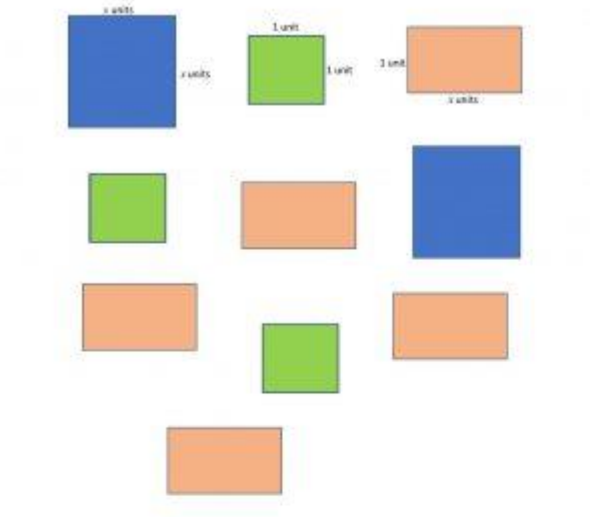
NOTES FOR TEACHERS

Title: ALGEBRAREA. Product of two brackets and area



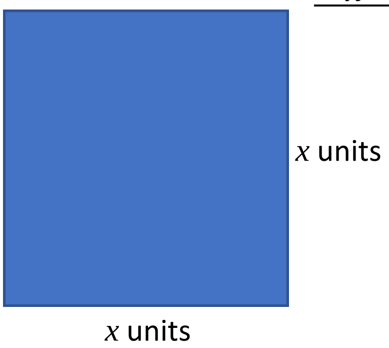
Group work:

1. Identify geometrical shapes from the pieces given to you?
2. Do you notice any pieces which are identical? If so, name them with the help of their colours.
3. Using all your pieces at a time, build one geometrical shape. Take note of the shape. Draw it in your notebook. Can you try to build another geometrical shape using the same pieces? How many more possible different shapes can you obtain? Try to discover as many as you can.
4. Of the three **different** pieces you are given, find their areas, in square units.
5. Find the areas of the different BIG shapes you just obtained. How are you calculating those areas? What do you notice about the areas of these geometrical shapes?
6. Of your BIG shapes, how else can you find their areas? Identify a shape whose area can be found in a different way? What special name is given to this shape? How can you find the area of this shape differently?
7. What mathematical relationship can you come up with from the area you just obtained and the area you calculated before, for this shape? How would you show that the two areas are in fact the same (since we are looking at the same shape)?
8. If (2 + 3) is multiplied by (4 + 1), show how you would workout (2 + 3) (4 + 1). Try different ways of coming up with the same answer.
9. Use your newly developed method to simplify the mathematical relationship you obtained above.

SOLUTION

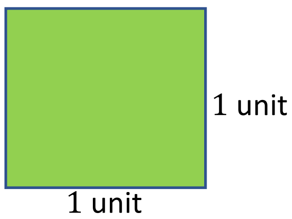
1. The geometrical shapes which can be identified are squares and rectangles (2D shapes):
   - 3 small squares (green)
   - 2 BIG squares (blue)
   - 5 rectangles (brown)
2. Yes, here are identical shapes (pieces). The blue squares are identical, the green squares are also identical, and the rectangles are identical. These identical shapes are said to be **congruent** (Teachers can first explore to find out if learners are familiar with the terminology).
3. Import the shapes which form part of the solution
4. Blue square: Area = x units $\times$ x units

   = $\underline{x^2 \text{ units}^2}$



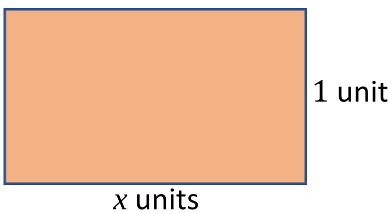
Green square: Area = 1 unit $\times$ 1 unit

