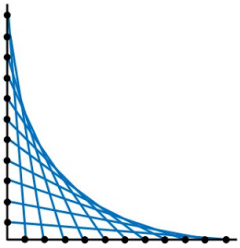
CONSTRUCT GEOMETRICAL PATTERNS

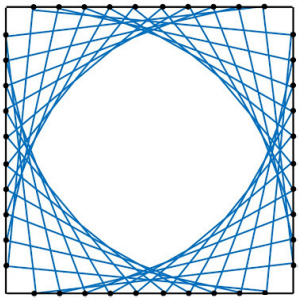


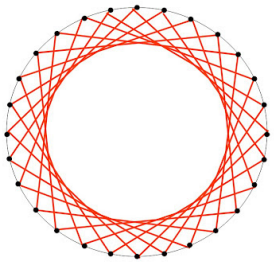
1. ENVELOPE PATTERNS WITH STRAIGHT LINES

Draw two base lines and mark the same number of points at equal distances along each line. Join the outermost point on one base line to the innermost point on the other base line. Work your way inwards on one base line and outwards on the other base line joining corresponding points.

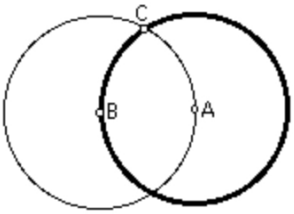
Experiment with different angles between the base lines, also using several pairs of base lines in the same diagram and using different colours.

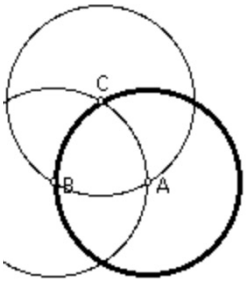
The straight lines form a family and the curves that you see, called envelopes, are tangent to each line in the family.



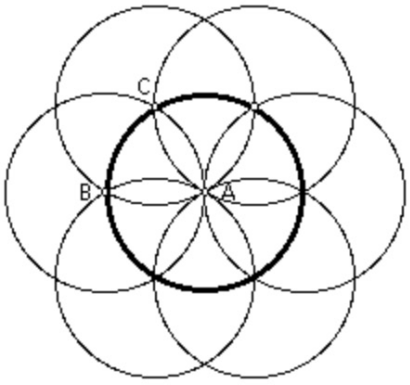


Similar envelope patterns can be drawn by marking points equidistant around a circle. The envelope shown has N=28 points around the circle. It is drawn using the rule $n \rightarrow n + 7$ by joining each point to the 7th point around the circle, missing 6 points between them where $n = 1, \ldots, N$.





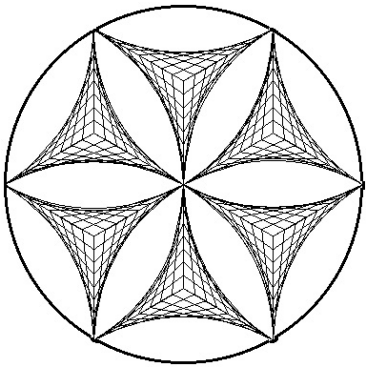
2. PATTERNS WITH CIRCLES



a. Draw 2 circles with equal radii, intersecting at C, so that each goes through the centre of the other.

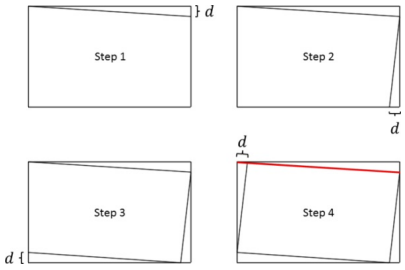
b. Draw a third circle of the same radius with centre C.

c. Draw 4 more circles with centres on the circle centre C and passing through A.

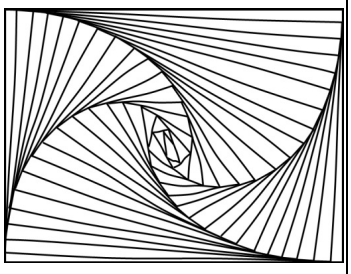


Experiment with this basic construction and also by drawing only the arcs of the 6 outer circles that lie inside the circle centre A.

This design is a clever combination of the methods used in parts 1 and 2.



3. MORE PATTERNS FORMED BY STRAIGHT LINES



a. Draw a rectangle, choose a small distance d and mark this distance along the edge of the rectangle as in Step 1. Mark the distance d along the next edge and draw the straight line as in Step 2.

b. Repeat this construction drawing straight lines spiralling inwards, marking the distance d along the next straight line that you will meet, as shown in Steps 3 and 4.

c. Continue this construction drawing straight lines spiralling inwards, marking the distance d along the next straight line that you will meet and working your way towards the centre of the pattern.

