# Mathematics Education for the 21st Century

**Toni Beardon**

**University of Cambridge and African Institute for Mathematical Sciences**

***Abstract***

*There is a widely held belief amongst teachers, educational researchers, policy makers, politicians and employers that teaching methods need to change to equip young people with 21st Century skills and competences and there is research evidence for this belief (Ananiadou and Claro, 2009). However there is limited experience of teaching and assessment methods, and few resources, particularly in developing countries, to support such a change. Some governments have incorporated into national educational standards sets of skills, competences and values that every student should attain by the end of compulsory schooling in order to function effectively in the workplace, as citizens and in their leisure time (e.g. Rwanda Competence Based Curriculum for Sustainable Development 2015). In her talk Toni will explore the nature of this change, how it might be brought about and explain her work in this context.*

*Bearing in mind the conditions that prevail across Africa of large classes, lack of resources and both teaching and learning in an additional language, Toni designs and develops training programmes, research, and free online open source learning resources for teaching and learning mathematics to support independent, active, inquiry based learning and to provide learning experiences that require deep thinking, develop 21st century skills and focus on what young people understand and can do rather than just memorize.*

## Introduction

This paper describes and compares different methods of teaching mathematics in schools and universities. It examines issues that have become causes of concern because the rapid growth of technology and its applications in the workplace have led to the disappearance of many jobs. In the lifetime of our students we face a future that we can barely imagine, so they will need a very different skill set from the one that would have served them well 40 or 50 years ago. As a teacher educator the author was surprised to be asked to address a conference for mathematicians but then most of the participants teach mathematics at some level, or will do so in the future. If mathematics is taught well in schools then the universities will be able to set higher standards, and our societies will benefit because mathematics underpins so many human activities on which the economic development of countries depend. In addition, education can provide a means for children to escape the poverty trap by enabling them to get a job that contributes to the development of their country and that they find personally fulfilling, and then they can help others from their own community.

The author has founded two projects that promote the learning of mathematics and the development of 21st century competences. NRICH the Online Maths Club [1] was born in 1996, at the University of Cambridge, to engage young people in problem solving and to enable older students to help younger students to understand mathematics better through the online forum 'Ask NRICH'. NRICH publishes new problems monthly together with *children's* solutions to the problems from the previous month. In 2003 AIMSSEC [2] was set up in South Africa with the objectives to advance educational opportunities for disadvantaged communities in Africa, to introduce new skills to

teaching and learning mathematics, and to raise the standards of mathematics teaching, making these opportunities accessible to teachers in rural areas.

AIMSSEC is the Schools Enrichment Centre of AIMS (the African Institute for Mathematical Sciences) established in 2003 in Muizenberg, South Africa which has developed into the AIMS Network [3] with higher education institutes in South Africa, Senegal, Ghana, Cameroon, Tanzania and Rwanda. AIMSSEC now teaches courses in South Africa, Uganda, Rwanda, Kenya and Tanzania. For all its work AIMSSEC depends entirely on sponsorship funding. It receives some funding from government sources in South Africa, from corporations and from private donors. The critical need for capacity building and the professional development of mathematics teachers in South Africa was identified at the inception of AIMS.

AIMSSEC is a community of academic mathematics teacher educators, researchers, teacher trainers, school teachers and subject advisers, all working in mathematics education, who share a commitment to lifelong learning and a dedication to improving the teaching and learning of mathematics. In 2012 AIMSSEC won the UNESCO-Hamdan Prize for Outstanding Practice and Performance in Enhancing the Effectiveness of Teachers in Developing Countries.

The AIMSSEC community involves an international team of over 40 academics working for AIMSSEC as unpaid volunteers, as well as the small team of Africans based in Cape Town, former students, and everybody who has taught on the AIMSSEC courses. There is a shared belief that we must learn from mathematics education research, and that teaching for the 21st century must be different from the past because different skills are needed in this radically changed and rapidly changing society.

The internet provides the means to reach young people and their teachers who do not have access to libraries and well resourced schools. In 2003 there was a big digital divide in Africa. AIMSSEC helps mathematics teachers working in the most disadvantaged communities, some in schools that do not have electricity. So whenever our courses introduce ideas for using ICT in teaching we always include other ideas for teaching the same topic in an ICT barren environment. Smartphones now enable teachers to benefit from technology as never before and, with satellite technology, they have made it possible to reach the most disadvantaged communities in Africa.

AIMSSEC produces high quality, teacher training materials; however access to them has traditionally been limited to those enrolled on courses. Now through the creation of the AIMSSEC App they are made much more widely available to anybody with a smartphone (a market which has more than doubled in the past 4 years alone, and is expected to reach nearly 50% of the population by 2022 [4]).

