

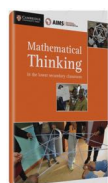
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP COVER PAGE

These guides are designed to empower teachers to run their own professional development collaborative learning workshops without the need for an expert leader, and to support teachers in developing more effective ways of teaching and a deep understanding of the mathematics they teach.

One teacher might organize the time, date and venue but once they are doing the activities together they should all participate on equal terms in discussion and reflection.

AIMSSEC Workshop Guides are available on the AIMSSEC App <http://aimssec.app>, on the AIMING HIGH Teacher Network website and in the Mathematical Thinking book. Teachers and teacher educators can use these guides on their own or as one of a group of teachers.

The workshops enable **teachers to open new horizons bringing new vision and new skills** to teaching and learning mathematics, to raise standards of mathematics attainment in the most disadvantaged communities and to improve life chances by giving children the education and skills needed to thrive in the 21st century.



Mathematical Thinking in the lower secondary classroom

Edited by Christine Hopkins, Ingrid Mostert and Julia Angileri

978-1-316-50362-1

Chapters in the AIMSSEC Mathematical Thinking Book are Lower Secondary Workshop Guides.

Buy the book online from [Amazon](http://amazon.com) or from <http://www.cambridge.org/za/education> search for AIMSSEC or for ISBN 9781316503621.

To order the book in South Africa go directly to <http://www.cup.co.za>

For reviews and curriculum maps see <https://aiminghigh.aimssec.ac.za/mathematical-thinking/>

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1


TITLE PAGE


Teaching strategy.
Curriculum content and learning outcomes
Summary of mathematical topic (FACT BOX.)


PAGES 2 & 3

WORKSHOP ACTIVITIES FOR TEACHERS

Two pages of activities for teachers to work through and discuss with colleagues.

For each activity there: is a list of resources needed .

suggestions for organising the activity (e.g. individual, pairs, whole class) .

suggestions for the time the activity will take .

when to pause, think and try the activity  and when to record your work .

PAGES 4 & 5

CLASSROOM ACTIVITIES FOR LEARNERS

Help for teachers on lesson planning. Advice on how long to allow for each activity, the resources needed and the key questions to ask to guide learning.

PAGES 6 TO 10

CHANGES IN MY CLASSROOM PRACTICE

Help for teachers on teaching strategies; additional resources and activities for use during or after the workshop; worksheets; solutions; templates; key questions for formative assessment; other key questions to guide learning and follow-up activities.

This workshop guide was written by the AIMSSEC Team



Number Games and Prime Numbers

Teaching strategy: Developing mathematical language and vocabulary

Curriculum content: Development of number sense: multiplicative patterns and relationships.

Prior knowledge needed: Secure counting skills and some knowledge of multiplication facts.

Intended Learning Outcomes: At the end of this activity teachers and learners will:

- ✓ Know how numbers are linked together in precise but different patterns and relationships;
- ✓ Understand these patterns and relationships follow certain rules and that these can be used and applied beyond the examples used;
- ✓ Be able to predict missing numbers and continue sequences where appropriate. Make reasoned judgements and use these to plan next steps;
- ✓ Identify factors, multiples and prime numbers;
- ✓ Have experienced mathematics learning in a fun and interactive way.

Fact box

A **factor** of a number is a number that can be divided into it exactly. 5 is a factor of 15.

Examples: The factors of 15 are 1, 3, 5 and 15.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 26.

A **multiple** of a number is made from adding the number to itself over and over again. So, for example, the multiples of 7 are: 7, 14, 21, 28....

A **prime number** is a number with exactly 2 factors. 2 is a prime number as it has exactly 2 factors, 2 and 1. 3 is a prime number as it has exactly 2 factors, 3 and 1. The number 1 is not a prime number as it has only one factor.

Example The factors of 12 are 1, 2, 3, 4, 6 and 12
and the **prime factors** of 12 are 2 and 3

Two numbers have a **common factor** when the same number is a factor of both. For example, 6 and 15 have a common factor of 3.

Resources:

1 – 100 grids for each participant or squared paper. Beans

Workshop Activities for Teachers

Activity 1: Multiplication Counting Patterns

 Whole group

 30 minutes

The Pattern Game All start sitting down.

All count in ones and emphasise the multiples of 3:

“1 ... 2 ... 3... 4 ... 5 ... 6... 7 ... 8 ... 9... 10 ... 11 ... 12...”

All stand every time a multiple of 3 is spoken and sit for the other numbers.