= $\underline{1 \text{ unit}^2}$

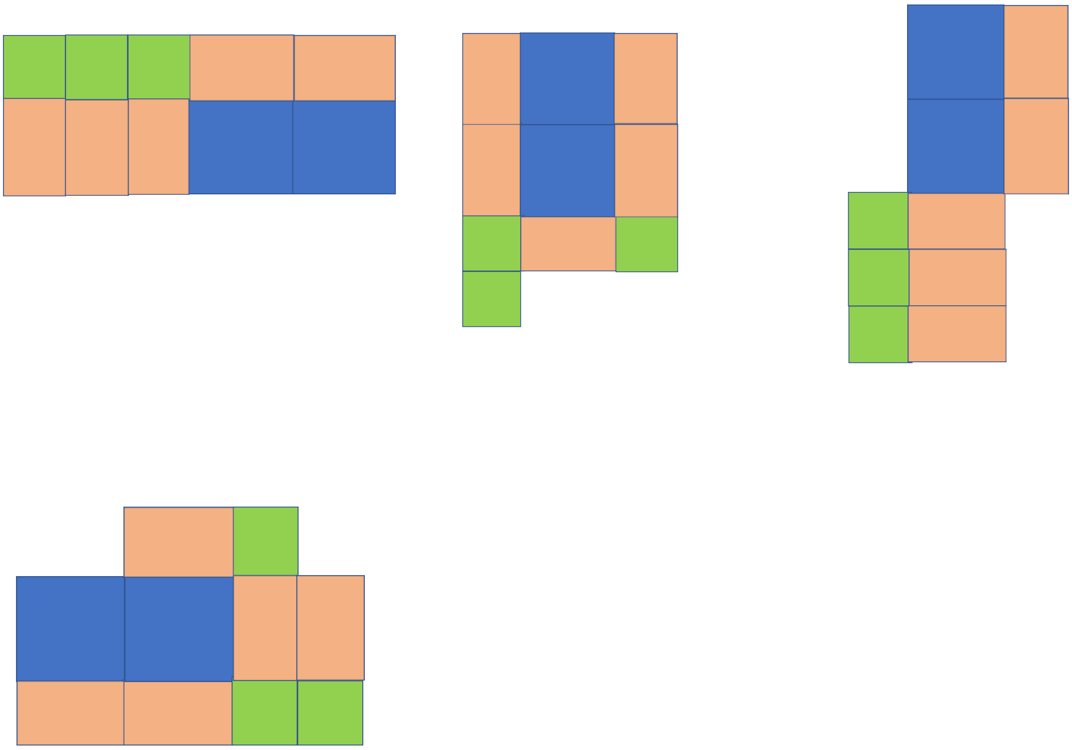


Brown rectangle: Area = x units $\times$ 1 units

= $\underline{x \text{ units}^2}$



5. Some images of possible combinations:



Let learners explore more options; there are still more combinations they can come up with. To find areas of these shapes, adding areas of the individual pieces can be a common strategy of getting the solution, for most learners.

Each shape should give us the following results:

Area $= x^2 + x^2 + x + x + x + x + x + 1 + 1 + 1$ units^2

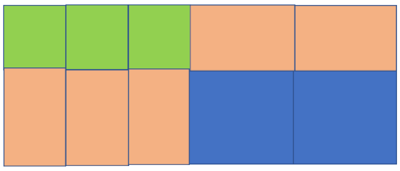
$\underline{= 2x^2 + 5x + 3 \text{ units}^2}$

Ideally, most learners should notice that ALL the shapes have the **same area**.

There is **conservation** of area in this case; logically, we are using the same pieces whose area remains the same.

6. Most of your learners should find it easy to notice that the **rectangle**, shown below can be used to calculate area using length and width of the shape. Look:

Area = Length x Width
= $(2x + 3)(x + 1)$



7. From question 5. above, the area was found to be: Area = $2x^2 + 5x + 3$

∴ From question 6 above, we can safely say: $\mathbf{(2x + 3)(x + 1) = 2x^2 + 5x + 3}$ which is the required mathematical relationship.

