



WORKSHOP GUIDES FOR TEACHERS TO LEARN TOGETHER LANGUAGE AND COMMUNICATION OF MATHEMATICAL CONCEPTS PRIMARY AND SECONDARY

Guide for your own self-help professional development workshop
and resources for inquiry based lessons

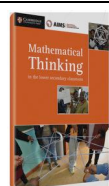
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to empower teachers to run their own professional development collaborative learning workshops without the need for an expert leader

The workshops enable **teachers to open new horizons bringing new vision and new skills** to teaching and learning mathematics, to raise standards of mathematics attainment in the most disadvantaged communities and to improve life chances by giving children the education and skills needed to thrive in the 21st century.

AIMSSEC Workshop Guides are available on the AIMSSEC App <http://aimssec.app> on the AIMING HIGH Teacher Network website and in the Mathematical Thinking book.

Teachers and teacher educators can use these guides on their own or as one of a group of teachers who meet together to talk about mathematics lessons as part of their professional development. Maybe one teacher will take the lead in organizing time, date and venue but once they are doing the activities together they should all participate on equal terms in the discussion and reflection.



Mathematical Thinking in the lower secondary classroom

Edited by Christine Hopkins, Ingrid Mostert and Julia Anghileri

978-1-316-50362-1

Chapters in the AIMSSEC Mathematical Thinking Book are Workshop Guides for Lower Secondary.

Buy the book online from Amazon or from <http://www.cambridge.org/za/education> search for AIMSSEC or for ISBN 9781316503621.

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EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1

TITLE PAGE

Teaching strategy.


Curriculum content and learning outcomes.


Summary of mathematical topic (FACT BOX.)



PAGES 2 & 3


WORKSHOP ACTIVITIES FOR TEACHERS

Two pages of activities for teachers to work through and discuss with colleagues.

For each activity there: is a list of resources needed ,

suggestions for organising the activity (e.g. individual, pairs, whole class) ,

suggestions for the time the activity will take , when to pause, think and try the activity ,

and when to record your work .

PAGES 4 & 5

CLASSROOM ACTIVITIES FOR LEARNERS

Help for teachers on lesson planning. Advice on how long to allow for each activity, the resources needed and the key questions to ask to guide learning.

PAGES 6 TO 10

CHANGES IN MY CLASSROOM PRACTICE

Help for teachers on teaching strategies; additional resources and activities for use during or after the workshop; worksheets; solutions; templates; key questions for formative assessment; and other key questions to guide learning.

Language and Communication of Mathematical Concepts

by Lindiwe Tshuma and Toni Beardon

Before moving to Witwatersrand University Dr Lindiwe Tshuma was an AIMSSEC primary specialist. Her interest in educational linguistics emerged from her experience as a primary mathematics and science teacher within the Deaf Section of a School for the Deaf and Blind in ThabaNchu, Free State Province. Having to present mathematics and science content through mediation of languages foreign to both the teacher and the learners made her realise the importance of harnessing subject content and the language of instruction instead of treating them as separate entities.



Toni Beardon has compiled this Workshop Guide using and adapting some of the material produced by Lindiwe and adding some of her own ideas and material.

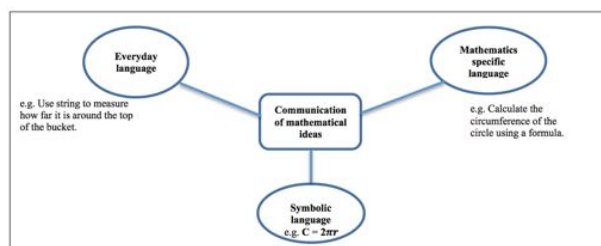
Language and Communication of Mathematical Concepts

Teaching strategy: Developing Mathematical Language and Vocabulary

Prior knowledge needed: Some experience of English (either as a Language of Instruction or mother tongue) and understanding of some mathematical terms.