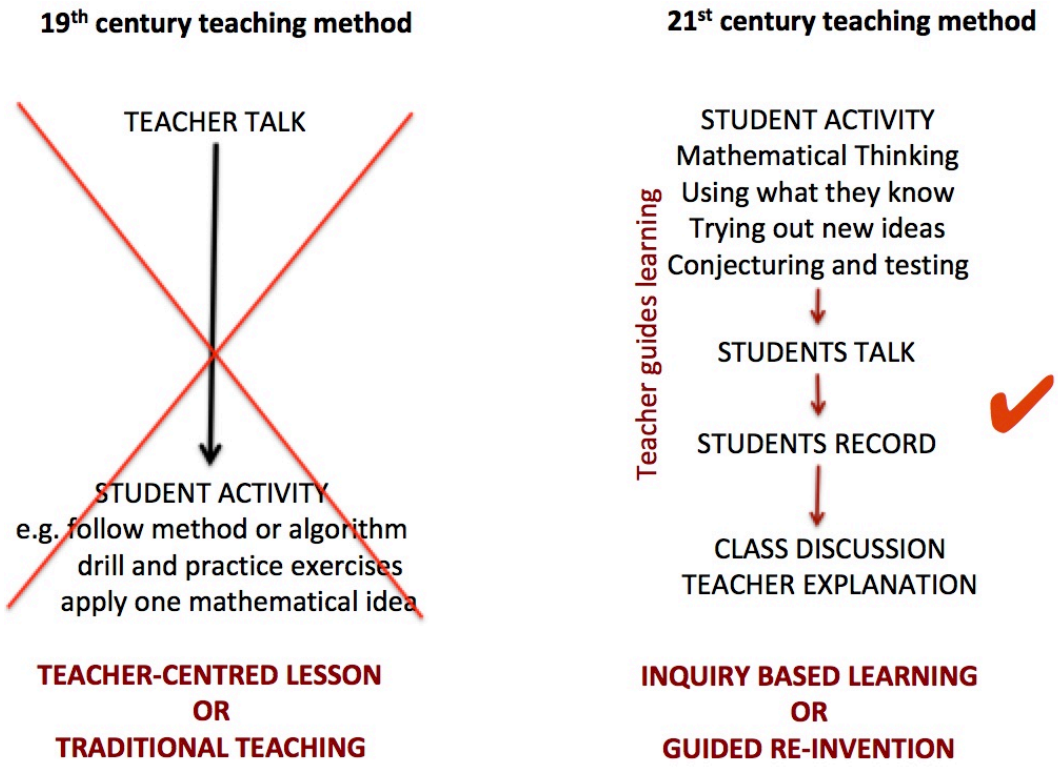
The App is also fully functional offline, removing barriers of access for many people who only have intermittent internet access. AIMSSEC workshop guides enable teachers to share what they have learned through AIMSSEC and to run ‘self-help’ professional development workshops for other teachers without the need of an expert to lead the group. Teachers use AIMSSEC lesson resources to support inquiry based learning to develop understanding and skills and then to share their experiences with each others and to give feedback to the resource developers [5].

**The AIMSSEC approach to teaching mathematics in school**

In a traditional ‘teacher-centred’ lesson, the teacher does most of the talking. But in a ‘student-centred’ lesson, rather than just listening to the teacher, taking notes and

remembering facts in order to pass exams, the students do most of the work on tasks that have been carefully chosen to introduce them to new ideas. The teacher is 'the guide on the side', not telling the students what to do but asking carefully chosen questions to guide the students' thinking, and to encourage them.

