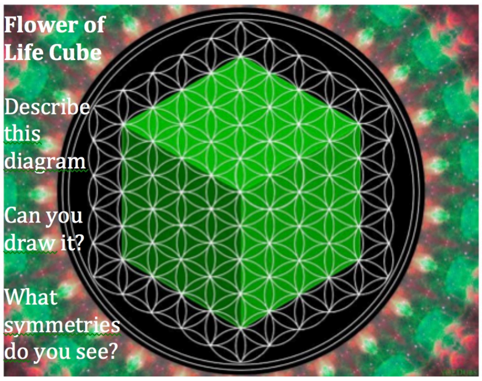
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

FLOWER OF LIFE

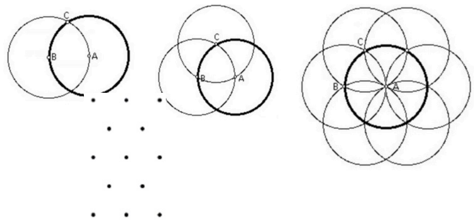
Describe the picture? What do you notice? Talk with your friends about it. What shapes can you see in it?



Describe the different symmetries. Can you explain how this 2-dimensional picture appears to show a 3-dimensional object? Draw the picture yourself. Draw your own designs using this idea.

Click the picture for a video to show how use paperclip compasses to draw the circles in this pattern.

You will find it helpful to draw the circles with their centres on the lattice points of an isometric grid. Start by first drawing 2 intersecting circles, then a third, then 4 more circles.



From the centre there is first one circle. The next flower pattern has 7 circles altogether and the next pattern has 19 so the sequence for the number of circles starts 1, 7, 19, …

Carry on this sequence of the number of circles as bands of circles are added around each pattern to make the next larger pattern in the sequence.

As an even more difficult challenge, could you describe this pattern to a friend over the phone who has not seen it so well that she is able to draw it from your instructions even though you have no way of showing her the picture?

Ask more questions about the geometric properties of this picture and provide your own answers.

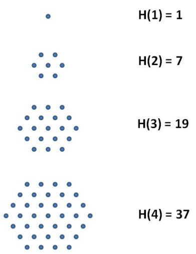
Make a display for your classroom wall.

SOLUTION

This is an open ended question and there are many possible answers. The picture has rotational symmetry of order 6, and 6 axes of symmetry (mirror lines) through the centre.

The sequence of the number of circles that make the flowers of life as the pattern grows is: 1, 7, 19, 37, 61, 91, … and there are 91 small white circles in this pattern.

<u>Method 1</u>



There are six extra circles in each successive band of circles to make a longer band of circles on the outside. So the number of circles is:

1 = 1
7 = 1 + 6
19 = 1 + 6 + 12
37 = 1 + 6 + 12 + 18
61 = 1 + 6 + 12 + 18 + 24
91 = 1 + 6 + 12 + 18 + 24 + 30

As this pattern continues, the sequence is 1 + 6(1 + 2 + 3 + 4 + 5 + …) so we add 6(n – 1) circles in the nth band. We can use the formula for the sum of the triangle numbers 1 + 2 + 3 + 4 + 5 + … = ½t(t + 1). But, noting that we add the first band of circles for n=2, the second band for n= 3 etc this means that the formula for the sequence of triangle numbers that we need to use, substituting t = n – 1, is ½(n – 1)n

The number of triangles for the n^{th} pattern is

$$C(n) = 1 + 6(1 + 2 + 3 + 4 + 5 + \ldots)$$
$$= 1 + 6 \times \tfrac{1}{2}n(n - 1)$$
$$= 3n^2 - 3n + 1.$$

Method 2
Assuming the sequence continues by adding 6(n-1) circles in each band then we can use the method of differences:

Sequence:	1	7	19	37	61	91	127…	
First differences		6	12	18	24	30	36 …	
Second differences			6	6	6	6	6 …	

So the formula for this sequence is a quadratic polynomial $C(n) = an^2 + bn + c$

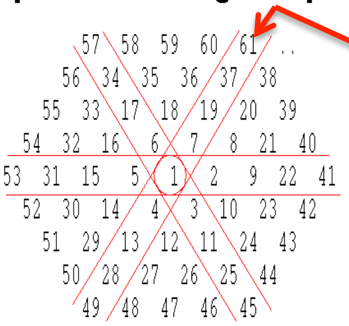
Substituting n = 1 gives $a + b + c = 1$ (i)
Substituting n = 2 gives $4a + 2b + c = 7$ (ii)
Substituting n = 3 gives $9a + 3b + c = 19$ (iii)

From (i) and (ii) $3a + b = 6$
From (i) and (iii) $8a + 2b = 18$

From these two equations $a = 3$, $b = -3$ and [substituting in (i)] $c = 1$.

