AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

ISHANGO BONE

The numbers 3, 5, 7, 11, 13, 17 and 19 are prime numbers. Can you make a total of 60 from four of these numbers?

One of three columns of tally marks on The Ishango Bone gives the answer to this question. Found in the Ishango region of the Democratic Republic of Congo, it dates from the Upper Paleolithic era, about 1800 to 2000 BC. More about this later.

Can you make a total of 28 from these prime numbers in two or more different ways.

If you now include the prime numbers 19 and 23, can you now make a total of 49 from these numbers in two or more different ways.

From the prime numbers up to and including 31, can you find prime numbers that make a total of 79 in two or more different ways.

From the prime numbers up to and including 41, can you find prime numbers that make a total of 118 in two or more different ways.

Can you find more patterns like this?

This problem solving activity was inspired by the Ishango Bone. It has a sharp piece of quartz at one end and appears to have been a writing tool. There are many interpretations of the markings on the bone, and many theories about their significance, but it is remarkable that our ancestors in Africa were using numbers in what appears to be a sophisticated way. See https://en.wikipedia.org/wiki/Ishango_bone

SOLUTION

11 + 13 + 17 + 19 = 60 as on the Ishango Bone. Notice: 11 + 19 = 30 and 13 + 17 = 30

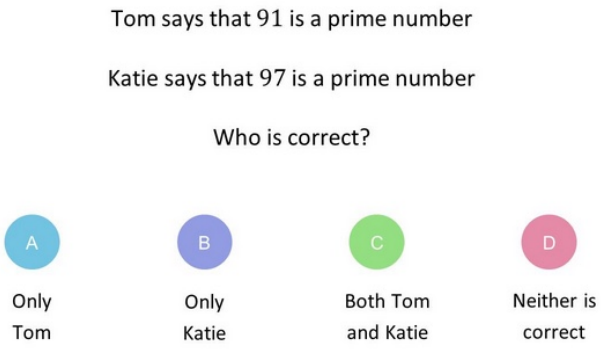
28 can be written as: 11+17 5+7+13+23	3+5+7+11+13+17+19+23=98 = 2 × 49 49 can be written as: 7 + 19 + 23 3 + 5 + 11 + 13 + 17 3 + 5 + 7 + 11 + 23 13 + 17 + 19	3+5+7+11+13+17+19+23+29+31 =158 = 2 × 79 79 can be written as: 19 + 29 + 31 3 + 5 + 7 + 11 + 13 + 17 + 23 3 + 5 + 11 + 29 + 31 3 + 11 + 17 + 19 + 29 5 + 7 + 13 + 23 + 31 5 + 7 + 17 + 19 + 31 7 + 13 + 17 + 19 + 23	3+5+7+11+13+17+19+23+29+31 +37+41 =236 = 2 × 118 118 can be written as: 3 + 7 + 13 + 17 + 37 + 41 5 + 11 + 19 + 23 + 29 + 31

Can you find any other solutions?

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



B. Katie is correct, 97 is a prime number.
91 = 7 × 13 so it is not a prime number.

A. Students giving this answer have failed to understand that 91 is a multiple of 7.

C. These students may be guessing or they may have failed to understand that 91 is a multiple of 7.

D. But 97 is a prime number as it is less than 10×10 and it is not a multiple of 2, 3, 5, or 7.

https://diagnosticquestions.com

Why do this activity?

This activity is accessible to both primary and secondary students as it only involves addition of whole numbers. It provides culturally significant information that can be followed up by students through further research online. The task given can be completed quickly as a lesson starter for older students to reinforce the concept of prime numbers but, as the prime numbers are given, it can also be done by primary students who may just take a bit longer.

Intended learning objectives:

Through this activity students should be able to:

- recognise prime numbers less than 50 and understand the definition of prime numbers
- appreciate something about the long history of the study of numbers and number patterns in African and other cultures.
- have experienced the need to work systematically to find all possible combinations from a set of numbers and hence to be sure that they have found all possible solutions.

Development of Competences

By making this a class project to find as many different solutions as possible this task can promote team working skills and also, as they discuss strategies for finding alternative solutions, the task helps students to develop communication skills

Suggestions for teaching

With younger learners it is not necessary to do the diagnostic quiz, rather start by showing a picture of the bone, telling the learners about it, and listing the numbers 3, 5, 7, 11, 13, 17 and 19. Then ask the students to make a total of 60 from adding four of these numbers.

For older learners, who have learnt about prime numbers, start with the diagnostic quiz. Then talk about the bone and ask the same question.

Then ask the learners to work individually or in pairs to find numbers that add up to 28 from the list. Make a chart with 4 columns on the board and ask the class to find numbers that add up to the four totals: 28, 49, 79 and 118 in as

many different ways as they can find. Whenever a learner finds a new solution write it up on the board. Ask the learner to explain how he or she found the new solution. This may lead to developing strategies that everyone can apply, like finding 2 or 3 primes in one solution that add up to another prime number so that this set can be exchanged between two solutions with the prime number that is equal to their total..

After a period of working in pairs you could ask the class to work in fours.

You might make a poster of solutions to put up on the classroom wall. If you think that the class has not found all possible solutions by the end of the lesson, encourage the learners to go on searching and follow this up for a short time in future lessons. Whenever a learner finds a new solution, praise their efforts and write the new solution on the poster.

Key questions

What are the numbers not in your list? What do they add up to?
Can you find the same total in a different way.
Can you find 2 or 3 smaller prime numbers that add up to 19?
Can you find 2 or 3 smaller prime numbers that add up to 29?
Can you find 2 or 3 smaller prime numbers that add up to 31?
Can you find any more solutions?
What is the total of all the numbers in this list?
What do you notice about the total that you are trying to make?

Possible extension

The next two prime number 43 and 47 could be included in the list and the students could look for similar combinations of numbers that add up to 163.

Possible support

You might arrange for students who are struggling to work in a pair with a more able student making it clear that both students should share the work. As the process is one of simple addition, both students should be able to contribute to the search.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**