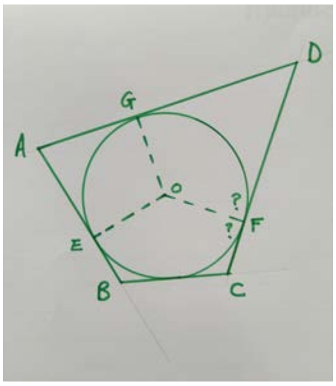
Name	Radius	AB	BC	CD	DA	AB + CD	BC + DA
Bob							
Najwa							
Buri							
Andiswa							

What do you notice?

Write down your own conjecture.

Can you prove your conjecture?

To help them ask leading questions:
Ask the class to draw the radii to the points of contact E, F, G & H, each in their own quad, and then to measure the angles.
What do you notice?



Now draw segment AO. Can you prove that Δ AEO Ξ Δ AGO?
Compare lengths AG and AE.

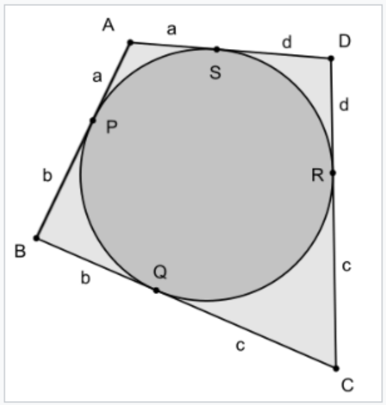
SOLUTIONS
AB + CD = AD + BC =

Possible conjecture:

In a tangential quadrilateral (i.e. one in which a circle can be inscribed) the two sums of lengths of opposite sides are the same.

Both sums of lengths equal the semiperimeter of the quadrilateral. A Google search led to Wiki

Proof of Pitot's Theorem

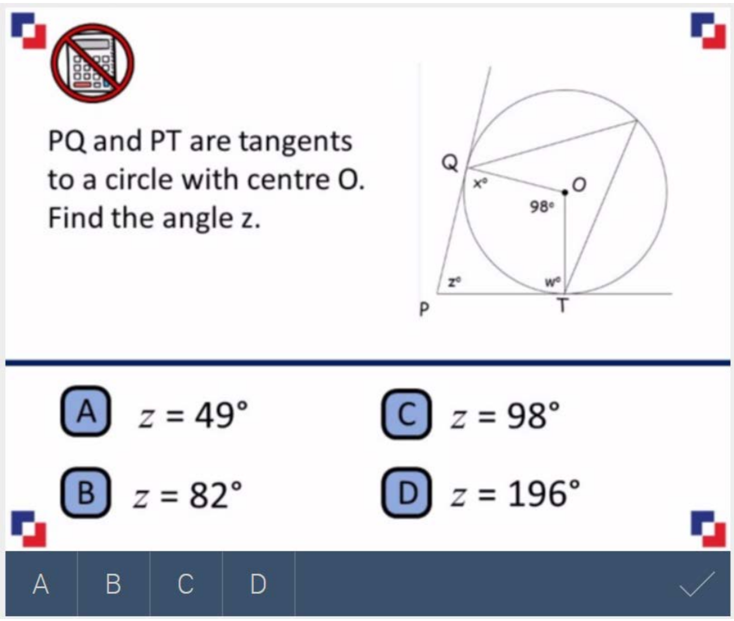


$$
\begin{aligned}
&|AB| + |CD| \\
&= (a+b) + (c+d) \\
&= (b+c) + (a+d) \\
&= |BC| + |DA|
\end{aligned}
$$

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write or show the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

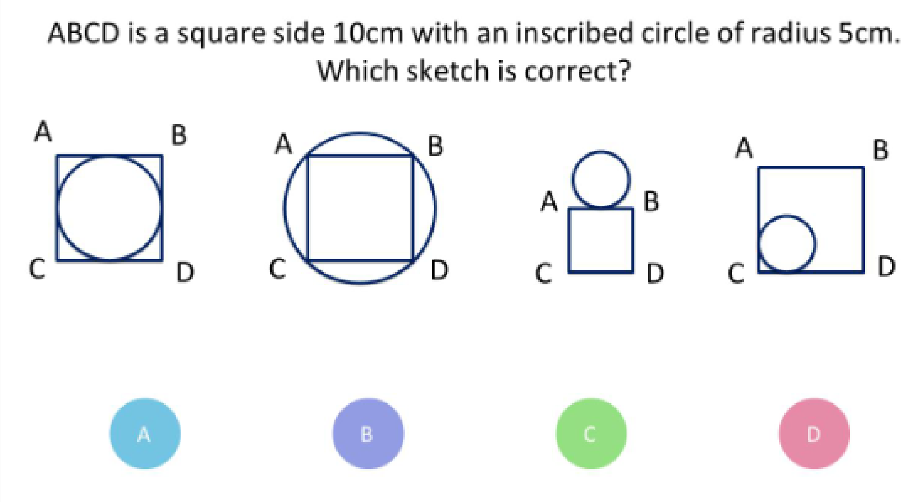
B. is the correct answer. $360^0 - 98^0 - 90^0 - 90^0 = 82^0$ as the angles between a radius and tangent $= 90^0$

Common Misconceptions

A z is not equal to 49^0, the angle on the opposite circumference which is half of the angle at the centre.
C z is not equal to 98^0 **,** the angle at the centre O
D z is not equal to 196^0 **,** which is double 98.