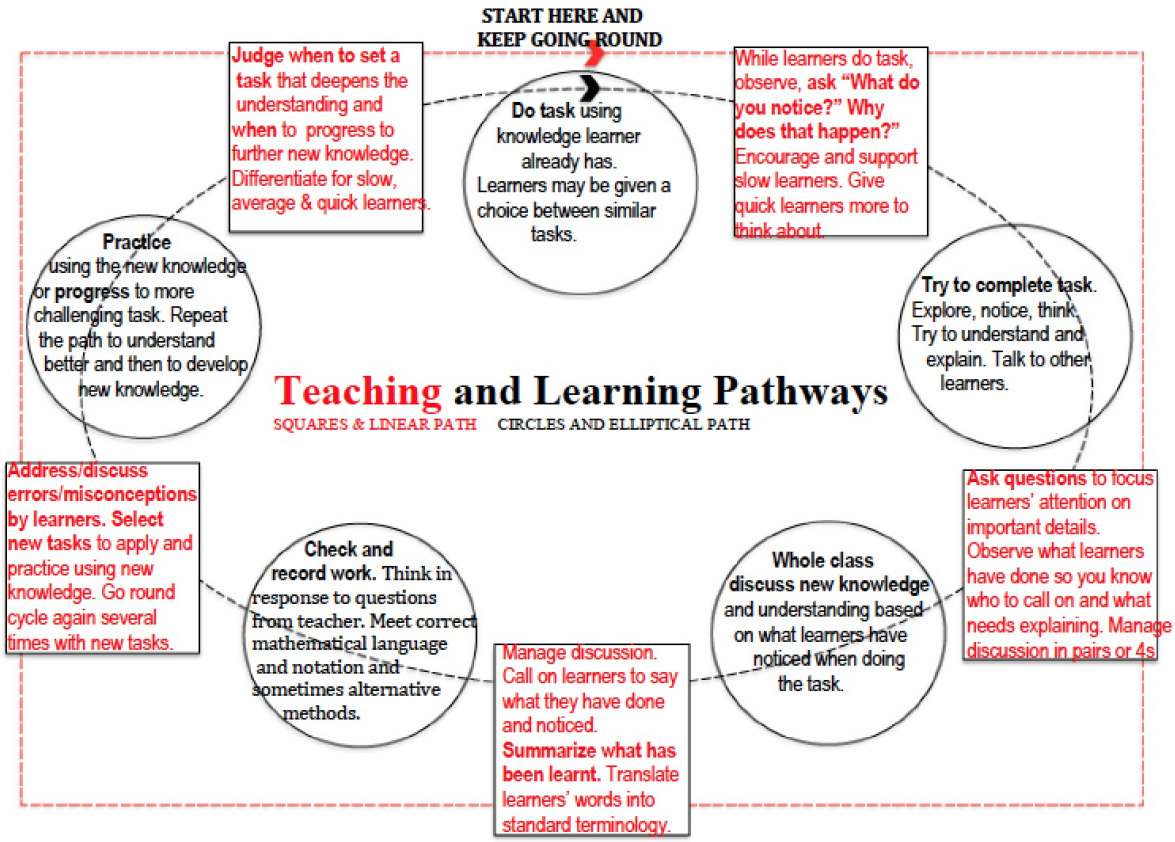
The diagram caricatures the two types of teaching but it is sometimes appropriate for a teacher to give some explanation before setting a task. In a university situation the same principle might be followed with preparatory tasks tackled in sessions between one lecture and the next that characterizes the problem solving Hungarian approach where students are often required to write on the board and to explain their work to the whole class.




In the student-centred or 21st century teaching method, students are sometimes encouraged to work alone so that they develop independent learning competences, but often they will compare answers and methods and learn to spot and correct mistakes, or learn that there are different ways to solve a problem or to prove a result. Working with a partner or in a group builds communication and team working competences.

When students have made a written record of their work some students explain what they have done to the whole class. They must give reasons for the steps in their working and explain, not just what they have done, but why; this builds communication competences of speaking in public and giving presentations. The explanation from the teacher, and the introduction of the standard vocabulary, formal definitions and notation, come after the students have worked on the activities and are based on the students' experience and the preparatory work that they have already carried out.

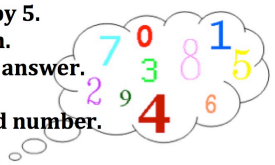
The second diagram illustrates a typical student-centred lesson. The ellipse shows the learning pathway or sequence of student activities and these activities are described in the circles. The rectangle shows the teacher's role in the lesson and the interactions with the students are described in the square boxes.




In planning the lesson the teacher chooses tasks designed to use and apply to new situations the knowledge that the learners already have and to help them to begin to understand a new concept, to develop a new insight or perspective, or to understand the need to extend their knowledge in order to accommodate a new idea. A very simple example is the activity of sharing in equal parts that can take young children beyond thinking that numbers are always whole numbers to instinctively understanding that there are other types of numbers.

This pre-algebra trick (adapted from an NRICH problem) is another AIMSSEC activity that serves to make children aware of a whole new world of mathematical ideas. It helps them to realize that it could be useful to represent numbers by letters and it introduces the concept of variables.

**Think of two whole numbers under 10.**
**Take one of them and add 1.**
**Multiply by 5.**
**Add 1 again.**
**Double your answer.**
**Subtract 1.**
**Add your second number.**
**Add 2.**
**Double again.**
**Subtract 8.**
**Halve this number and tell me your answer.**
**I can work out both your numbers. How?**




By just subtracting 9 from their answer the teacher can work out the two numbers that any child in the class has used. If a child uses 2 and 7 she will get the answer 36 and the teacher will subtract 9 to get 27 and know her answer. This seems to the children to be magic. After playing the game a few times the teacher asks the class to work out how it is done. The teacher might use secret numbers given by 2 different children and call each number by the first letter of the child's name. The children then write down in any way they can what they get at each step and, after repeating this a few times, the teacher can help them to tidy their recording into conventional algebraic notation to arrive at 10A+K+9 for ten times Asha's number plus Kwame's number plus 9. Children's creativity is developed through AIMSSEC activities that, as in this example, ask them to make up similar tasks of their own.