Everyone could clap on the multiples as an alternative to standing up and sitting down

Repeat this game until everyone feels secure with this counting. For a challenge try counting back from 40 standing for each **multiple** of 3.

Now repeat but stand each time a multiple of 5 is spoken: This time count:

“1... 2... 3... 4... 5(stand) ... 6... 7... 8... 9... 10(stand) ... 11... 12... 13... 14... 15(stand) ...” All stand for every **multiple** of 5

The Two Pattern Game

Now the fun begins!!

Identify half the group to stand when multiples of 3 are spoken and the other half stand when multiples of 5 are spoken.

“1... 2... 3(half stand) ... 4... 5(other half stand)... 6(first half stand)... 7... 8... 9(first half stand)... 10(other half stand)... 11... 12(first half stand) ... 13... 14... 15(all stand) ...”

When everyone stands these numbers are **common multiples** of both 3 and 5.

Notes:

Topic: Multiples

Start slowly so that no one is left behind. It may be helpful to have the counting numbers written on a board or use a large one hundred squares (see page 11) for everyone to see. The layout on the one hundred square will help identify number patterns, particularly multiples of 5.

After each game, ask:

What are the first numbers you stand on called? (They are the **multiples** of 3. They all have a **factor** of 3.) What are the second numbers you stand on called? (They are the **multiples** of 5. They all have a **factor** of 5.)

The game can be played again with different numbers. Easier numbers could be half stand for every multiple of 3 and others stand for every multiple of 10.

Activity 2 Making Factor Bugs

 10 minutes

First draw a body and a head for the bug. Write a number in the body.

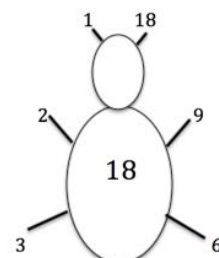
For each **factor** draw a leg and the other leg in the factor pair.

Write the factors on the legs.



Work through the numbers 2, 3, 4, 5, ...systematically so that no factors are missed. Finally draw antennae with the number itself and the number 1 as the factor pair.

Any bug with no legs and only antennae will be a prime number.

How many legs does Bug 48 have?



Activity 3: Patterns on 100 grid

 Give each teacher a 1-100 grid from the photo-copiable sheet on page 8, or use squared paper or one large grid for everyone to see.  30 minutes

Fill in the numbers 1 to 100 on squared paper if you don't have a grid.
Cross out the number 1 as it is not a prime number.
Circle the number 2. Put a line through every multiple of 2 up to 100.
Is there a pattern? What do you see?
Circle the number 3. Put a line through every multiple of 3 up to 100.
Is there a pattern? What do you see?
What do you notice about multiples of 4?
Circle the number 5. Put a line through all the multiples of 5 up to 100.
What do you notice about multiples of 6?
Circle the number 7. Put a line through all the multiples of 7 up to 100.
What do you notice about multiples of 8, 9 and 10?
What can you say about the numbers that aren't crossed out?
You might wonder why you don't cross out the first number of a table. That's because the numbers that aren't crossed out are the **prime numbers** – that have no factors other than themselves and 1. The numbers 2, 3, 5 and 7 are prime numbers.
There are lots of patterns to look for – numbers crossed out twice have two prime factors, numbers crossed out three times have three prime factors etc.
A Greek mathematician called Eratosthenes invented this systematic method of finding prime numbers. It is known as the Sieve of Eratosthenes. You could try extending it to higher numbers.

Notes

For another activity for younger learners, multiples can be coloured on separate grids to illustrate multiples of different numbers. See Multiple Patterns

<https://aiminghigh.aimssec.ac.za/multiple-patterns/>

The purpose of Activity 3 is to produce the prime numbers from 2 to 100. The class could discuss why the process stops with multiples of 7. (It's because when any number up to 100 has a factor greater than 10 it also has a factor less than 10 so that number has been shaded already). Colours can be used.

Discussion of pedagogical issues:

The Pattern Game is suitable for any year group and the Two Pattern Game can be introduced in Grade 3. Discuss with other teachers which multiplication tables you would introduce in each school year. Activity 3 is suitable for Grades 5 and above and, as an introduction, younger learners can shade the multiples of 2, 3, 4, 5,... on separate number grids and talk about the patterns made.

All three activities emphasise the same idea that some numbers are prime, some have few factors and others have lots. Doing different kinds of activities involving sight, hearing and movement to achieve the same learning outcome, gives all learners reinforcement and a chance to learn in the way which is most natural to them.

