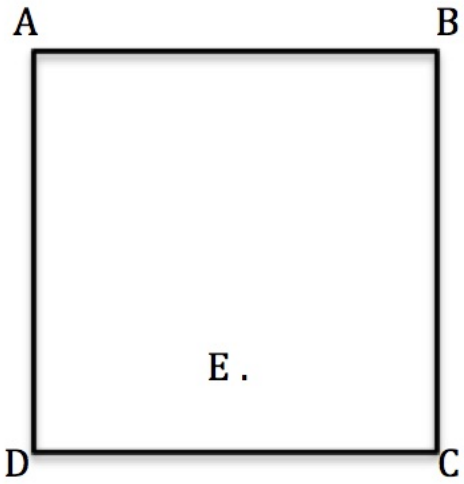
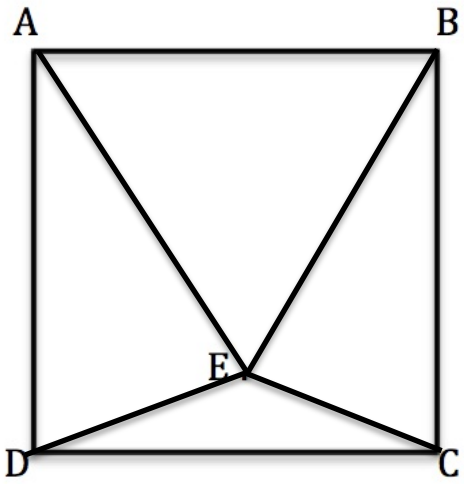
GEO PROB



The point E is inside the square ABCD and AEB is an equilateral triangle. Find the angles EBC, BEC and ECD and give reasons for your answers.

SOLUTION



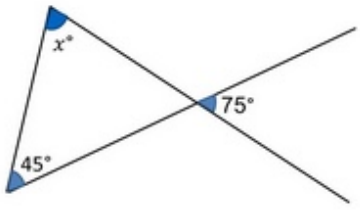
Triangle AEB is equilateral so all its angles are 60^o
ABCD is a square so triangles AED and BEC are isosceles.
Angle ABC is an angle of the square so angle EBC is $90^o - 60^o = 30^o$
As triangle BEC is isosceles, the base angles are equal so angle BEC = angle BCE = $\frac{1}{2}(180^o - 30^o) = 75^o$.
Angle ECD is $90^o - 75^o = 15^o$.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

What is the size of the angle marked x?



A 45°

B 75°

C 105°

D 60°

D. is the correct answer.

Common Misconceptions

A. Students often give 45° when the diagram looks as if that is a possibility and when they don't know how to find the answer.

B. Students may assume the triangle is isosceles.

C. Students may be confused and making some incorrect connection with adjacent angles on a straight line.

Why do this activity?

This question could be a lesson starter. It gives practice in drawing a diagram from information that learners have to read for themselves and the diagram provided helps them to get started. The problem can be used as a means to encourage learners to read the question and **to make sure that they have used all the information given**. Often teachers say that their learners can answer well in class and then they do badly on written tests. Perhaps these teachers underestimate how different the experience can be for learners when they have to read the question for themselves in a test.

Learners benefit from being given more practice in reading questions and deciding for themselves how to proceed. In this problem learners often miss the clue that two of the triangles are isosceles. They may use the information about ABCD being a square to identify the four right angles, but perhaps they don't use the information that this gives about the lengths of the edges.

This problem naturally leads into further work on angles in triangles and symmetry and it can also be used in a revision lesson planned as exam preparation.

Intended learning outcomes

Practice in solving geometry problems involving properties of triangles.

Suggestions for teaching

Write the question on the board and ask the learners to do it **on their own**. Be sure to tell them to copy and complete the diagram from the information given.

After a few minutes, for those who are having trouble getting started, you might ask "what information does the question give you about any of the angles in the diagram?" and 'is there anything

that you notice in the diagram that might help you?" If you avoid giving additional help at this stage you will still leave the initiative to the learners.

Some learners may spot that the angles of the equilateral triangle are 60^o and that, looking at the corners of the square, they can find angle EBC.

Having given the learners a few minutes to do the problem on their own, and when you see that some learners have got answers, you could ask learners to check with a partner and share ideas. Learners benefit from explaining their reasoning to each other.

To help learners who don't spot the isosceles triangles you might say "read the question again, is there any information given there that you have not made use of?"

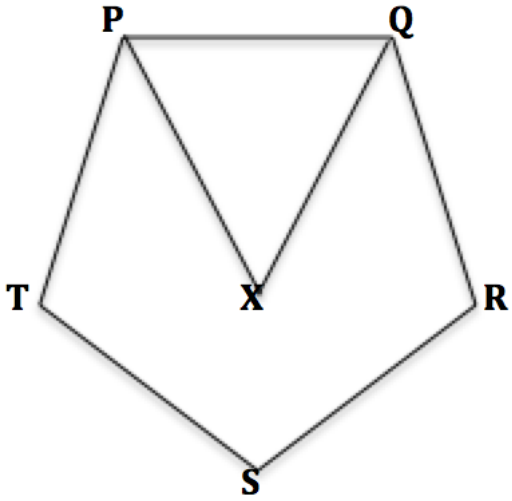
You could ask a pair of learner to come to the board and explain their method to the class. Having a pair of learners to do this means that while one is writing on the board the other can do the explaining.

Key questions

What do you know about squares that could be useful here?
What do you know about equilateral triangles that could be useful here?
Read the question again, is there any information given in the question that you have not made use of?
Is there anything that you notice in the diagram that might help you?
Can you explain your method to me?

Possible extension

Ask the learners to find the angles in triangle DEC. Ask what they can say about the symmetry in the diagram.



Here is a similar question based on a regular pentagon PQRST and equilateral triangle PQX.
Join XT, XR and XS and find all 15 angles in the diagram.
Beware: don't assume that TXR is a straight line.
What can you say about the symmetry in the diagram?

Possible support

For the learners who are struggling you could ask them to find which lengths are equal in the diagram and then ask them if this tells them anything about triangle BEC.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**