## 21st Century Skills And Competences For New Millennium Learners

This section heading is taken from the title of an OECD Directorate for Education research paper reporting on the findings of a questionnaire study conducted in 17 OECD countries and other relevant background material such as white papers or curriculum documents from which the following quotation is taken.

"The findings of the questionnaire survey show that most countries or regions cover 21st century skills and competencies in their regulations, guidelines or recommendations for compulsory education. However, there are few specific definitions of these skills and competencies at national or regional level and virtually no clear formative or summative assessment policies for these skills. The only evaluation regarding their teaching is often left to external inspectors as part of their whole school audits. Similarly there are few teacher training programmes that target the teaching or development of 21st century skills, although there exist several teacher training initiatives that focus on developing teachers' ICT pedagogical skills, most of them optional. The paper discusses the implications of these findings especially with regard to the particular role of ICT in the development of these skills and competencies, and issues related to assessment practices and teacher training." (Ananiadou & Claro 2009)

This confirms what I have been able to find out from other sources and in my study of national curricula for mathematics across East Africa, and in South Africa as well as in the UK. The latest UK national curriculum (2014) only aims to ensure that all pupils "become fluent in the fundamentals of mathematics, can reason mathematically and solve problems" and does not specifically mention other transferable skills or competences. We seem to have gone backwards in this respect from an earlier version of our national curriculum that had a strand entitled 'Using and Applying Mathematics' which is now only mentioned in a non-statutory guidance paper.

The emphasis is now on problem solving. Recent GCSE papers (the UK public examination for 16 year olds) are much harder than papers from the past because they are designed to assess problem solving skills. As an example, here is a question from one of this year's GCSE papers, which was set as quoted below with no hints to the candidate:

**"The sum of the first 48 terms of an arithmetic series is 4 times the sum of the first 36 terms. What is the sum of the first 30 terms?"**

The reader can judge for him or herself whether this is a good test of problem solving skills for 16 year olds. It is now published on the AIMING HIGH Teacher Network website with notes for teachers giving solutions and suggestions for teaching. [5]

In 2015 the Rwandan government introduced a Competence Based Curriculum [6] across all subjects. It defines generic competences to promote the development of **higher order thinking skills** helping students to **deepen their understanding** of subjects and **apply their subject learning** in a range of situations.

My contention is that mathematics provides ideal learning experiences for the development of these *generic* competences as specified in the Rwanda Curriculum. In this paper I will endeavour to describe how AIMSSEC teaching methods are designed to help teachers to foster the development of these competences in their students.