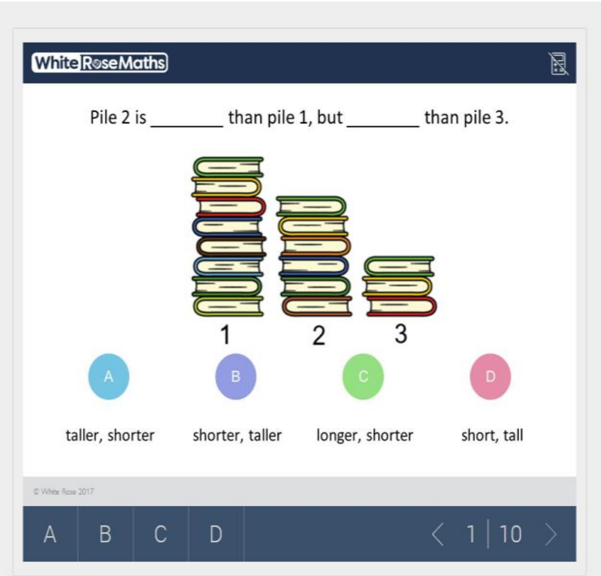
8. $(2 + 3)(4 + 1) = (5)(5) = 25$
$(2 + 3)(4 + 1) = 2(4 + 1) + 3(4 + 1) = 2(5) + 3(5) = 10 + 15 = 25$

9. $(\mathbf{2x} + \mathbf{3})(x + 1) = \mathbf{2x}(x + 1) + \mathbf{3}(x + 1)$
$= 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

<u>$\therefore \mathbf{(2x + 3)(x + 1) = 2x^2 + 5x + 3}$</u>

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is B and possible misconceptions:

A. The student may have compared pile 2 and 3 for the first comparison.
B. Correct answer. First focus on pile 2 and 1, pile 2 is shorter. Secondly focus on pile 2 and 3, pile 2 is taller
C. The student may be confusing length with height
D. The student may be confusing pile 1 and 3 in comparison with pile 2

https://diagnosticquestions.com

Why do this activity?

This activity is designed to develop conceptual understanding of the product of two binomials, connected to the area problems in some common 2D shapes like squares and rectangles. It links algebra with area problems. Learners develop skills of manipulating a product of two binomials to obtain a quadratic expression; work which prepares them for solution of quadratic equations which follow in subsequent lesson activities. The activity consolidates learner's knowledge on 'transformation' and 'tessellation' in geometry, especially when they are playing with the pieces to obtain different 2D shapes. For those who will go into industry, construction, for example, the work will resonate with problems of 'tiling floors'. The activity is a preparation for work which involves the application of quadratic equations in solving area problems in 2D shapes; we focus on the connections between areas of simple geometrical shapes like squares and rectangles and product of two binomials.
This activity becomes useful as prior knowledge for learners when they get into topics like 'nets' of 3D shapes. The connection between geometry and the algebraic terrain can not be over emphasised.

Learning objectives

- Construct quadratic expressions through area problems in 2D shapes
- Calculate areas of squares and rectangles using dimensions given in algebraic form.
- Establish a mathematical relationship between area (algebraic expression) and product of two binomials
- Find the product of two brackets.

Generic competences

- develop deep conceptual understanding in order to make sense of Mathematics (construction of algebraic expressions by the product of two binomials)
- develop algebraic manipulative skills that recognise the equivalence between different representations of the same relationship.

Suggestions for teaching

- Try to make the lesson as practical and engaging as possible. Learners can either work individually in their exercise books or do it in small groups of two to four students per group.

Key questions

- Can you subdivide the geometric shapes into simpler subunits? The sum of the areas of such units give rise to the integral area of the geometric shape in question.
- Can you apply the distributive law to multiply a binomial by another binomial?

Possible extension *(Note – this is provision for the abler students)*

- Encourage learners to workout more problems involving expansion of brackets of the form: $(a+b)(c+d)=ac+ad+bc+bd$, say:
  - $(3x+2)(2x+3)$
  - $(2x-3)(2x+4)$
  - $(3x-2)(4x-3)$

Possible support *(Note – this is provision for students who have difficulties)*

- Assist learners with difficulties to realise the importance of subdividing the geometrical shapes into simpler units and then finding areas of these. More worksheets with ready drawn geometrical shapes may help.
- Simplify simple numerical expressions like: $(3+4)(2+5)$; connect this to $(x+4)(x+2)$

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6