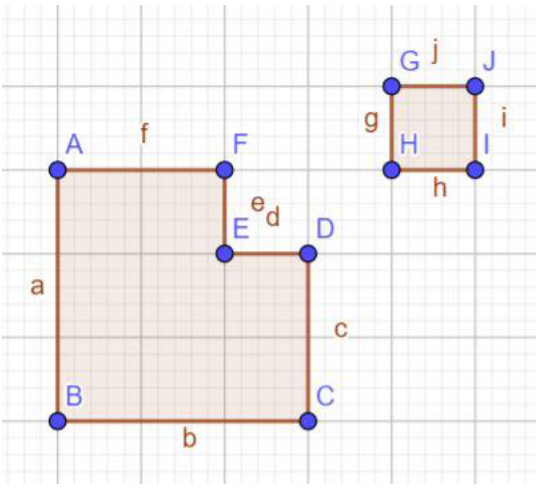
GRAPHING QUADRATICS

Graph (sketch) $y = x^2 - 2x - 1$.
What type of an equation is this?
Remember the general form of an equation of this nature: $y = ax^2 + bx + c$
When $a > 0$ the graph is ∪ − shaped and when $a < 0$ it is ∩ − shaped.
What is the situation in our problem?
Does the graph of $y = x^2 - 2x - 1$ intersect with the x-axis?
What is the value of y when this graph crosses the x-axis?
Can you solve $x^2 - 2x - 1 = 0$?



Think of the method of completing a square for solving quadratics.

Geometrically, you are thinking of adding shape GHIJ to the bigger shape to make it a complete square, then solving the problem.
What two values of x are solutions to $x^2 - 2x - 1 = 0$?

From $x^2 - 2x - 1 = (x - 1)^2 - 2 = 0$, notice that the graph is above the x-axis for $(x - 1)^2 \geq 0$ for all values of x. The minimum value of $(x - 1)^2$ is zero and this happens $x - 1 = 0$, that is $x = 1$. What is the value of y when $x = 1$?
Comment on this point.

Parabolas are symmetrical shapes, what is the line of symmetry of this graph?

At what point(s) does the graph cross the x-axis?

Taking into consideration all the special points you have obtained so far, draw and label axes and plot all the special points.

Using the special points sketch the graph of $y = x^2 - 2x - 1$.

HELP

It's important to find the coordinates of special points, that is points where the graph cuts the y–axis (when $x = 0$ which is easy to find) and where it cuts the x–axis (when y = 0 corresponding to the solution of an equation).

The other special points are where the graph 'changes direction', where it has a maximum or minimum value (which you can often spot from the symmetry of the graph) and also where the graph has an S-bend (called a point of inflection).

NEXT

It is necessary for you to learn to sketch graphs from your knowledge of the type of equation without first making a table of values and plotting points from the table. This is a skill you will need as you go on with your studies in mathematics and science, for example in topics like calculus and trigonometry.

(1) Complete the square in this quadratic expression: $f(x) = x^2 - 4x - 1$ and find the solution of $x^2 - 4x - 1 = 0$.
(2) Find the coordinates of the turning point of $f(x)$ and the point where the graph cuts the y-axis.
(3) Sketch the graph of the function without using a table of values or graphing software.
(4) If you have the software on your device you can use GeoGebra or some other graphing software to check your work.

Read the following if you need help.
To determine whether the graph is ∪ shaped with a minimum turning point or ∩ shaped with a maximum turning point try the following:

In our problem, the quadratic function is $y = f(x) = x^2 - 2x - 1 = (x - 1)^2 - 2$.
We saw that it is a minimum when $x = 1$ and $f(1) = 1^2 - 2(1) - 1 = -2$.
So (1 ; -2) is a **minimum point and the graph is ∪ shaped.**

The function $f(x) = -(x)^2 + 2x - 1 = -(x - 1)^2 \geq 0$ for all x and $f(x) = 0$ for $x = 1$.
So (1 ; 0) is a **maximum point and the graph is ∩ shaped.**

In general, the completed square form of a quadratic expression $f(x) = a(x + p)^2 + q$ can help sketching quadratic graphs if we understand that the turning point (-p, q) occurs when $x = -p$, and if we understand that we can decide whether it is a maximum or a minimum depending on the sign of the coefficient a.

If $x + p = 0$ and $a > 0$ then then (-p, q) is a minimum and the graph is ∪-shaped.
If $x + p = 0$ and $a < 0$ then then (-p, q) is a minimum and the graph is ∩-shaped.