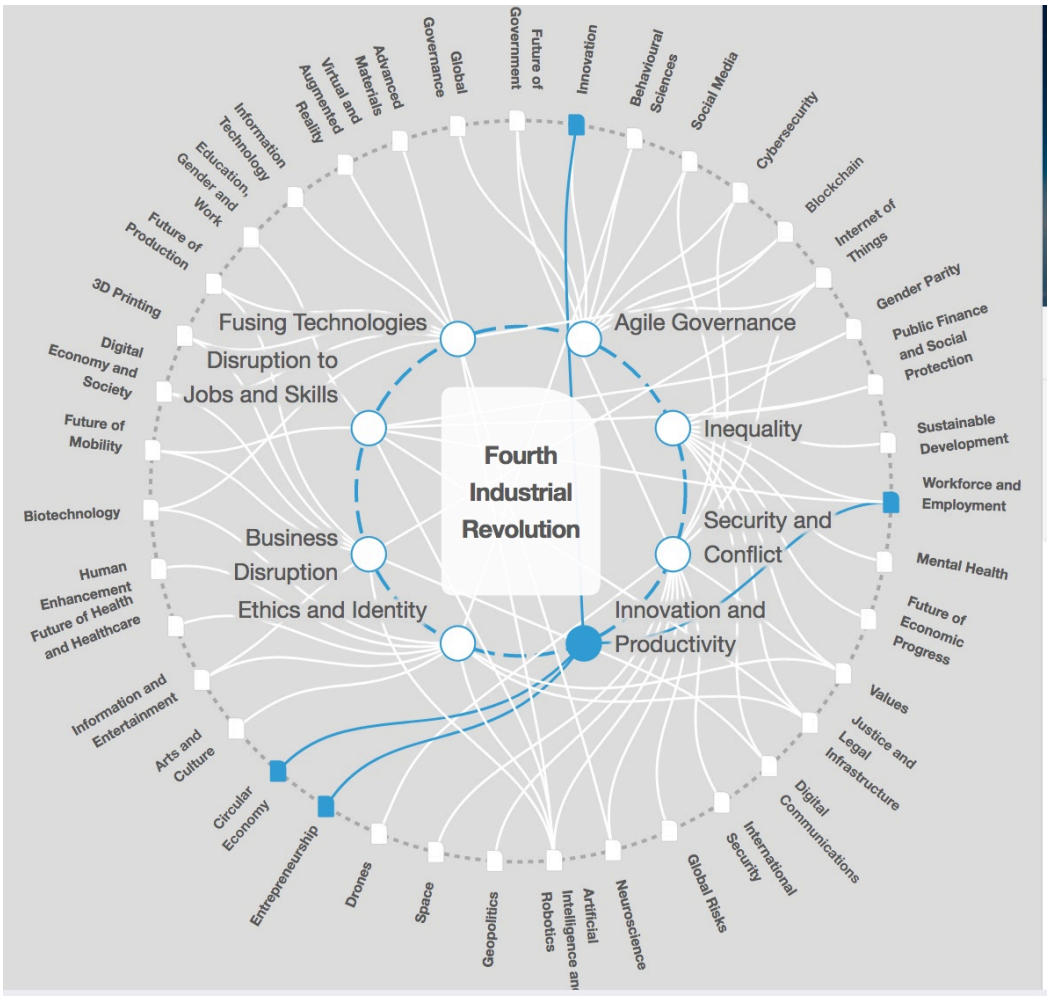
- Critical thinking
- Logical reasoning
- Creativity and innovation

- Research and problem solving
- Communication
- Co-operation, interpersonal relations and life skills
- Independent learning/Life long learning

The terms 'competence', 'competency' and 'skill' are used interchangeably but one useful difference is given in this quotation:

"A competency is more than just knowledge and skills. It involves the ability to meet complex demands, by drawing on and mobilising psychosocial resources (including skills and attitudes) in a particular context. For example, the ability to communicate effectively is a competence that may draw on an individual's knowledge of language, practical IT skills and attitudes towards those with whom he or she is communicating" (Rychen & Salganik, 2003).

Another often quoted definition is found in an article written for human resources professionals [7]. "A competence is a measurable pattern of knowledge, skills, abilities, behaviours, and other characteristics that an individual needs to perform work roles or occupational functions successfully." To say a person is competent generally means he can do something but is not an expert; to say someone is skillful generally implies expertise. Never-the-less this paper uses the term competency rather than skill with the above definitions in mind.




The above image of an interactive transformation map published by the World Economic Forum [8] gives an, albeit static, representation of the complexity that we and

our students face with the fourth industrial revolution. Our educational systems, and all of us responsible for educating young people, should be preparing them for this new world. Knowledge of facts without understanding, the ability to follow algorithms and the skills needed in the last century are obsolete or inadequate today because machines can perform frequently repeated tasks much more efficiently than people. Education must foster students' imagination, creativity, vision and inspirations; these are attributes that machines do not have.

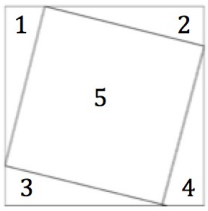
**How does mathematics as a discipline relate to these competences?**

It is unarguable that doing mathematics requires logical reasoning unless it is just regurgitating facts learned by rote. AIMSSEC promotes mathematical thinking which we claim is 'critical thinking in the context of mathematics'. AIMSSEC emphasizes mathematical thinking on all its courses and in all its lesson resources and workshop guides.

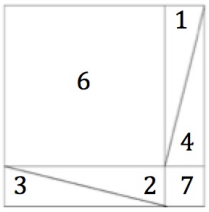
AIMSSEC courses prepare teachers to run local, collaborative learning, professional development workshops for teachers in their own communities, without the need of an expert leader. In the AIMSSEC book 'Mathematical Thinking in the Lower Secondary Classroom' (Hopkins et al, 2016) each chapter is a workshop guide and, importantly, many more workshop guides for teaching students of all ages from 9 years old to school leavers are freely available online [9] published under a creative commons licence. Each guide focuses on one of many teaching strategies to help teachers to develop their own teaching repertoire.

These guides, along with many other lesson resources, can be downloaded using the AIMSSEC App [10] and used without connection to the internet. Each workshop guide is designed so that groups of teachers working together, can discuss approaches, deepen their understanding of the related mathematics, and try out teaching strategies for learner-centred lessons that often involve problem solving and guided re-invention.