https://diagnosticquestions.com

Other possible questions also taken from https://diagnosticquestions.com could be:

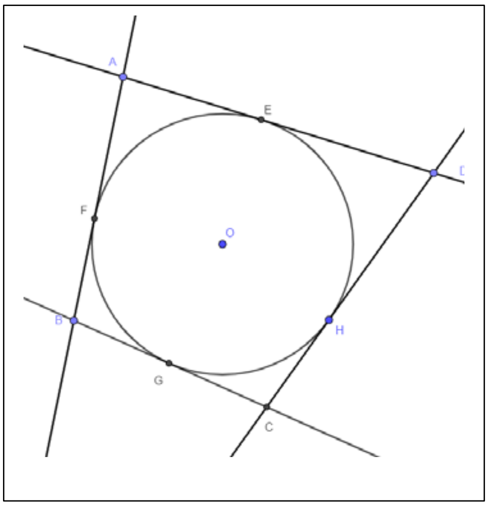


A is the correct answer. B, C and D does not fulfil the requirements.
Is the square a special case of this theorem?

Why do this activity?
This is an extension of the school syllabus in South Africa, but it could be given as an investigation or task. This activity provides experiences of discovering Pitot's theorem. Constructing the tangents to a circle do not prove the theorem, but give ideas for the formal proof.

Intended learning outcomes

- These activities illustrate the properties stated in the theorem and the activities lead to visualization that helps learners:
- Gain a deeper understanding of the geometrical properties;
- Build their vocabulary about circle theorems;



Suggestions for teaching

If you are familiar with GeoGebra, then you could demonstrate the following:
Start by drawing any circle with centre O,
then draw 4 tangents to the circle to form a quadrilateral ABCD with contact points EFGH.
Measure all the segments AE and AF. What do you notice? CE and CH, etc. Can you think of another way to draw an inscribed circle to a quadrilateral?
Calculate (AF + FB +CH + DH) and (AE + ED + BG + GC).
What do you notice? Can you convince your friend?
Complete: Each bracket adds up to …
Now formulate the theorem in your own words.

Explore the ready-made app on GeoGebra for Pitot's theorem for convex and concave quads - follow the link:
https://www.geogebra.org/m/ZbQFQhvu
and drag the points of contact.

Key questions

Can you explain what you have done so far?
Do you think that this would work with other sketches or drawings?
Did you use any new words today? What do they mean? How would you spell them?
Does all quadrilaterals have inscribed circles?
Draw any quadrilateral and bisect two of its angles.

Possible extension

Will the converse theorem also be true?

Extension activity:

State and prove the converse theorem

We want to show that in a convex quadrilateral $ABCD$, if $AB+CD=BC+DA$, then we can inscribe a semicircle that is tangential to all 4 sides.

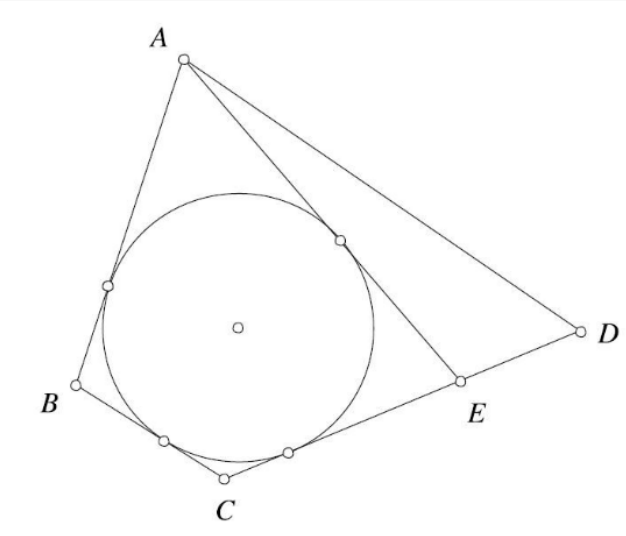
Note that we can always draw a circle tangent to AB, BC and CD. The center of that circle is the point the angle bisectors of $\angle ABC$ and $\angle BCD$ intersect at. This point always exists and so does the circle.

All we have to do is to show that this circle is tangent to DA as well.

We're going to do a proof by contradiction.

Assume that the circle is not tangent to AD. Now draw a tangent to the circle from A and let E be the point that it intersects with CD.

Here's a picture where E lies in the interior of CD.



Since the circle is inscribed in the quadrilateral $ABCE$, from Pitot's theorem we have $AB+CE=AE+BC$. Remember that we had $AB+CD=BC+DA$ or $AB+CE+ED=AD+BC$. But that implies $ED+AE=AD$, which is impossible since $\triangle AED$ is non-degenerate.

So our previous assumption was wrong and the circle is tangential to all 4 sides of $ABCD$. $\square$

https://brilliant.org/wiki/pitots-theorem/#proof-of-converse

Possible support

The teaching strategy should be to guide the learners through these practical investigations, encouraging them to notice certain properties and to make conjectures about what might be true in general. Teachers may give the ready-made drawing and the learners must measure the lengths of the opposite sides. It is important to avoid the misconception that observation of even a large number of cases supporting the conjecture proves it is true. The supporting evidence does not prove that the conjecture is true in all cases and a formal proof is needed.

Another way to reinforce the benefits of visualization, and to help learners to remember these concepts, is for the class to make a large poster of the theorem for the classroom wall.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**