TABLE OF TABLES

1	2	3	4	5	6	7	8	9	10		
2	4	6	8	10	12	14	16	18	20		
3	6	9	12	15	18	21	24	27	30		
4	8	12	16	20	24	28	32	36	40		
5	10	15	20	25	30	35	40	45	50		
6	12	18	24	30	36	42	48	54	60		
7	14	21	28	35	42	49	56	63	70		
8	16	24	32	40	48	56	64	72	80		
9	18	27	36	45	54	63	72	81	90		
10	20	30	40	50	60	70	80	90	100		

What patterns do you notice in this table?

Continue the patterns to fill the empty squares.

Write a list describing all the patterns that you see.

What do you notice about the numbers on squares of the same colour?

Would the patterns continue for ever to the right and downwards?

What can you deduce from the patterns that you see?

What do these patterns tell you about the sum of consecutive odd numbers and can you prove this result in general?

SOLUTION

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The rows give sequences of multiples of 1, 2, 3, …so the nth row is n, 2n, 3n …

The columns give sequences of multiples of 1, 2, 3, …so the nth column is n, 2n, 3n …

The diagonal from top left to bottom right gives the sequence of square numbers.

The table has reflective symmetry in the diagonal from top left to bottom right.

Numbers on squares of the same colour are all multiples of the same number and the nth set contains 2n – 1 numbers.

The patterns can be continued indefinitely to the right and downwards.

PROOF THAT THE SUM OF CONSECUTIVE ODD NUMBERS IS SQUARE NUMBER

Notice that 1 + 3 = 4; 1 + 3 + 5 = 9; 1 + 3 + 5 + 7 = 16; 1 + 3 + 5 + 7 + 9 = 25 and so it seems that the sum of consecutive odd numbers 1 + 3 + 5 + 7 + … + (2n-1) + … is the square number n^2.

To prove this result note that the nth odd number in the sequences is (2n – 1).

If the result that we are trying to prove is true then the sum of the first (n – 1) odd numbers is $(n-1)^2$.

Adding the next odd number gives the sum of the first n odd numbers:

$(n-1)^2 + 2n - 1 = n^2 - 2n + 1 + 2n - 1 = n^2$.

This shows that if the statement is true for (n – 1) then it follows that it is also true for n.

The table shows that the statement is true for n = 1, 2, 3 … 10 so it follows that it must be tru for 11 and then for 12 and so on for all the natural numbers.

This method of proof is called proof by Mathematical Induction.

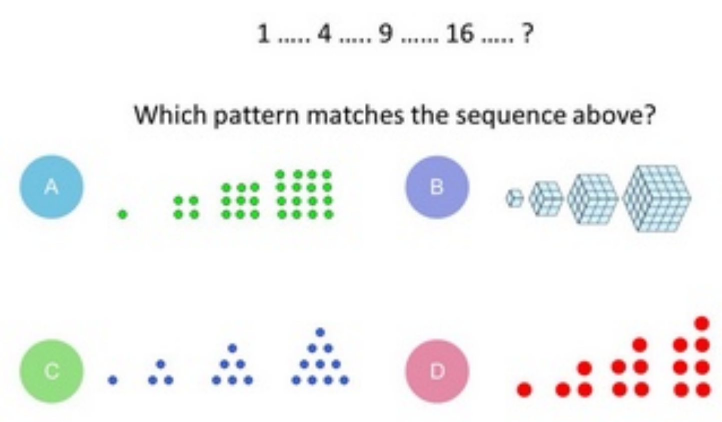
Note: Proofs by Mathematical Induction are generally on the syllabus for the school leaving examination necessary for students to qualify to study STEM subjects at University.

NOTES FOR TEACHERS

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



A. is the correct answer giving the square numbers n^2

The other sequences are:

B. 1, 8, 27, 64 … giving the cube numbers n^3.

C. 1, 3, 6, 10 … giving the triangle numbers ½n(n + 1)

D. 1, 3, 5, 7, … giving the odd numbers (2n – 1)

https://diagnosticquestions.com

Why do this activity?

This is a low threshold high ceiling challenge suitable for students in primary school as well as in the school leaving year when they would be asked to give the formulae for the general terms of all the sequences and to prove the formula for the sum of consecutive odd numbers. The activity is very suitable for mixed ability classes because all learners can have some success and notice and describe some of the patterns while there are challenges for the most able students. While it is true that human students should be learning to carry out tasks that they can do better than computers rather than doing routine calculations, never-the-less there are many occasions when number sense is very useful. Moreover many older students do not have instant recall of the number facts in the multiplication tables and they waste time working out products so this task helps them to develop their number sense in an interesting way.

Intended learning outcomes

1. Development of numeracy and number sense.
2. Development of recognition of patterns and symmetry.
3. Development of logical reasoning and ability to make generalisations.
4. Recognition and understanding of sequences and their general terms.

Suggestions for teaching

Start with the diagnostic question. This will help the learners to 'tune in' to the recognition the pattern of square numbers. Older students could be asked for the general terms for the other three sequences.

This activity works well in a mixed ability class where the students sit in pairs with a learner that struggles with mathematics partnered by a more able learner. In the very large classes that are common in developing countries this is a good teaching strategy. The more able learners benefit from explaining ideas and methods, which helps them to reflect on their own understanding and to develop their communication skills. It must be emphasised that each individual learner must fully understand what s/he is doing, and each individual must do the work for themselves, while at the same time learners can assist their partner at times.

In the latter half of the lesson the teacher might conduct a class discussion and ask students to present their findings to the class. If pairs of students do the presentations together this can give both the weaker and the stronger student equal responsibility and an equal sense of achievement.

Key questions

What do you notice?
Can you tell me the next number in that row?
Can you tell me the next number in that column?
Can you tell me the next number on that diagonal?
Can you tell me the 100^{th} number in that row?
Can you tell me the 100^{th} number in that column?
Can you tell me the 100^{th} number on that diagonal?
Could those rows continue for ever to the right?
Could those columns continue for ever downwards?
Does that always work? Can you prove that result?

Possible extension

What about continuing the patterns to the left?

Actually the top row gives the positive numbers on the real number line. Continuing the table and all the patterns to the left gives, on the top row, the integers on the real number line.

There can be discussion about vectors and equivalence of vectors on the line to the real numbers, also the equivalence of addition and subtraction of vectors to addition and subtraction of numbers. This paves the way for the introduction of complex numbers as equivalent to 2 dimensional vectors.

Possible support

The support necessary is built into the task that is based on a chart produced by Montessori for very young children.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6