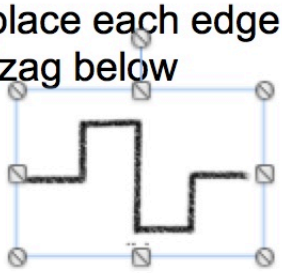
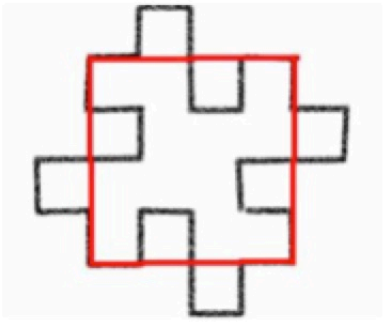
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

SQUAREFLAKE FRACTAL

1. Start with a square (stage 0)
2. Replace each edge with the zig-zag below

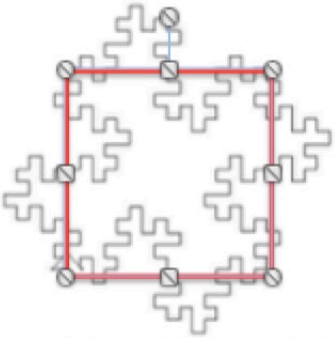




Squareflakes stage 0 (red) and stage 1

3. Repeat step 2 infinitely often.

What happens to the area?
How does the perimeter change?



Squareflake stage 2

1. If the red square has area 1 square unit. What is the area enclosed by the stage 1 squareflake?
2. What is the area enclosed by the stage 2 squareflake? What about the stages 3 and 4 squareflakes?
3. If replacing each line segment on the edge with the zig-zag is repeated infinitely often what is the area enclosed by the fractal squareflake?
4. What is the perimeter of the stage 0 squareflake?
5. The length of each line segment is exactly one quarter of the length of the line segment replaced by the zig-zag. What is the perimeter of the stage 1 squareflake?
6. What is the perimeter of the stage 2 squareflake?
7. What is the perimeter of the stage 3 squareflake?
8. What do you notice about the number pattern for the perimeter as the process is repeated?
9. If replacing each line segment on the edge with the zig-zag is repeated n times what is the perimeter of the stage n squareflake?
10. If replacing each line segment on the edge with the zig-zag is repeated infinitely often what is the area enclosed by the fractal squareflake?

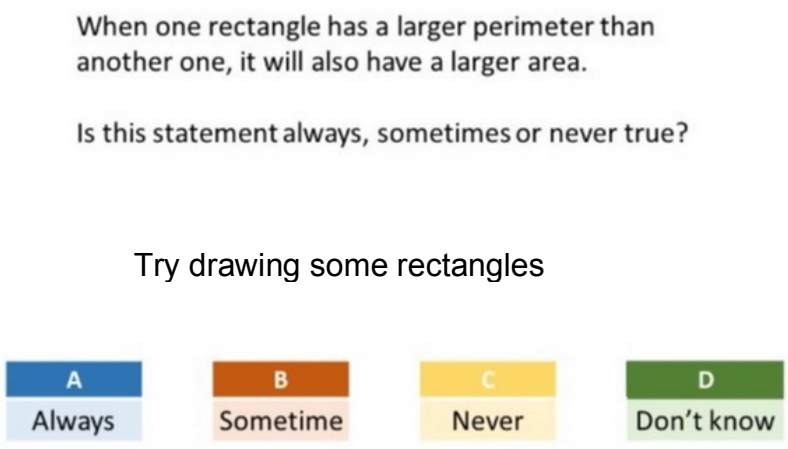
SOLUTION

1. The area enclosed by the stage 1 squareflake is 1 square unit because the areas of the indentations are equal to the areas of the protrusions. So the area remains constant.
2. The area enclosed by the stage 2 squareflake is 1 square unit. The areas for stages 3 and 4 are the same.
3. If replacing each line segment on the edge with the zig-zag is repeated infinitely often the area enclosed by the fractal squareflake is 1 square unit.
4. The perimeter of the stage 0 squareflake is 4 units.
5. The perimeter of the stage 1 squareflake is 8 units.
6. The perimeter of the stage 2 squareflake is 16 units.
7. The perimeter of the stage 3 squareflake is 32 units.
8. The number pattern for the perimeter as the process is repeated is the sequence 4, 8, 16, 32, … which is a geometric sequence with common ratio 2.
9. If replacing each line segment on the edge with the zig-zag is repeated n times the perimeter of the stage n squareflake is 2^{n+2}.
10. If replacing each line segment on the edge with the zig-zag is repeated infinitely often the area enclosed by the fractal squareflake is 1 square unit.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is B, see these examples:
1 by 20 rectangle has perimeter 42 and area 20
3 by 9 rectangle has perimeter 24 and area 27
4 by 6 rectangle has perimeter 20 and area 24
4 by 5 rectangle has perimeter 18 and area 20