Intended Learning Outcomes At the end of this activity teachers and learners will:

- Identify words and phrases in English that cause confusion to learners meeting them in mathematics.
- Understand the relationship between everyday language and the use of language in mathematics
- Appreciate the existence of a variety of word meanings in mathematical contexts
- Have experienced the use of games, puzzles and creative writing in mathematics.




Types of mathematical language


Fact Box: Examples of vocabulary misconceptions

Source of misconception	Examples	Source of misconception	Examples
Words shared by mathematics and standard English but with different meanings.	Right angle Right answer Reflection: flipping over a line Reflection: thinking about something Table: organising information Table: piece of furniture	Irregularities found in English spelling and usage	Four has a 'u' Forty does not
Words shared by mathematics and standard English with similar meanings, but with a more precise mathematical meaning.	Difference: answer to a subtraction problem Difference: general comparison Even: divisible by 2 Even: smooth or level	Specialist terms: mathematical terms only found in mathematical contexts	quotient decimal denominator isosceles
Mathematical terms sharing the same pronunciation as standard English words, but spelt differently.	Divide in mathematics means to separate into parts. In geography the continental divide is a ridge separates east/ west flowing waters. Variable in mathematics is a letter that represents possible numerical values; variable clouds in science describe a weather condition. Element in mathematics is a member of a set. In chemistry an element is one of the specific substances in the periodic table.	The same mathematical concepts expressed in more than one way	One quarter / one fourth / $\frac{1}{4}$
Informal everyday English words that are sometimes used incorrectly as if they were mathematical terms.	Round: a circle  Round: a number rounds off... 243 rounds off to 240 Square: a shape  Square: a number times itself $2^2 = 4$ Second: a measure of time Second: a location in a set of ordered items $1^{st} 2^{nd} 3^{rd} 4^{th} \dots$	Related mathematical words whose distinct meanings may be confused	Hundreds and Hundredths Numerator and Denominator Mode, Mean and Median LCM Lowest common multiple HCF Highest common factor
Mathematical terms sharing the same pronunciation with standard English terms	Sum / some Arc / ark Pi / pie	Imprecision: adapting an informal term and using it as a mathematical term	Diamond for rhombus Corner for vertex Cross for intersection Turn for rotate

Workshop Activities for Teachers

Activity 1: Ambiguous terms in mathematics

 Pairs or small groups, then whole group.

 1 hour

The Language and Learning Challenge

Language and Mathematical Learning are a double challenge in that learners must develop competence in both the language of instruction and also in the language of mathematics. Development of mathematical learning and understanding is interrelated with language capability. In doing this activity participants will think about mathematical words and their mathematical meanings and about common meanings of the same words in everyday English. Words have precise meanings in mathematics but, when they occur in everyday language, a word often has several different meanings, sometimes with one meaning close to the mathematical meaning but used less precisely. This can be very confusing for learners.

Instructions: Form 4 groups. Each group will consider a situation in which learners face difficulties from the use of words in mathematics that, in common usage, have several different meanings. Each group will work on different examples so as to introduce as many examples as possible in a limited time. Each group will make a poster using flip chart paper and board markers, then explain their poster to the class. If your group is large then share the work, each of the subgroups making their own poster. The collection of posters will make a wall display. **The beauty of this activity is that it gives teachers a strategy to use in their lessons when new words arise, using the examples provided and other words. The ‘two definitions and two sentences’ format can be used by the students and teachers to make a class resource reference bank.**

Group 1. Words shared by mathematics and standard English with different meanings.

In everyday English a word often has many meanings. Here we have just picked common meanings.

Work in a pair or small group to make a poster. Exactly as in the examples below of the words ODD and POWER write definitions of the mathematical meaning of *some* of the following words and also giving a common English meaning. *If you don't know the meaning then you could use a dictionary or look the word up on the internet.* Make up sentences using the words as in the examples. You will have 20 minutes to finish your task, then a representative of your group must explain what you have done to the class and stick your poster up on the wall.