Extension

Create some designs of your own.

Help

The straight line constructions in blue are the easiest so start with them.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

If this month is April what month will it be in 18 months time?

A. November **B.** July **C.** January **D.** October

The correct answer is D. After 12 months it will be April again so count 6 months on from April.

Possible misconceptions:
Perhaps learners who get this wrong don't know the months or made a mistake in counting. Follow up with some oral practice of similar problems involving also weeks, hours or minutes.
https://diagnosticquestions.com

Why do this activity?

This activity introduces the concepts of envelopes and of clock (modulus) arithmetic in a simple and enjoyable way and to apply their knowledge of mathematics in creative ways. The activities gives learners practice in using a ruler, protractors and compasses to draw accurate geometrical constructions. To make the patterns learners will need to measure lengths and angles, to draw circles, to follow instructions and to draw accurately. The activity offers opportunities for talking about the geometrical properties of the shapes.

Teachers may plan to use these lesson ideas at one time, perhaps with different learners drawing different designs, or they may plan for using different parts of the activity at different times. Teachers can plan to meet the needs of learners of different abilities by giving learners different patterns to draw. The activity may improve learners' attitude to mathematics by appealing to some learners who have negative feelings about mathematics or about their own ability to do maths. The activity encourages and helps to develop creativity and learners will enjoy experimenting with different ways to adapt the designs to make their own patterns.

Learning objectives

In doing this activity students will have an opportunity to:

1. gain practice in following instructions and in accurate drawing including measuring angles;
2. work with functions in a non-standard application.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

1. think flexibly, be creative and innovative - to apply knowledge and skills;
2. develop the skill of interpreting and creating visual images to represent concepts and situations;
3. co-operate - to collaborate/work in a team.

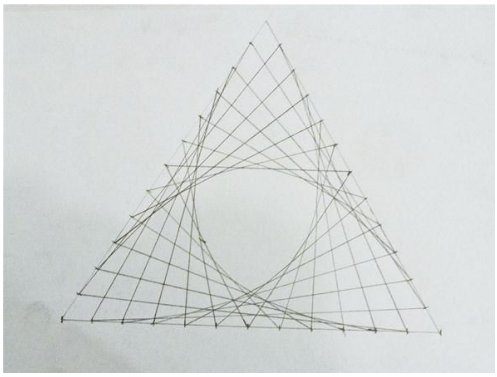
Suggestions for teaching

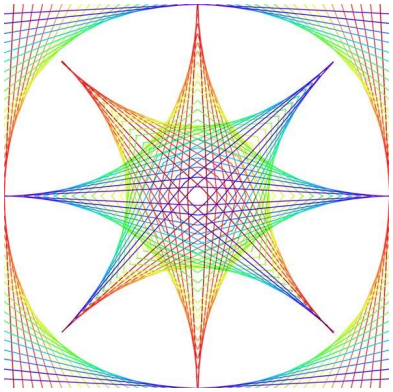
Start with the whole class and show them some pictures see page 4). Give out geometrical instruments. Start everyone on the envelope starting with 2 base lines. Show them how to draw the first pattern by drawing it on the board. When they have copied some designs ask them to describe the geometrical properties.

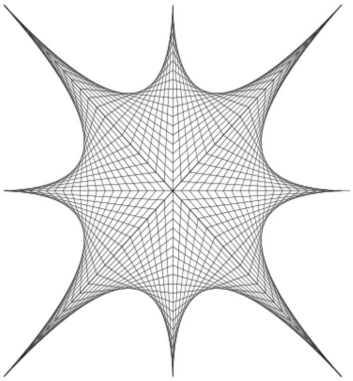
Tell the class that they are going to copy some of the designs and then to create some of their own. The class could work in groups and each group could make a poster showing examples of all the different types of patterns including some that they have created themselves.

Tell the class that this is an exercise that requires each group to work as a team, to plan their project and to share the work so that everyone contributes to the final result. Give them a deadline when the poster must be completed that includes some time in class and some homework time.

Alternatively the teacher can decide to give learners different designs to work on (perhaps chosen to suit their abilities). It works well to have pairs of learners working on the same design but each learner making their own drawing. All the designs can be used in a wall display for the classroom.







Key questions

Can you explain the rule that are you using for drawing that pattern?

