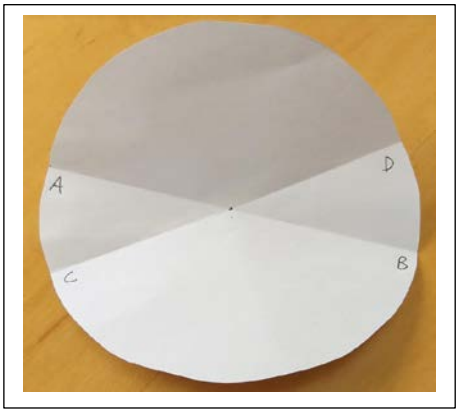
TITLE: Investigating circle theorems

Activity 1: Finding the centre of the circle



You will need a cut-out paper circle (about 12cm diameter).

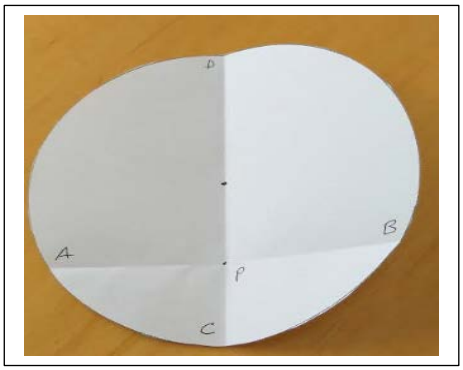
(i) Use a circle that does not show the centre.

(ii) Fold your circle along a diameter. Mark the ends A and B.

(iii) Now unfold your circle and then fold again along a diameter and mark the ends C and D. Also mark the point where AB and CD intersect as O. Why is this the centre?

(iv) Is there another way of finding the centre?

Activity 2: Looking at the perpendicular bisector of a chord.



You will need a cut-out paper circle with centre O marked (about 12cm diameter).

(i) Fold over a segment of the circle to make a chord. Mark the ends A and B.

(ii) Fold the circle so that A and B come together. Unfold and mark the points where this line cuts the circle as C and D.

(iii) If AB and CD intersect at P, what do you know about AP and PB? What about angle CPA?

(iv) How is CD related to AB?

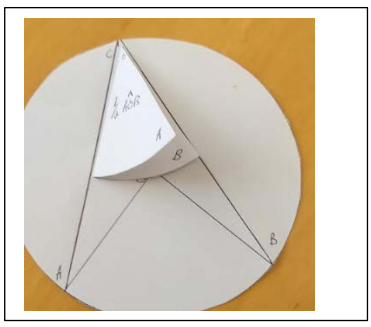
(v) Is CD a diameter of the circle?

Activity 3: Angles at the centre and circumference subtended by the same arc/chord.

- Each person needs a cut-out paper circle with centre O marked (about 12cm diameter).
- Work in pairs if possible

(i) You and your partner should have identical circles by placing the two circles on top of one another and mark identical points. Mark on both circles two points A and B one side of O and a point P the other side of O.

(ii) One of you folds along the chords AP and BP, the other along AO and BO.



(iii) By folding again so that AO and BO are aligned, make the angle ½ AOB.

(iv) Compare this angle ½ AOB with ∠APB.

In question(iii) above, it may be easier to cut out ∠AOB.

SOLUTIONS

Activity 1: O is the centre because the intersection point of the two diameters bisect each diameter therefore OA, OB, OC and OD are all radii.

Another way is to fold a semi-circle again into a quarter circle to get the centre.

Activity 2: CD is the perpendicular bisector of a chord AB

CD is the diameter as it goes through the centre.

Activity 3: The angle at the centre is twice the size of the angle at the circumference - if they are both subtended by the same arc or chord.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write or show the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

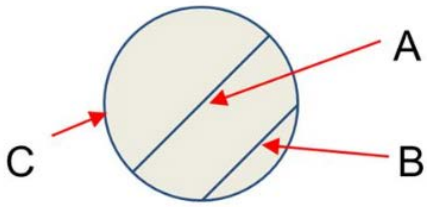
Which of these is not true of the circle?

A A is the diameter

B B is a chord

C C is the circumference

D A is the radius



2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

D. is the correct answer.

Common Misconceptions

A Is the diameter – the question was which is not true.

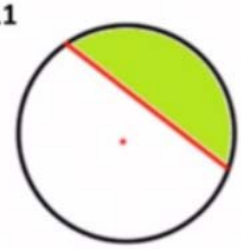
B Is the chord - the question was which is not true

C Is the Circumference - **the question was which is not true**

https://diagnosticquestions.com

Other possible questions also taken from https://diagnosticquestions.com **could be:**

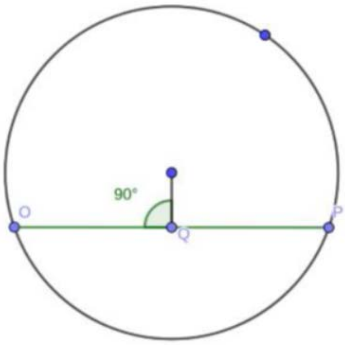
Question 11



What is the name of the circle part in green?

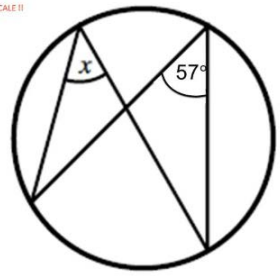
A) Tangent
B) Sector
C) Segment
D) Chord

The line segment QP is 7cm. What is the length of the chord OP?



A 7cm | B 14cm | C 49cm | D Don't know

!! NOT TO SCALE !!



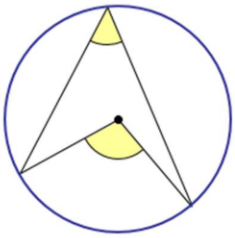
What is the size of angle x?