Right, Even, Table, Reflection, Mean, Volume, Operation, Value, Point, Base

Can you think of more mathematical words having different meanings in everyday English?

WORD	ODD
Mathematical meaning	A whole number that is not divisible by 2. A number that is not a multiple of 2.
Mathematical sentence	The numbers 13 and 15 are odd because when divided by 2 the remainder is 1.
Common English meaning	Unusual, strange or peculiar. Not like other things of the same type.
Example sentence	He is very odd, he has painted yellow and purple stripes on his face and clothes.
WORD	POWER
Mathematical meaning	How many times a number is multiplied by itself, as in square, cube, the power 4 etc.
Mathematical sentence	Two squared is 2×2 . Two cubed is $2 \times 2 \times 2$. ‘Two to the power 4’ is $2 \times 2 \times 2 \times 2$
Common English meaning	Being able to direct or influence the behaviour of others.
Example sentence	He is a powerful leader. He had a stroke and lost the power of speech.

Group 2. Words shared by mathematics and standard English with comparable or similar meanings, but with a more precise mathematical meaning. In everyday English the words have many meanings and we have picked common meanings. What other meanings do you know?

Work in a pair or small group to make a poster. Exactly as in the examples below of the words PARALLEL and ADDITION give a common English meaning that is slightly different from the mathematical meaning and give the mathematical meaning of *some* of the following words. *If you don't know the meaning then you could use a dictionary or look the word up on the internet.* Make up sentences using the words as in the examples. You will have 20 minutes to finish your task, then a representative of your group must explain what you have done to the class and stick your poster up on the wall.

Relation, grid, similar, property, circular, sign, place, average, mode, square, product

Can you think of more mathematical words having similar meanings in everyday English?

WORD	PARALLEL
Mathematical meaning	Two lines are parallel if they never meet even when continued on indefinitely.
Mathematical sentence	The opposite edges of a rectangle are parallel. They are always the same distance apart.
Common English meaning	A person or thing that is similar or analogous to another.
Example sentence	She draws a parallel between achievement and hard work.

WORD	ADDITION
Mathematical meaning	The process of finding the sum of numbers or other quantities under specific rules.
Mathematical sentence	By addition we find that the sum of 12 and 5 is 17, and the sum of $3x^2$ and $5x^2$ is $8x^2$.
Common English meaning	The process of including something else in a collection or the thing that is added.
Example sentence	It's very cold there so a warm coat would be a useful addition to your wardrobe.

Group 3. Mathematical terms sharing the same pronunciation as standard English words, but spelt differently.

The English words may have many meanings, we have picked common meanings. What other meanings do you know?

Work in a pair or small group to make a poster. The following words sound similar in mathematics and in everyday English. Exactly as in the examples below of the words SOME and SUM give a common English meaning and a mathematical meaning, and this time give the meaning of the word that only sounds similar to the mathematical term as in the example. *If you don't know the meaning then you could use a dictionary or look the word up on the internet.* Make up sentences using the words as in the example. You will have 20 minutes to finish your task, then a representative of your group must explain what you have done to the class and stick your poster up on the wall.

Ark and Arc. Pie and Pi.

Can you think of more everyday English and mathematical words that sound the same?

WORD	SUM
Mathematical meaning	A sum is the total found when quantities are added.
Mathematical sentence	The sum of 12 and 5 is 17, and the sum of $3x^2$ and $5x^2$ is $8x^2$.
Common English meaning	A sum often means a particular amount of money or something else.
Example sentence	He only knew the names of two birds, that was the sum of his knowledge about birds.
	SOME
Common English meaning	An unspecified amount or number of... or referring to a person who is not named.
Example sentences	He made some money selling tickets. I talked to some teacher about education.

Group 4. Informal everyday English words that are sometimes used incorrectly as if they were mathematical terms. This can cause confusion for learners but learners need to learn the correct mathematical terms.