As an example of a lesson activity using guided re-invention to develop logical thinking and the confidence and ability to prove mathematical theorems, consider the activity below. [11] Students are required to give reasons for all their answers.



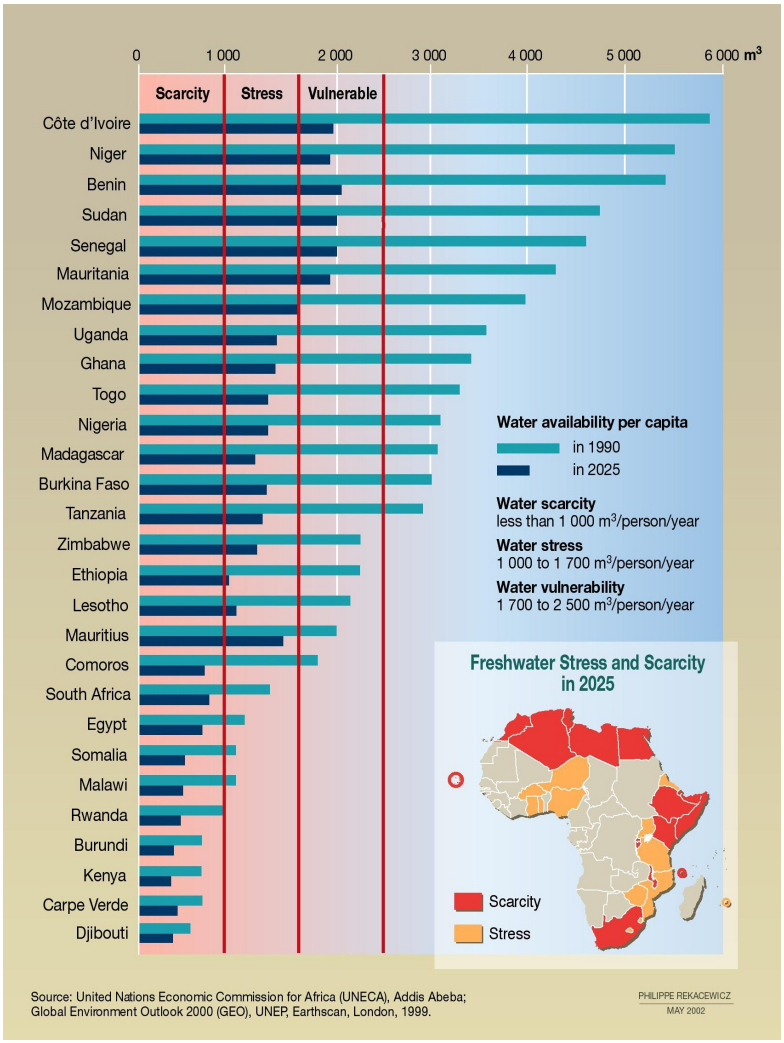

1. Cut 4 congruent right angled triangles from some scrap paper.
2. Arrange them as shown in diagram 1 in your notebook.
3. What do you notice about the quadrilateral labelled 5?
4. Draw the square frame that this arrangement fits into exactly.
5. How can you be sure that the frame is a square?




6. Place the 4 triangles inside the square frame as in diagram 2.
7. What do you notice about the quadrilaterals labelled 6 and 7?
8. What can you deduce about the areas of quadrilaterals 5, 6 & 7.
9. Does the same hold for any 4 congruent right angled triangles?
10. What have you proved?

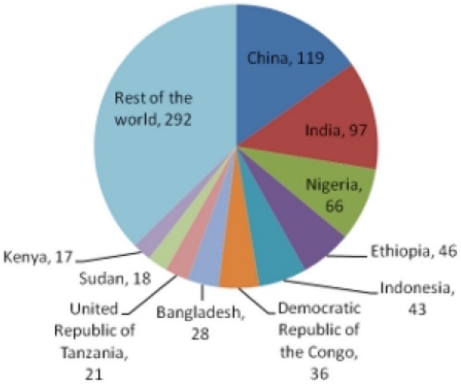
AIMSSEC lesson activities are designed to promote all the competences. Babies are expert independent learners who learn to walk and talk without being taught; there is research evidence that babies between 7 and 33.5 months can learn a second language in just one hour a day over 18 weeks [12]. Most children go to school full of curiosity, asking frequent questions and keen to learn. Good teaching can keep that independent

learning competence and curiosity alive, and also ensure that children do not become maths-phobic.




One of the competences specified in the Rwandan curriculum is '**be able to apply their subject learning in a range of situations'.**

This AIMSSEC lesson activity 'Drinking Water' [13] engages children in applying the maths they learn in school to problems in the real world, links to what they learn in geography and science, calls on them to analyse the information, to discuss reasons and remedies, and to be better able to evaluate data presented in the media.