Possible misconceptions

A. Students often believe that increasing perimeter will increase area or vice versa. Neither is true.

C. Shows a guess or poor understanding of area and perimeter.

D. Shows lack of confidence.

https://diagnosticquestions.com

Why do this activity?

This activity engages learners in thinking about numerical and geometrical patterns and sequences and helps to give them an appreciation of how the maths they are learning connects to real life applications to science, conservation issues, nature and art.

Learning objectives

- Number patterns and sequences.
- Geometric patterns and visualisation.
- Experience of imagining an infinite process that follows a simple set of rules.
- A deeper understanding of area and perimeter.
- Awareness of applications of the maths they learn to real world applications.

Generic competences

- to think critically/mathematically
- to reason logically
- to solve and interpret problems in a variety of situations
- to collaborate/work in a team
- to apply knowledge and skills

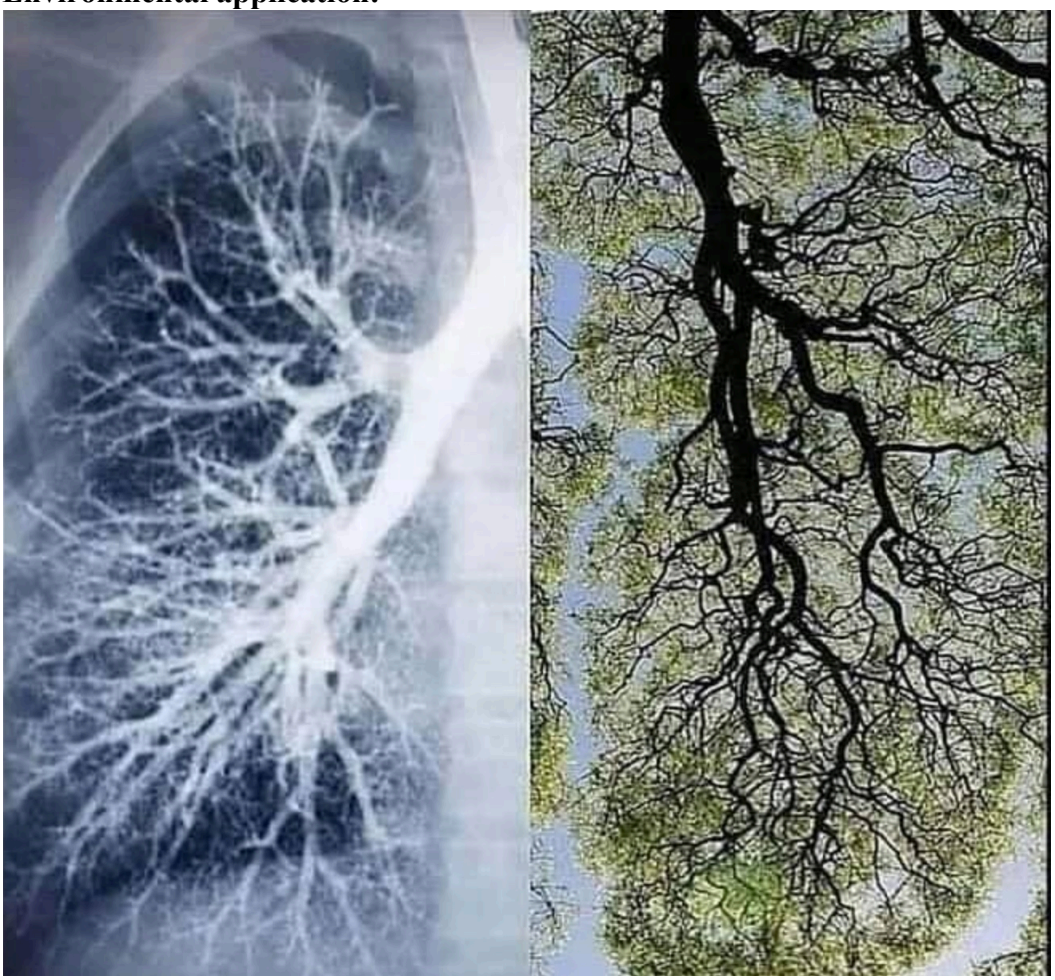
Suggestions for teaching

Start with the diagnostic quiz and if necessary review the concepts af area and perimeter.

