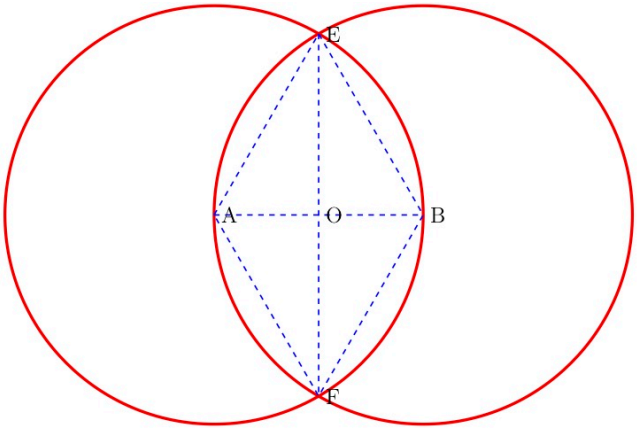
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

A MATHEMATICAL LENS



STEP 1

1a) Double click on the picture to start the movie showing how to draw a circle using the paperclip compass.

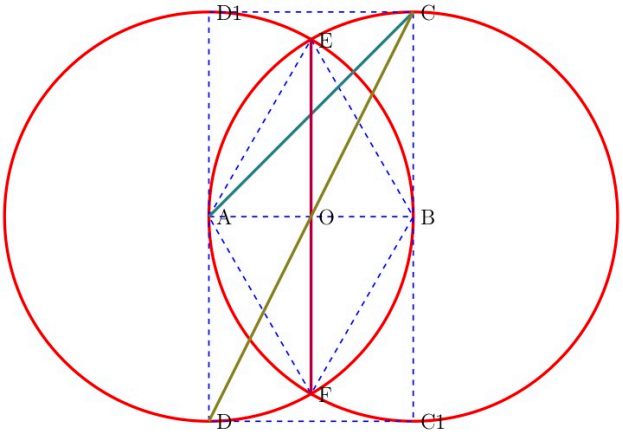
1b) Draw 2 circles of the same radius with centres A and B so that each circle goes through the centre of the other circle.

1c) Explain how you know that triangle ΔAEB is equilateral.

1d) Suppose AE = 1 unit. Give the length of EF leaving the answer as a surd.

1e) Using a calculator work out the length of EF to one decimal place with your value of AE. Now measure EF in your diagram. Do your two answers agree?

STEP 2

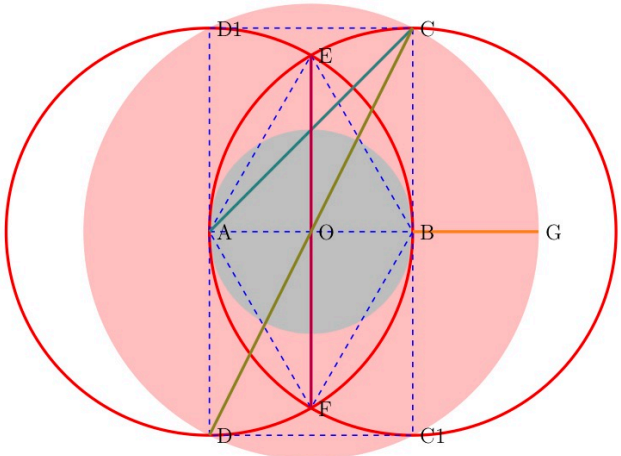


2a) Draw the extra lines in your diagram as shown here and label all the points.

2b) Calculate the lengths CA and CD for circles with AE = 1 unit giving your answers as surds.

2c) Using a calculator work out the lengths of CA and CD to one decimal place with your value of AE. Now measure CA and CD in your diagram. Do your measurements agree with your answers?

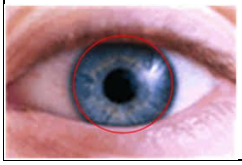
STEP 3



3a) You have worked out the radius of the pink disc in STEP 2 and now you can draw the circles around the pink and grey discs in your own diagram.

3b) Calculate the length BG with AE = 1 unit leaving your answer as a surd.

3c) The Golden Ratio $\varphi = \frac{1}{2}(1 + \sqrt{5})$ is a very important number in mathematics. Have you found that $BG = \varphi - 1$? If not go back and check your working in STEP 3b).



STEP 4

Find out from the internet what you can about this mathematical lens, so called because it is shaped like the human eye. The diagram, called Vesica Piscis, appears in many ancient mathematical writings and it has been considered a sacred symbol for thousands of years.