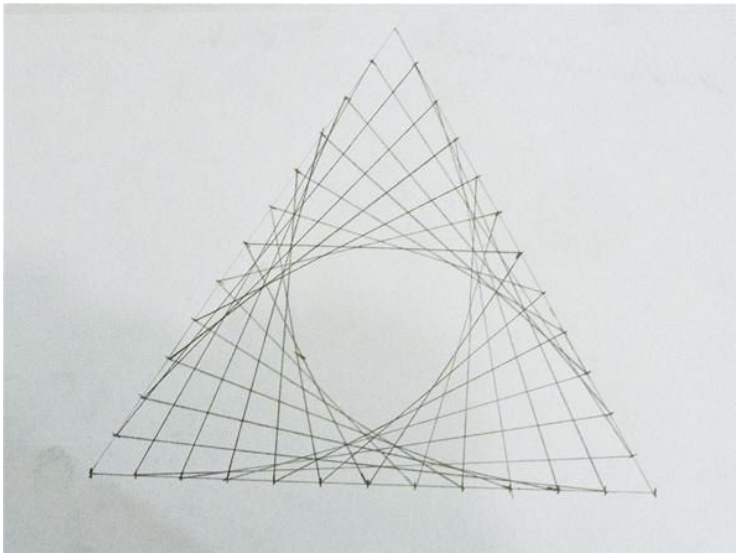
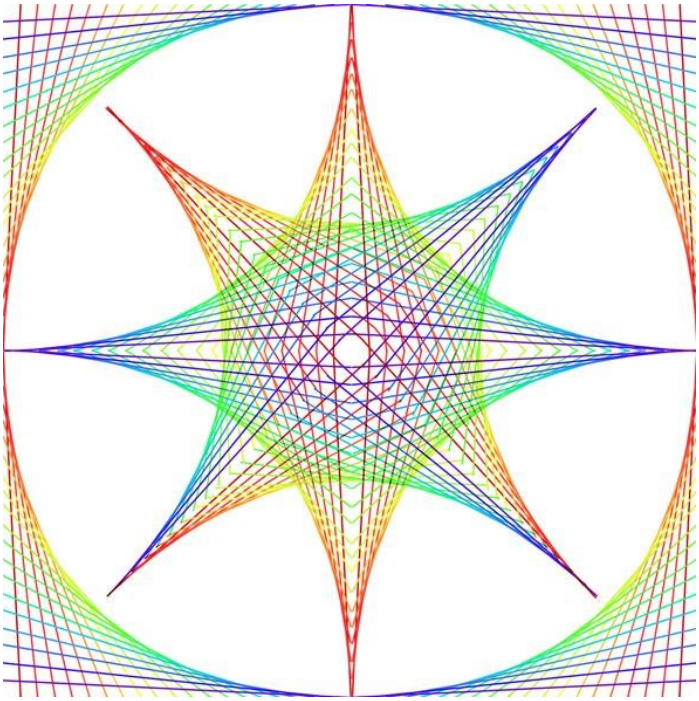
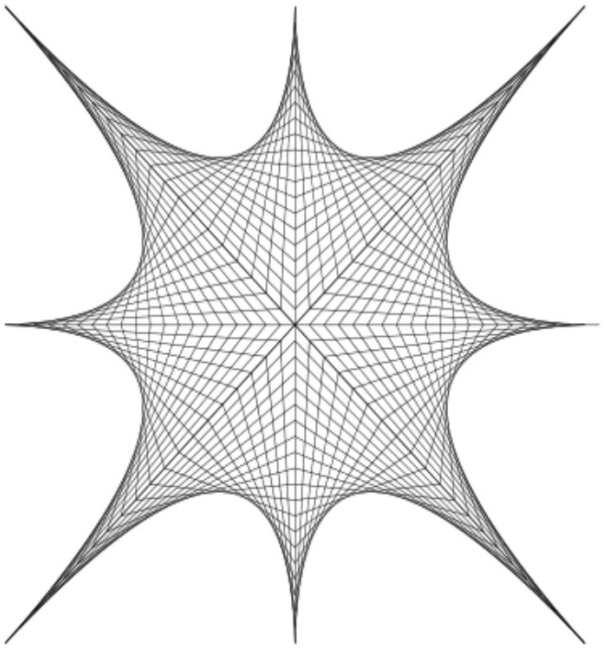
For groups:

- Have you **designed your poster** – which designs are to be included and how they will fit into your poster?
- Have you made a **list of all the jobs** to be done and who will do each one?
- Have you got all the **equipment** you need?
- What about the **timeline** – have you decided when the designs must be finished so that they can be glued onto the poster?

Follow up

For more information:
https://aiminghigh.aimssec.ac.za/years-7-to-9-constructions-with-lines/
https://aiminghigh.aimssec.ac.za/years-7-to-9-constructions-with-circles/
and https://aiminghigh.aimssec.ac.za/years-7-to-9-construct-circle-and-line-patterns/

See also https://aiminghigh.aimssec.ac.za/years-10-to-12-clock-arithmetic-and-envelopes/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6