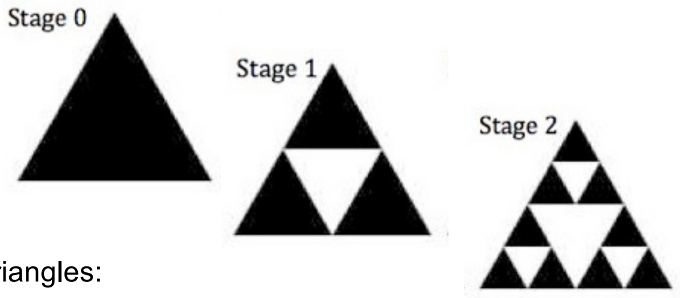
FANTASTIC FRACTALS **Short Explanation and History of Fractals**

What is a fractal?

"Great fleas have little fleas upon their backs to bite `em,
and little fleas have lesser fleas, and so ad infinitum.
And the great fleas themselves, in turn, have greater fleas to go on,
while these again have greater still, and greater still, and so on."
Augustus De Morgan (1806 – 1871) Adaptation of poem by Jonathan Swift (1667 – 1745)

Roughly speaking, a fractal is a shape that contains, within itself, smaller and smaller copies of itself. These smaller copies may be rotated and moved to different parts of the shape but they must always be similar copies of the shape itself.



Let us construct a fractal. We start with a triangle:

Then remove a smaller `upside down' triangle within it:

Then remove three more such triangles within the outer triangles:

Now suppose that we continue this process for ever. Of course, we cannot draw this resulting shape (in a finite time), but we can imagine this fractal shape which is called the **Sierpinski gasket** after the Polish mathematician Waclaw Sierpinski (1882 – 1969).

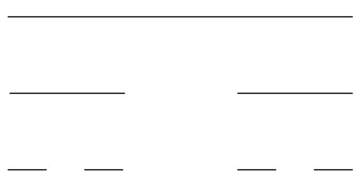
Fractal like forms occur naturally in nature for example trees, lungs, the vascular system, broccoli, coastlines etc. The efficiency of the forms often seems to depend on their fractal-like structure. For example the exchange of gases across the surface of lungs is optimised by a very large 'fractal' surface area despite the lung's limited volume.

Once modern computers were available, they could be used to draw fractals, and it was realised that fractals could be used to represent features in real life, for example, mountain ranges, clouds, rivers, and so on. As a striking example of this, we have the **Barnsley fern**, not a real fern, but a computer generated fractal image of a fern that can be made to look more realistic by introducing small random variations,

With the high speed of modern computers, it takes much less memory to generate a picture, say of a mountain range, by using fractals than it does to store the entire picture, so fractals are often used in computer graphics. Fractals arise in modern, advanced mathematics, they are used in computer graphics, and they provide us with some really beautiful abstract pictures!

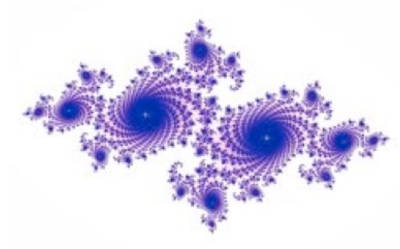
A brief history of fractals

Benoit Mandelbrot (1924 – 2010) popularised fractals in the 1980's, when computer graphics first became readily available, but fractals were really part of mainstream mathematical thought long before this. For example, Georg Cantor (1845 – 1918), who laid the foundation for infinite processes in mathematics, introduced what is now known as the **Cantor middle-third set** which is a fractal that can be described by the sequence of pictures as follows.

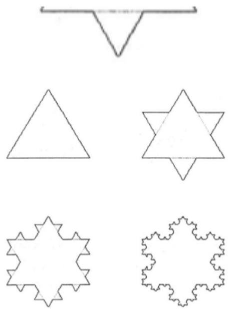


We start with a segment; then remove its middle third; and then the next two middle thirds; and so on. The early steps in this process are illustrated here Again, we must imagine the outcome; known as **Cantor dust**; for example, the endpoints of the initial segment (and many other points too) are part of the `dust', but the dust does not contain any segment nor any isolated points!

Some of the most beautiful fractals of all arise from a process known as **iteration** (that is, repeating a rule over and over again). This happens in the subject known as complex dynamics that started its active life around 1915 with the works of the French mathematicians Gaston Julia (1893 – 1978) and Pierre Fatou (1878 – 1929). Here is a picture of one of the many intricate fractals (known as **Julia sets**) that arise in this subject.



Here is another way to produce fractals: we begin with a curve made up of straight line segments and then replace each segment by another curve which is slightly more complicated, and repeat this process again and again (iterate the process). For example, at each iteration every line segment is replaced by a zig-zag curve (as shown here) made up of 4 pieces with each piece one third of the length of the line segment it replaces. If we start with a triangle and replace each segment by the curve we obtain the **Von Koch snowflake curve** devised by the Swedish mathematician Helge von Koch (1870 – 1924). The length of the curve tends to infinity but the area enclosed by the curve finite, a 2-dimensional example rather like the example of mammalian lungs described above.



Fractals occur naturally in many branches of mathematics, including the theory of numbers, dynamical systems and probability theory. They were studied long before computer graphics became available, so in

earlier times, mathematicians understood much about their structure, but without ever being able to see the real complexity of these fascinating objects.

In 1919 Felix Hausdorff (1868 – 1942) introduced a new way to measure the size of a set. We are all used to the idea of a line being one-dimensional, (and talking about its length), a disc being two-dimensional (and speaking of its area), and a solid ball being three-dimensional (and speaking of its volume). However, Hausdorff went far beyond this and introduced the idea of ***measuring a fractal in any dimension at all***. For example, some curves, if they wiggle infinitely often, have a dimension greater than 1 like the snowflake curve above (dimension approximately 1.26). The Peano Space Filling curve has dimension 2.

References

Jonathan Swift 1667 - 1745	Georg Cantor 1845 – 1918	Felix Hausdorff 1868 – 1942	Helge von Koch 1870 – 1924	Pierre Fatou 1878 – 1929
Waclaw Sierpinski 1882 – 1969	Gaston Julia 1893 – 1978	Benoit Mandelbrot 1924 – 2010	Interactive Mandelbrot set	Explanation of Mandelbrot set

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