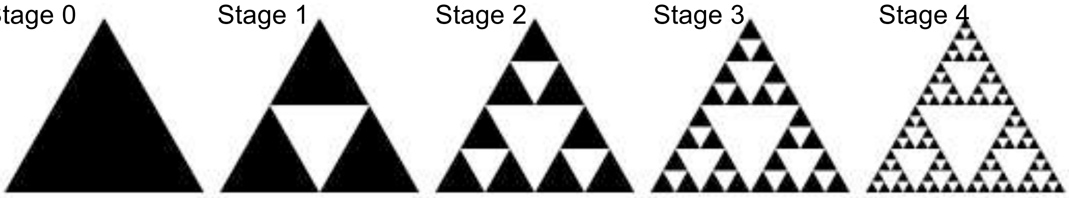
FANTASTIC FRACTALS FOLLOW UP

More questions and answers on Number Patterns, Lengths, Areas and Fractions

Notice that the Sierpinski triangles contain copies of themselves at a smaller scale.

Take a triangle, reduce it so that the lengths of the edges are halved, make two copies and arrange the three triangles with two side by side and one on top as shown. Repeat this process on and on forever to form a fractal. ***Alternatively*** join the midpoints of the edges of the black triangles and remove the inverted triangles at each stage leaving a white space.

Stage 0 Stage 1 Stage 2 Stage 3 Stage 4



Here are some notes to help teachers to use this as a starting point for mathematical activities and investigations. There is scope at all levels. Young learners can develop language and visualization skills by just describing what they see. Learners can investigate number patterns, spot similar triangles and work out scale factors, investigate areas and the numbers of triangles in the pattern and even sum geometric series.

Some questions for a mathematics lesson

This activity should help learners to appreciate how fractals are self similar repeating at different scales.The activity produced a fractal pattern growing outwards getting bigger. A true fractal is the result of a similar process that repeats infinitely by making triangular holes (like the white triangles) at smaller and smaller scales inside itself. As humans we cannot actually complete a task that goes on forever, or see the infinitesimally small structure inside the fractal, but we can try to imagine it.

1. How many triangles are shaded at each stage of the Sierpinski Gasket?
2. At each stage compare the smallest shaded triangle to the whole gasket.
   - compare the lengths of the edges
   - compare the areas
3. What fraction of the area of the gasket is shaded at each stage? What fraction is unshaded?

In answering these questions what number patterns do you notice?
What happens as the process goes on indefinitely?

STAGE	0	1	2	3	4	5	n
NUMBER OF TRIANGLES	1	3	3^2=9	3^3=27	3^4=81	3^5=243	3^n
EDGE LENGTH Now keeping the outer boundary the same and halving the length of the edge of each triangle.	1	½	$(½)^2$	$(½)^3$	$(½)^4$	$(½)^5$ =0.03125	$(½)^n$
AREAS OF EACH TRIANGLE SHADED COMPARED TO THE WHOLE GASKET A= ¼ √3	A	¼ A	$(¼)^2$A	$(¼)^3$A	$(¼)^4$A	$(¼)^5$A	$(¼)^n$A
FRACTION SHADED	All	¾	$(¾)^2$	$(¾)^3$	$(¾)^4$	$(¾)^5$	$(¾)^n$
FRACTION UNSHADED	0	¼	7/16	37/64	175/256	781/1029	$1/(¾)^n$