A 125°

B 57°

C 26.5°

D Not enough information



What is the correct name of this circle theorem?

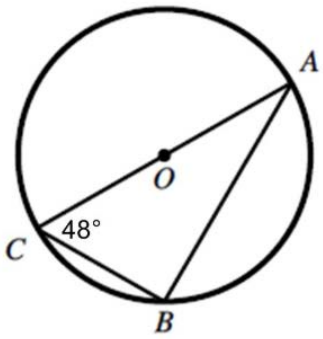
A 'Opposite angles in a cyclic quadrilateral sum to 180°'

B 'The angle at the centre is twice the angle at the circumference'

C 'Angles in the same segment are equal'

D 'Alternate segment theorem'

!! NOT TO SCALE !!



Angle ACB = 48°

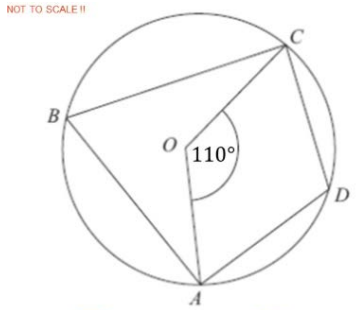
What is the size of angle CAB?

A 52°

B 90°

C 48°

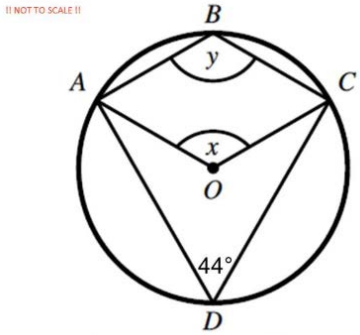
D 42°



Angle AOC = 110°

What is the size of angle CDA?

A	B	C	D
125°	110°	55°	70°



Angle ADC = 44°

What is the size of angle x?

A	B	C	D
136°	22°	88°	44°

Why do this activity?

These paper folding activities provides experiences of discovering the properties of circles theorems. Folding cut-out paper circles does not prove the theorems, but ideas for the formal proofs are given.

Intended learning outcomes

- Visualise the positions and relationships between the lines and angles in the circle theorems. These activities illustrate the properties stated in the theorems and the activities lead to visualization that helps learners;
- Gain a deeper understanding of the geometrical properties;
- Build their vocabulary about circle theorems;

Suggestions for teaching

Similar paper folding activities can easily be devised that illustrate Theorem 6: the tangents drawn to a circle from a point outside the circle are equal in length and Theorem 7: the angle between a chord and the tangent to the circle at the endpoint of the cord is equal to the angle subtended by that chord in the alternate segment.

Key questions

Can you explain what you have done so far?
Do you think that this would work with other sketches or drawings?
Did you use any new words today? What do they mean? How would you spell them?

Possible extension

When the angle subtended at the centre is on the diameter itself so it is 180°, then since half of this is 90°, we should expect the angle at the circumference to be 90°.

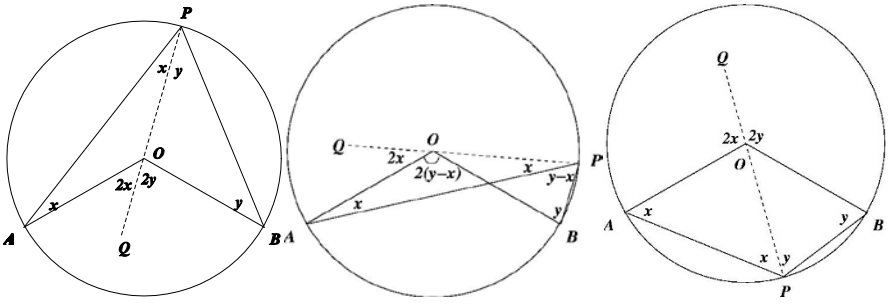
Extension activity: The opposite angles of a cyclic quadrilateral are supplementary.

- Each person needs a cut-out paper circle with centre O marked (about 12cm diameter).
- Work in pairs.

(i) Both partners need to mark 4 points on the circumference so they have identical cyclic quadrilaterals, ABCD.
(ii) Fold along the chords AB, BC, CD, DA.
(iii) Using one angle from each quadrilateral, place ∠A adjacent to ∠C. What do you notice?
(iv) Using one angle from each quadrilateral, place ∠B adjacent to ∠D. What do you notice?

After the facts and concepts have been established the theorems can be proved.

Theorem 3 The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference of the circle.



Proof Three diagrams are necessary here to show the possible configurations but the proof is the same in each case.
As the radii are equal and the base angles of an isosceles triangle are equal:
∠OAP =∠OPA = x and ∠OBP =∠OPB = y.

As the exterior angle of a triangle equals the sum of the two interior opposite angles:
∠AOQ = 2x and ∠BOQ = 2y.

Hence ∠AOB = 2∠APB.

Possible support

The teaching strategy should be to guide the learners through these practical investigations, encouraging them to notice certain properties and to make conjectures about what might be true in general. Another way to gather evidence to support the conjectures is for the learners to draw diagrams and to measure lengths and angles. Teachers may give the drawing and measurement reinforcement activities for the other theorems as extra support to learners who struggle with this topic. It is important to avoid the misconception that observation of even a large number of cases supporting the conjecture proves it is true. The supporting evidence does not prove that the conjecture is true in all cases and a formal proof is needed.

Another way to reinforce the benefits of visualization, and to help learners to remember these concepts, is for the class to make a large poster of the seven theorems for the classroom wall.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. **Note:** The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase **Age 5 to 9**	**Upper Primary** **Age 9 to 11**	**Lower Secondary** **Age 11 to 14**	**Upper Secondary** **Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**