This United Nations Childrens' Fund pie chart [14] shows the global locations of 783 million people who do not have safe drinking water. The total world population in 2017 is 7.5 billion (7,500,000,000). What percentage of the world population is without safe drinking water?

The world population is forecast to rise to 8.2 billion by 2025. What percentage increase is this from 2017 to 2025? Do you think the same percentage of people will be without water in 2025, or more people or fewer people?

The angle in the pie chart for China is 55o. How would you work this out? Calculate the angle in the pie chart for Nigeria.

The second chart [15] compares water availability per person across Africa in 1990 with the expected availability in 2025. Study this chart. What does it tell you?

Which countries will be vulnerable in 2025, having just enough water, that is more than 1700 cubic metres of water per person per year, but being at risk of shortages.

Use the red lines in the chart to find the number of countries with plenty of water, the number vulnerable, the number stressed and the number with water scarcity in 1990 and 2025 ?

Why do you think that the problem of water shortage is getting worse? Write a short report on this problem.

**Barriers that deprive many children in Africa of access to quality education**

The AIMSSEC Team get to know the teachers who come on our residential courses very well and, from what we observe, and what they tell us, we know that the following barriers deprive children in many schools in Africa of access to quality education.

1. Isolation of rural communities.
2. Lack of mathematically qualified teachers.
3. Lack of experience of, and lack of resources to support, modern teaching methods.
4. Large classes, typically 60 or more.
5. Language of instruction being a challenge for both teachers and learners.
6. Lack of equipment and basic amenities in schools.
7. Unequal provision for educating girls e.g. teenage pregnancy, families who cannot afford school fees for government schools sending sons not daughters to school.
8. Disaffection of teenagers due to poverty and lack of opportunity.
9. Recent conflict that has led to interrupted education.
10. Poverty and malnutrition affecting the children.
11. Long distances from home to school.
12. Children having to work in the home or fields rather than go to school.

AIMSSEC designs its learning activities with all this in mind. In particular, our courses and resources are tackling barriers 1 to 6. One of the AIMSSEC courses focuses on the issues relating to developing fluency in the communication of mathematical concepts where the students and the teacher seldom speak English outside the classroom. AIMSSEC lesson activities give many ideas for using cost free, and often scrap materials, for learning aids. AIMSSEC activities are also designed to be possible in large classes.

There is much less that AIMSSEC can do about the other barriers but it is important for the AIMSSEC Team to be aware of the difficulties and challenges faced by the teachers we work with, and to tailor our support to them individually according to the conditions under which they are working. We do this by having small tutor groups of 8 to 10 teachers on our residential courses, and making it a priority to get to know them individually, for example, by sitting with them at mealtimes and having tutor group meetings. Many AIMSSEC assignments require teachers to plan, conduct, and write reflective reports on lessons for their students, and on workshops for other teachers. Tutors assess their own tutee's assignments according to a criterion referenced matrix and give feedback and support taking into account each teacher's working conditions.

**Names and descriptions of some teaching methods and styles.**

The AIMSSEC approach to teaching mathematics in school, incorporates most of the features of the various teaching methods listed in this section. It involves flexibility, and adaptation to both the needs of the class and the constraints of the learning environment, while striving to provide quality education as well as varying the teaching strategies and learning activities to motivate the students.

Mathematics lectures to undergraduates at university level are usually 'teacher-centred' with the lecturer doing most, or maybe all, of the talking, and the students listening, trying to understand, and taking notes. There are notable exceptions to this methodology. **The Moore Method** is named after Robert Lee Moore, a well known topologist. Students had to formulate their own definitions and use them to prove theorems that they had to explain and present to the class. Moore would suggest counterexamples to ensure that the students arrived at workable definitions. Not as

much subject content can be taught in this way as in a conventional lecture course, but students generally have a deeper understanding of what they have created. As a mathematician colleague of mine once said "I certainly can teach more mathematics in the set number of lectures but it is not clear that the students will learn more".

The **Hungarian Problem Solving Method** promotes problem solving, mathematical creativity, and communication. There is a strong problem solving culture in Hungary. Problems from the very popular monthly Kömal Magazine [16], published since 1894, are often a family talking point, school student's own solutions are published, and the top problem solvers earn places at the top universities. In a typical problem solving lesson the teacher poses problems chosen to foster students' reasoning, problem solving and proof writing competences. Typically the teacher will be at the back of the classroom, and students write on the board. Students learn concepts by working on carefully sequenced problems that have the complexity and structure to promote mathematical thinking and perseverance. A spirit of cooperation must be fostered so that nobody is made to feel foolish if they make a mistake; indeed, mistakes are seen as learning opportunities. Students are encouraged to ask questions of each other and suggest different methods, some of which will work and others perhaps not.