Use the 1 – 2 – 4 – more teaching strategy.

1. Give students time to read the question and work **independently** on it.
2. Then ask them to work **in pairs**. You may want to pair students who are likely to struggle with more able students making it clear that both should be able to report on and explain the work done by the pair, giving reasons.
3. Later in the lesson ask **pairs to team up with another pair** and compare methods and answers.
4. Then have **a class discussion** in which learners present their work to the class. After each presentation, even if mistakes have been made, don't correct or add to explanations until after asking the rest of the class if they agree or would like to add anything.

Environmental application:



Explain how, like the squareflake fractal, trees and human lungs have a fractal structure designed to have a large surface area and a limited volume.

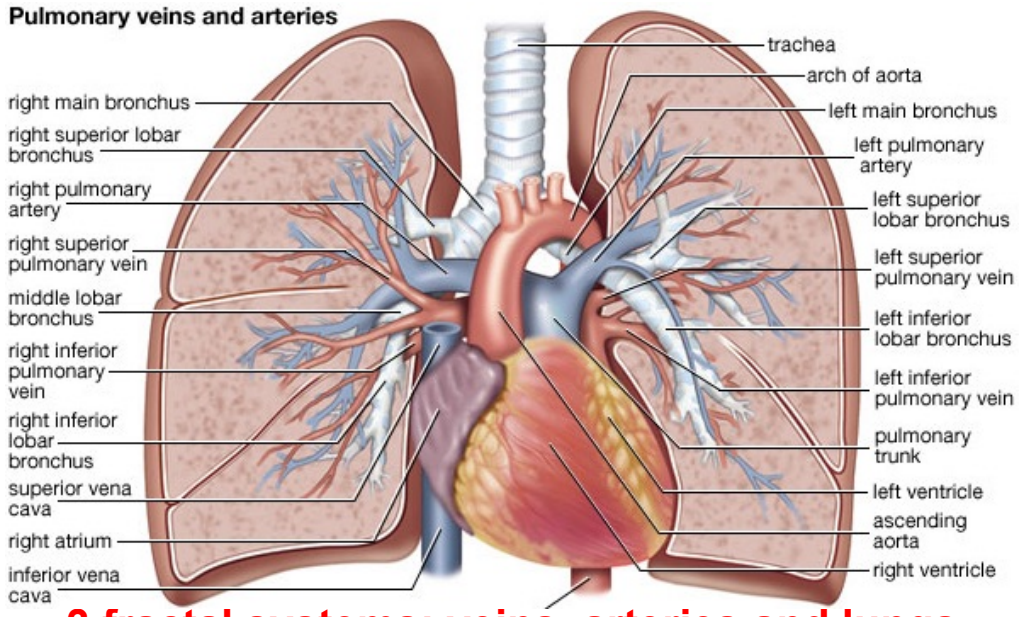
Explain that trees absorb carbon dioxide and emit oxygen so they are vital to the environment.

People breathe in oxygen which they need for life and they breathe out carbon dioxide.

Evolutionary development has produced these fractal forms to maximise the surface area for the exchange of gases.

Application to human biology that applies to each learner and is important for each of them

Fractals in Nature



3 fractal systems: veins, arteries and lungs.

These structures, with limited volumes and very large surface areas, provide the surfaces for the exchange of oxygen from the lungs to the bloodstream and carbon dioxide back into the lungs.

Explain how, like the squareflake fractal, human lungs and vascular systems (both arteries and veins) have a fractal structure designed to have a large surface area and a limited volume.

Evolutionary development has produced these fractal forms in mammals to maximise the surface area for the exchange of gases.

Key questions

- What do you notice?
- Can you explain why that happens?
- What changes the area when the edge is replaced by the zig-zag?
- Does that square going inwards increase or decrease the area?
- Does that square going outwards increase or decrease the area?
- What changes the perimeter when the edge is replaced by the zig-zag?
- What can you tell me about that number pattern?
- Is that increasing or decreasing or staying the same?

Possible extension

https://aiminghigh.aimssec.ac.za/years-4-to-10-make-a-von-koch-poster/

Possible support

Provide squared paper and suggest that the student draws the stage 1 squareflake starting from a stage 0 square measuring 4 units by 4 units.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6