SOLUTION Pythagoras Theorem is used to calculate the following lengths:

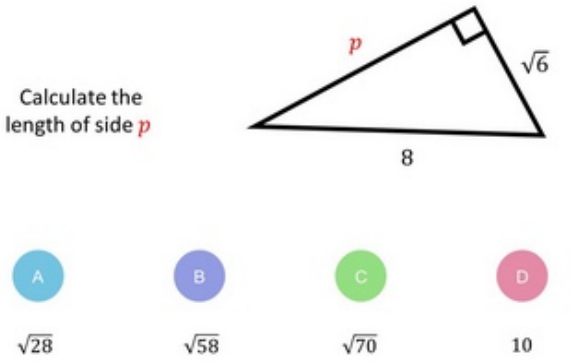
EF = $\sqrt{3}$, CA = $\sqrt{2}$, CD = $\sqrt{5}$, BG = ½($\sqrt{5}$ – 1) = φ - 1

NOTES FOR TEACHERS

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



B. is the correct answer.

Common Misconceptions

A. As 36 – 8 = 28 learners may have been using these numbers not knowing how to use the squares and square roots correctly.

C. Here the mistake is calculating $8^2 + (\sqrt{6})^2 = 70$ instead of $8^2 - (\sqrt{6})^2 = 58$

D. Here there are 2 different mistakes to get the answer 10: $\sqrt{[8^2 + (\sqrt{6})^2]} = \sqrt{[64 + 36]} = \sqrt{100} = 10$

https://diagnosticquestions.com

Why do this activity?

This activity caters for learners of different abilities. Every learner should be able to have some success while, at the same time the activity provides challenges for the most able. It gives practice in making accurate constructions with straightedge and compass, in applying Pythagoras Theorem, in making accurate measurements and in working with surds. It also has the advantage that learners can correct for themselves any mistakes that they make by measuring lengths in their own diagrams and comparing the measurements with their calculated results.

Intended learning outcomes

To become more proficient in making accurate geometrical constructions with straightedge and compasses.
To practice applying Pythagoras Theorem.
To develop the ability to work independently and to check one's own work.

Suggestions for teaching

Start with the diagnostic question and, if necessary, review Pythagoras Theorem.

Give each pair of learners a copy of page 1 so that they can work through the activity without the teacher telling them what to do. Give each learner a paperclip and, if they have not used paperclip compasses before, make sure that they know how to draw circles using the paperclip. If possible show the video to the class.

Tell the learners that everyone should draw their own diagrams, so their own calculations and check their own answers but that they can help their partner.

To cater for all abilities you might pair more able learners with less able learners but do not allow the more able learners actually to do the work for their partners. In your class there should always be an atmosphere of cooperation and a willingness on the part of everyone to help others and it is the responsibility of the teacher to create this atmosphere and type of learning environment. Mistakes should be recognised as experiences that help people to understand more, and they should not be thought of as failure. All the learners should recognise that they are studying entirely for their own benefit, that copying somebody else's work is of no benefit to them because they need to be able to make progress and to understand everything for themselves.

While the learners are working on the tasks the teacher should go around asking questions to guide the progress of the students as needed.

In the last half of the lesson learners could be asked to come and present their work. If there is access to computers learners could research the Viscera Piscis symbol and its significance, and possibly the Golden Ratio as well, and present their research findings to the class.

Key questions

Show me which lengths in the diagram are equal. How do you know?
What are the angles in that triangle. Can you explain why?
Describe the symmetry that you see in the diagram.
What do you know about the lengths AO and OB?
Show me a right angled triangle in the diagram.
If you know the lengths of two edges of a right angled triangle how do you find the length of the third edge.
Have you measured that length to see if it agrees your calculation?
What have you found out from the internet about Visera Piscis?
What have you found out from the internet about The Golden Ratio?

Possible extension

Constructing Circle and line patterns
https://aiminghigh.aimssec.ac.za/grades-7-to-10-constructing-circle-and-line-patterns/
Pythagoras Jigsaw
https://aiminghigh.aimssec.ac.za/grades-8-to-12-pythagoras-jigsaw/
Riding on Pythagoras
https://aiminghigh.aimssec.ac.za/grades-8-to-10-riding-on-pythagoras-1/

Possible support

Constructions with Circles
https://aiminghigh.aimssec.ac.za/grades-7-to-10-constructions-with-circles/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6