So the formula is $C(n) = 3n^2 - 3n + 1$

Spokes of a hexagonal spiral



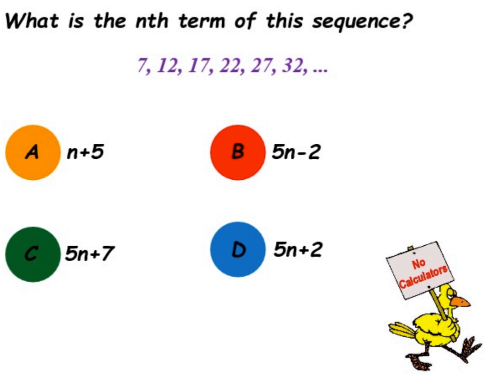
Comment Method 2 uses the standard numerical methods procedure for the method of differences to find a polynomial formula for a sequence. However Method 1 uses more insight and understanding of the way that the sequence of numbers arises from the construction of the sequence of patterns.

Method 3 This clockwise hexagonal spiral diagram shows how to count the number of small white circles moving out to the next band as each band is completed.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson tollow, explain the right answer or give a remedial task.



The correct answer and possible misconceptions.

A. These students may have recognised that the common difference is 5 but they are confused about how to find the n^{th} term.

B. These students have probably recognised that the common difference is 5 but they don't understand how to check that their formula is correct for n = 1 and 2.

C. These students have probably recognised that the common difference is 5 but their formula starts with n=0 then n=1, 2, 3 etc.

D. This is the correct answer

https://diagnosticquestions.com

Why do this activity?

This is a low threshold high ceiling activity perfect for mixed ability classes where everyone can achieve success working within the same context and learners of all abilities can be challenged at a level suitable for them. In addition this activity can be used in a primary school (without the part on counting circles and finding formulae) and it can also be used with students at the school university interface as an exercise in summing series. Competences are developed by this task, in particular communication and creativity, and students can also follow up the cultural and historical information about the symbolism in this pattern.

Learning objectives

For primary: students should be able to describe the pattern and its symmetries
For lower secondary: students should be able to draw the pattern and describe its symmetries
For upper secondary: students should be able to draw the pattern, describe its symmetries and find the formula for the sequence of the number of circles that make up the pattern as it grows.

Generic competences

This activity helps students to develop communication skills as describing the pattern presents a significant challenge. The activity also provides an opportunity for creativity and critical and logical thinking in order to derive the formula for the series.

Suggestions for teaching

The diagnostic question given above is meant for upper secondary students who have met arithmetic sequences and series.

These pictures of flowers makes a good lesson starter for all ages and abilities. As a test of prior knowledge, ask the students to describe what they see in these pictures. Then ask questions about line symmetry (reflections) and rotational symmetry.

Start the main part of the lesson by asking the learners to work in pairs to make a list of everything that they notice about the Flower of Life pattern of circles. Then conduct a class discussion in which the learners explain to the class what they have noticed.

Then, according to the age of your class select challenges from those offered in the question.

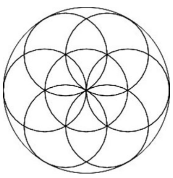
Key questions

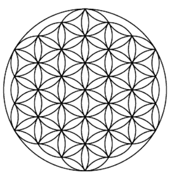
What do you notice?
What symmetries can you see in the pattern?
How does the pattern grow from 7 circles to the next flower in the sequence as it spreads outwards.
How many arcs are there around the edge of the hexagon?
What insect does this pattern make you think of?
Why is the pattern of white circles not a hexagon when the green shape is a hexagon?
Could this pattern grow bigger and bigger if you added more bands of circles around the outer edge?
How could you find the total number of small white circles.

Possible extension

See this Wikipedia article https://en.wikipedia.org/wiki/Overlapping_circles_grid for information about the Flower of Life Pattern and copy some of the designs in the article.

See also https://aiminghigh.aimssec.ac.za/grades-7-to-10-constructions-with-circles/





Possible support

The use of an isometric grid is very helpful for drawing the pattern and also to see stage 1 and stage 2 as shown here. The pattern given in the question is stage 4.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see https://nrich.maths.org/12339 **Note:** The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**