Work in a pair or small group to make a poster. The following informal terms from everyday English are sometimes used in place of the correct mathematics words. Give the meaning of the mathematical term and the meaning of the word from everyday English, as in the example DIAMOND for RHOMBUS. *If you don't know the meaning then you could use a dictionary or look the word up on the internet.* Also make up sentences using the words as in the example:

**Corner for vertex. Crossing point for intersection. Turn for rotate.
Side for face. Side for edge.**

Can you think of more everyday English words that are sometimes used in place of the correct mathematical words?

WORD	DIAMOND FOR RHOMBUS
Mathematical meaning	A rhombus is a quadrilateral with 4 edges of equal length.
Mathematical sentence	A square is a special type of rhombus, the only rhombus whose angles are 90° .
Common English meaning	A diamond is a precious stone used in jewellery and also in cutting instruments.
Example sentence	One of the suits in a pack of cards is diamonds. A baseball field is called a diamond.

Activity 2: Playing with words

⌚ 10 minutes

Teaching strategies for language and communication development.

Language and communication are essential in teaching and learning mathematics. Playing with words and making jokes is a good strategy for engaging students in language learning, exploring meanings and spelling.

Discuss: (1) How you might use the following jokes in your lessons.


(2) Can you think of other suitable puns or play on words or perhaps jokes that use 2 languages?


- Question: Why was 6 afraid of 7? Answer: Because 7, 8, 9
- Question: What did zero say to 8? Answer: Nice belt!
- Question: Why won't Ada drink a glass of water with 8 pieces of ice in it? Answer: It is too cubed?
- Question: Why don't you do arithmetic in the jungle? Answer: Because if you add $4+4$ you get ate.
- I ordered a takeaway from the Chinese last night. I ordered a 23, 13, 31 and 79. I had to send them back. Why? Answer: Because they tasted odd.
- Did you hear about the mathematician who's afraid of negative numbers? He stops at nothing to avoid them
- Surgeon: I have so many patients, who do I work on first? Nurse: Use the order of operations.

Activity 3 Final group discussion The activities in this guide are not designed for a series of lessons but to be used at different times with different age groups in primary and secondary mathematics lessons. If you have time work through and discuss the remainder of this document.

Classroom Activities for Learners

Activity 1 Creative writing in mathematics

 Individual

 30 minutes

Read the poem, then write a poem of your own by completing the two given statements, stating at least three reasons for your choice. Instead of writing about shapes you may write about any other concept like number, graphs, functions etc. This makes the activity adaptable to different topics.

If I were a shape, I would be a *circle*
I would have neither end nor beginning
I would be *infinite*
You would see me everywhere
You would see me every day
On your watch, the shape of the Sun
Even when you eat
My *circumference* loves *pi*
But my *radius* loves the *inverse of pi* half as much as my *circumference*

If I were a shape, I would not like to be a *square*
Everything would always be the same
Everyone would know me wherever I go
Even when I am stretched,
I would still just be a *special square*
You would find me in any *area*... how boring!


Althea Baartman SP teacher (2015) Aloe Junior High School, Cape Town


If I were a ... I would be a ... because ...

If I were a ... I would *not* be a ... because ...

Note: Once learners formulate the first two sentences, they build up more impressive lines and begin to play with the rhythms created by lengthening sentences. Poetry writing following this format, or more freely, or writing short stories or songs, strengthens vocabulary learning in mathematics. Creativity can include cartoons, jokes and illustrations and learners of all abilities can benefit, choosing their own theme or writing on a particular mathematical concept. Short creative writing sessions can be introduced to add variety to any lesson or can span a longer period of time to allow learners to find more information on the concept.

