Expansion of brackets. Area problems leading to solution of quadratic equations.

Find the area of a rectangle whose dimensions are: $(2x + 3)$ by $(x + 1)$.

Sketch and label the rectangle whose dimensions are: $(2x + 3)$ by $(x + 1)$

How do we calculate the area of a rectangle?

Can partitioning (subdividing) the rectangle into simpler units help us? Find the areas of these subunits.

What is the total area obtained from the subdivided rectangle?

Can the rectangle be further subdivided into simpler squares and rectangles? Try this out.

What do you obtain as the area of the original rectangle?

How else would you have obtained this area without subdividing the rectangle?

Can you multiply $(2x + 3)$ by $(x + 1)$?

Does your method yield the same results?

If the numerical value of the area of the rectangle is given as 21 square units, form an equation in terms of x. Hence solve it to find the dimensions of the rectangle.

Does the area of the rectangle equal 21 square units? Check the validity of your solutions.

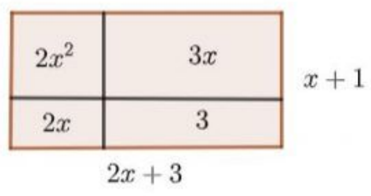
On a Cartesian plane, sketch the graphs of each of the functions you formed.

What are the x-intercepts of the graph? What do you notice between these values and the solutions you obtained above?

Using the same method or otherwise, find the area of a rectangle whose dimensions are: $(2x + 1)(x + 2)$.

If the numerical value of the area of this rectangle is 5 square units. Find the dimensions of the rectangle.

SOLUTION



Area = $2x^2 + 3x + 2x + 3$

$= 2x^2 + 5x + 3$

Area of a rectangle = Length x Width

$= (2x + 3)(x + 1) = 2x(x + 1) + 3(x + 1)$

$= 2x^2 + 2x + 3x + 3 \quad = 2x^2 + 5x + 3$

If the area of the rectangle is 21 square units

$2x^2 + 5x + 3 = 21$

$2x^2 + 5x - 18 = 0$

$(2x + 9)(x - 2) = 0$

$x = -4.5$ or $x = 2$

Discuding $x = -4.5$ and considering $x = 2$

The dimensions are: length $= 2(2) + 3$ Width $= 2 + 1$

$= 7$ $= 3$

∴ The length of the rectangle is 7 units and the width is 3 units.

Hence the area is 7(3) square units = 21 square units

From $(2x + 1)(x + 2) = 2x^2 + 5x + 2$

If the area is 5 square units ∴ $2x^2 + 5x + 2 = 5$

$\Rightarrow 2x^2 + 5x - 3 = 0 \Rightarrow (2x - 1)(x + 3) = 0 \Rightarrow x = 0.5$ *or* $x = -3$

∴ The dimensions are:

length $= 2(0.5) + 1$ Width $= 0.5 + 2$

$= 1 + 1$ $= 2.5$ units

$= 2$ units

Hence the area is 2(2.5) square units = 5 square units

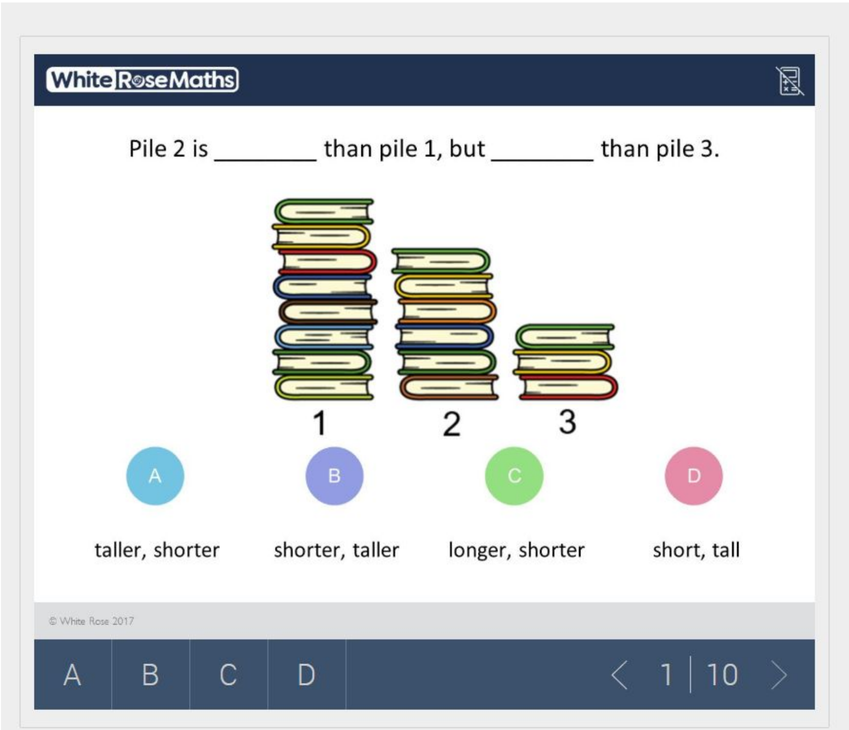
NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.

2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is B and possible misconceptions:

A. The student may have compared pile 2 and 3 for the first comparison.

B. Correct answer. First focus on pile 2 and 1, pile 2 is shorter. Secondly focus on pile 2 and 3, pile 2 is taller

C. The student may be confusing length with height

D. The student may be confusing pile 1 and 3 in comparison with pile 2

https://diagnosticquestions.com

Why do this activity?

This activity involves the application of quadratic equations in solving area problems in 2D shapes; we focus on the connections between areas of simple geometrical shapes like squares and rectangles and solution of quadratic equations. In this activity learners develop skills in constructing quadratic equations given the linear dimensions in algebraic form and the known area values. They develop skills of manipulating a product of two binomials to obtain a quadratic expression, hence, evaluate this expression through substitution to obtain a numerical value of it.The quadratic equations are solved using factorisation.

Learning objectives

- ❖ use the distributive law to multiply two binomials (product of two binomials)
- ❖ apply algebra in finding areas of 2D shapes given linear dimensions in algebraic form
- ❖ determine the numerical value of an algebraic expression through substitution
- ❖ solve quadratic equations using factorisation

Generic competences

- ❖ develop deep conceptual understanding in order to make sense of Mathematics (application of quadratics in solving area problems)
- ❖ develop algebraic manipulative skills that recognise the equivalence between different representations of the same relationship.

Suggestions for teaching

- ❖ Try to make the lesson as practical and engaging as possible. Learners can either work individual in their exercise books or do it in small groups of two to four students per group.

Key questions

- ❖ Can you subdivide the geometric shapes into simpler subunits? The sum of the areas of such units give rise to the integral area of the geometric shape in question.
- ❖ Can you apply the distributive law to multiply a binomial by another binomial?
- ❖ Can you solve a quadratic equation of the form $ax^2 + bx + c = 0$ through factorisation?

Possible extension *(Note – this is provision for the abler students)*

- ❖ Encourage learners to try and plot or sketch the graphs of quadratic equations, obtain the values of the the x-intercepts (and y-intercepts). What do they notice about the values of the x-intercepts and the solution of the quadratic equations through factoring? Notice that this helps learners to establish connections the factorisation method and the graphical method of solving quadratics.

Possible support *(Note – this is provision for students who have difficulties)*

- Assist learners with difficulties to realise the importance of subdividing the geometrical shapes into simpler units and then finding areas of these. More worksheets with ready drawn geometrical shapes may help.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**