This teaching method occurs all over the world and is particularly common in Eastern Europe. AIMS students come from 42 different African countries to take the Masters in Mathematical Sciences course. Interestingly, over the last 15 years, students from Madagascar, who consistently do very well, describe to me year after year, their own experiences of learning mathematics in school and university, that are very similar to the Hungarian Problem Solving Approach. I suspect, from asking AIMS students from the other 41 countries similar questions, that this is not true for other African countries.

The founder of **constructivism** was Jean Piaget, a Swiss psychologist whose ideas on child concept development widely influenced teaching methods in the middle of the 20th century. Vygotsky, who wrote "learning is a necessary and universal aspect of the process of developing culturally organized, specifically human psychological function" ([17] Vygotsky 1978, p. 90), regarded constructivism as learning from experience that can be influenced by teachers and society and may precede and promote development.

**Guided reinvention,** that led to **Realistic Mathematics Education,** is based on the concept, introduced by the Dutch mathematics educator Hans Freudenthal (1905 – 1990) of mathematics as a human activity in which students can be guided by a sequence of questions to re-invent the mathematics for themselves. The following quotation explains the ideas behind Freudenthal's philosophy and methods.

"No mathematical idea has ever been published in the way it was discovered. Techniques have been developed and are used, if a problem has been solved, to turn the solution procedure upside down, or if it is a larger complex of statements and theories, to turn definitions into propositions, and propositions into definitions, the hot invention into icy beauty. This then if it has affected teaching matter, is the *didactical* inversion, which as it happens may be *anti-didactical*. Rather than behaving anti-didactically, one should recognise that the learner is entitled to recapitulate in a fashion of mankind. Not in the trivial matter of an abridged version, but equally we cannot require the new generation to start at the point where their predecessors left off." [18]

**Inquiry based learning** is related to the Discovery Based Learning of the 1960s which is also known as the **Heuristic Method**. It has the following features:

- Inculcates a habit of inquiry and research;

- Enables students to discuss, ask, listen, explain;
- Emphasises understanding;
- Students obtain and explain evidence to support their answers;
- Students create arguments and justify their explanations;
- Students also create questions of their own;
- Develops a scientific attitude.

In this method, students are put in the place of an independent discoverer. The teacher sets a problem for the students and then asks them to discover the answer. Here students get only a little help from the teacher. Students are excited when they can find solutions by experiments, group discussions, or any other means.

## The use of ICT in the learning and teaching of mathematics

The majority of teachers who come on AIMSSEC courses have never used a computer before. Some teachers come from schools where the government has supplied tablets or computers or smartboards but there has been no training or very minimal training. Most of these teachers are unaware of the educational software that their learners might use or how to load such software onto their devices.

All the courses in the AIMSSEC 7-course 2½ year subject leader training programme have 3 strands: pedagogy, mathematics and ICT. Teachers connect to wi-fi via their smartphones and engage in professional discussions and uploading and downloading files within the AIMING HIGH Teacher Network [9], the AIMSSEC App. Moodle and WhatsApp. The ICT courses cater for all levels of IT experience and knowledge, from the most basic skills such as engaging with a network, and using features of mathematical software that are particularly useful in teaching mathematics, to more advanced skills of selecting, evaluating and using free mathematical software like GeoGebra, using a statistics packages that can handle big data, and coding. They also learn how to use smartboards and are able to take the International Computer Driving Licence in Education [19].

It is very important that teachers receive training in ICT, are able to make the best possible use of hardware and software available and know how to apply for refurbished computers and other equipment for their schools to charities such as Computers for Schools and ASOL [20] and to companies in their home regions.

## Conclusion

Considering the importance of education, and the importance of mathematics in particular as it underpins all the other sciences, it is time that teaching became a more respected and desirable job that attracted more of the brightest young people and that the quality of teaching both in schools and universities was given higher priority.

Schools and universities of the future will inevitably be very different from schools and universities today as students increasingly use technology to advance their learning, often following individual learning pathways rather than being taught at the pace of slower learners in their class. The role of the teacher responsible for guiding students' learning pathways in schools and universities will be significantly more demanding and require significantly higher levels both of subject knowledge and of management skills. A good school or university in the future will also be a social community offering group learning activities both academic and also in sport, music, and drama as well as in arts and crafts and those accomplishments in which human beings can still outperform

machines. However students will be able to learn at home and may only engage with a school or university for assessment and to obtain a recognized qualification. Let's hope this all happens.

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