Classroom Activities for Learners

Activity 1: Counting Games



Whole class



Either game can be a 10 minute lesson starter often repeated

The Pattern Game

All start sitting down.

All count in ones and emphasise the multiples of 3:

“1 ... 2 ... **3**... 4 ... 5 ... **6**... 7 ... 8 ... **9**... 10 ... 11 ... **12**...”

All stand every time a multiple of 3 is spoken and sit for the other numbers.

Repeat this game until everyone feels secure with this counting. For a challenge try counting back from 40 standing for each multiple of 3.

Now repeat but stand each time a multiple of 5 is spoken: This time count:

“1...2...3...4...**5**(stand) ... 6... 7... 8... 9... **10**(stand) ...11...12...13...14...**15**(stand)...”

All stand for every multiple of 5

The Two Pattern Game

Now the fun begins!!

Identify half the group to stand when multiples of 3 are spoken and the other half stand when multiples of 5 are spoken.

“1... 2... **3**(half stand) ... 4... **5**(other half stand)... **6**(first half stand)... 7... 8... **9**(first half stand)... **10**(other half stand)... 11 ... **12**(first half stand)... 13... 14...**15**(all stand) ...”

When everyone stands these numbers are **multiples** of both 3 and 5.

Notes:

Start slowly so that no one is left behind. Sitting and standing is good fun but learners can remain seated and clap instead of standing.

After each game, ask: What are the first numbers called? (They are the **multiples** of 3 They all have a **factor** of 3.) What are the second numbers called? (They are the **multiples** of 5. They all have a **factor** of 5.) When everyone stands these numbers are **common multiples** of both 3 and 5.

Ideas for Teaching

Activity 1 is suitable for any yeargroup. With the youngest learners count only in 2's and 3's. Then introduce the Two Pattern Game and multiples of higher numbers in later years. It may be helpful to have the one hundred square for learners to see. The layout will help identify number patterns, particularly multiples of 5.


These are very simple games for some learners but cause more difficulties for others. They are useful for learners beginning their multiplication tables and for second language learners to practice saying the numbers in English. Actively involving the learners in standing will make the learning fun.

Can the learners tell you in advance which will be multiples of 3? Can they point to them on a one hundred square? You don't have to start counting at 1. You could start at 10: 10 ... 11 ... **12**... Or you could start at 20 and count backwards: 20 ... 19 ... **18**...

The two-pattern game will always cause a laugh as it becomes complicated focusing on both numbers at the same time. Talk about the number patterns using the words **multiple** and **factor**. Get learners to point to them on a one hundred square. Choose different numbers to help identify different patterns. For example, multiples of 3 and multiples of 6 would mean all stand for every multiple of 6. Reinforcing that all multiples of 6 must also be multiples of 3.

Activity 2: Grab it

 Beans (ideally each pair will have about 50 beans)

 15 minutes

Learners should play the game in pairs. They should take it in turn to grab a handful of beans. The aim of this game is to make as many different rectangular arrays as you can from your handful of beans.


Putting all the beans in one line does not count as a rectangular array. When you cannot make a rectangular array, how many factors does the number have?


For each array a player makes (and that the partner agrees with), score one point. Keep a record of the numbers you have grabbed and the rectangles made.

Some numbers give you lots of points. They are the numbers with lots of **factors**, such as 24 and 30.

Prime numbers can't make a rectangular array so the player scores zero on that turn.

Activity 3: Patterns on 100 grid

 One large 1-100 grid for all to see. Each learner needs to draw a grid on squared paper or a copy of the photo-copiable sheet (page 8).

 40 minutes

If squared paper is being used fill in the numbers 1 to 100.

Cross out the number 1 as it is not a prime number.

Circle the number 2. Count on 2 squares at a time and put lines through every multiple of 2 up to 100.

Is there a pattern? What do you see?

Circle the number 3. Count in 3s to put lines through every multiple of 3 up to 100.

Is there a pattern? What do you see?

What do you notice if you count in 4s giving the multiples of 4?

Circle the number 5. Count in 5s to put lines through all multiples of 5 up to 100.

What do you notice about the multiples of 6 if you count on in 6s from 6 to 96?

Circle the number 7. Put a line through all the multiples of 7 up to 100.

What do you notice about multiples of 8, 9 and 10?


What can you say about the numbers that aren't crossed out?

You might wonder why you don't cross out the first number. It's because the numbers that aren't crossed out are the **prime numbers** – that have no factors other than themselves and 1. The numbers 2, 3, 5 and 7 are prime numbers.

There are lots of patterns to look for – numbers crossed out twice have two prime factors, numbers crossed out three times have three prime factors etc.

A Greek mathematician called Eratosthenes invented this systematic method of finding prime numbers. It is known as the Sieve of Eratosthenes. You could try extending it to higher numbers.

Activity 4: Making Factor Bugs

 10 minutes

As described in the Teacher Workshop Activities on page 6. This is a good lesson starter. The teacher can ask the class to draw a factor bug for 30 (or whatever number she chooses), and then ask the class to hold up their drawings to show her. Or a learner could come and draw it on the chalkboard. Assign numbers to learners according to their knowledge of numbers.



PRIMARY NUMBER GAMES AND PRIME NUMBERS DEVELOPING MATHEMATICAL LANGUAGE AND VOCABULARY

Guide for your own self-help PD workshop. Resources for inquiry-based lessons.

Changes in my classroom

The Fizz Buzz version of The Pattern Games.

The Pattern Game and The Two Pattern Game (Activity 1) can be made a little more challenging for learners if, instead of saying the numbers aloud, they say FIZZ or BUZZ for every multiple. For example:

1, ... 2, ... FIZZ, ... 4, ... 5, ... FIZZ, ... 7, ... 8, ... FIZZ, ... 10, ... 11, ... FIZZ, ... 13, ... 14, ... FIZZ, ... and
1, ... 2, ... 3, ... 4, ... BUZZ, ... 6, ... 7, ... 8, ... 9, ... BUZZ, ... 11, ... 12 ... 13, ... 14, ... BUZZ, ...

Then the learners talk about when they heard FIZZ, when they heard BUZZ and when they heard both FIZZ and BUZZ at the same time.

Mental activities

Starting each lesson with mental activities will make the learners alert and ready for connecting new ideas to the knowledge they already have. Counting patterns for multiplication will give the learners a feel for numbers as some crop up frequently, such as 12, while others don't often appear, such as 13. Counting also goes beyond the times tables so that learners see numbers such as 48 have a factor 3.

Pedagogical Issues

Familiarity with patterns comes through oral and visual experiences. Involving learners actively in standing and sitting will make the learning fun and more memorable.

Helping learners remember

Making resources: Photocopy the one hundred square on the following pages and use a laminator to make a more durable classroom resource. A marker pen can be used to circle particular numbers, for example, multiples of 3. A large one hundred square displayed in the classroom will help learners see patterns of numbers.

Follow up activities

From the AIMING HIGH website.

You can also download the activities on your smartphone using the AIMSSEC App

<http://aimssec.app>

Prime Sieve: <https://aiminghigh.aimssec.ac.za/prime-sieve/>

Multiple Patterns: <https://aiminghigh.aimssec.ac.za/multiple-patterns/>

Path to the Stars: <https://aiminghigh.aimssec.ac.za/path-to-the-stars/>

Wholesome Rectangles: <https://aiminghigh.aimssec.ac.za/wholesome-rectangles/>

There are many good articles and learning activities on the NRICH website.

Read the article: Using arrays to explore numbers <http://nrich.maths.org/2466>

For more activities on multiples and factors see:

Factor Track <http://nrich.maths.org/7468>

Counting Cogs <http://nrich.maths.org/6966>;

Table Patterns Go Wild <http://nrich.maths.org/6924>

For similar activities to What's my number see:

I Like <http://nrich.maths.org/6962>

Which Numbers 1 and 2 <http://nrich.maths.org/6948> and <http://nrich.maths.org/6949>



Key questions

1. What is the link between the clapping (or standing) game and the patterns on the 100 square? Why would you use both activities in the lesson?
2. When will the learners use the words 'factor' and 'multiple'?
3. When multiples of 2, 3, 5 and 7 have been crossed out on the **Patterns on 100 grid activity**:
 - (a) What do you notice about the multiples of 8, 9 and 10?
 - (b) What can you say about the numbers that are not crossed out?
 - (c) The number 4 has 3 factors: 1, 2 and 4.
How many factors does 5 have?
List the factors of the other circled numbers.
 - (d) Why do you get all the prime numbers between 1 and 100 by marking all multiples up to multiples of 7? Why is it not necessary to cross out multiples of 11?

2. Using the 1 to 200 grid

Find all the prime numbers up to 200 by crossing out the multiples of 2, 3, 5, 7, 11 and 13.
Why is it not necessary to cross out multiples of 17 or any higher numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100