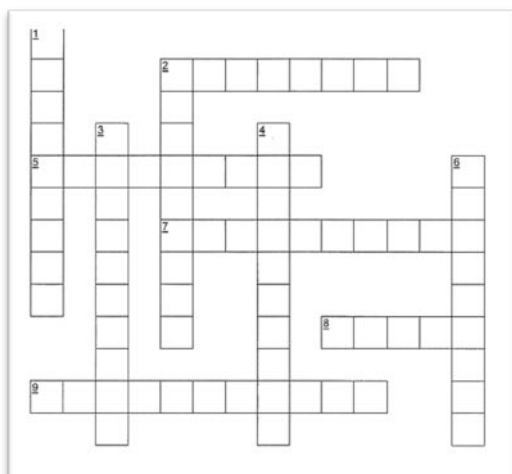
Activity 2: Crossword puzzles

 Individuals or Pairs

 30 - 40 minutes

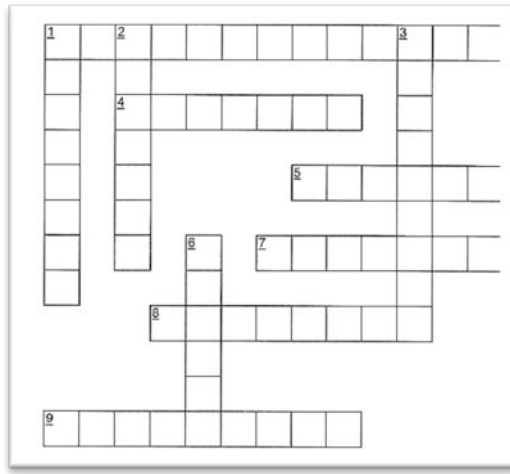
A: Crossword puzzle: Double digit addition.

Instruction: Work out the sums and use the answers to fill in the crossword puzzle below:



B: Crossword puzzle: Geometric shapes

Instruction: Use the given clues to fill in the crossword puzzle below:



Across		Down	
2	53 + 8	1	57 + 25
5	13 + 13	2	28 + 36
7	10 + 37	3	37 + 39
8	23 + 17	4	41 + 12
9	20 + 17	6	29 + 63

Across		Down	
1	A four-sided polygon with two pairs of parallel sides.	1	A five-sided polygon.
4	An eight-sided polygon.	2	A four-sided polygon with all four sides of equal length.
5	A quadrilateral with four equal sides and four right angles.	3	A quadrilateral with four right angles and two sets of parallel opposite sides that are equal.
7	A six-sided polygon.		
8	A polygon whose sum of interior angles equals 180°	6	A collection of points in a plane that are the same distance from a centre point.
9	A four-sided polygon with one pair of parallel sides.		

WORD BANK:		WORD BANK:	
eightytwo,	fiftythree,	circle,	hexagon,
seventytwo,	forty,	octagon,	triangle
fortyseven,	ninetytwo,	parallelogram,	pentagon,
seventysix,	sixtyfour,	rectangle,	rhombus,
sixtyone,	thirtyseven,	square,	septagon,
twentysix.		trapezium.	

Activity 3 Word Searches



Pairs

⌚ 30 - 40 minute

A: Word Search: Multiplication table of 2				B: Word Search: Addition up to 20			
Instruction: Work out the multiplications and cross out the answers on the word search below. Words can be horizontal, vertical or on a diagonal and they can go in any direction.				Instruction: Can you do these simple addition sums and spell the number words correctly? Work out the sums and cross out the answers on the word search below:			
	3 × 2	11 × 2	2 × 2		4 + 14	10 + 8	7 + 4
	7 × 2	6 × 2	8 × 2		11 + 4	8 + 6	1 + 8
	10 × 2	1 × 2	12 × 2		12 + 7	2 + 5	15 + 2
	5 × 2	9 × 2	4 × 2		14 + 2	13 + 3	15 + 5
					11 + 2	11 + 1	10 + 10

Ideas for Teaching: This is a good activity for learners to practise spelling. For different levels of challenge only provide the word bank for younger or weaker learners or only for learners to check answers.

<p>A M C N R H J N L T E S</p> <p>D I O E S L U E W D I J</p> <p>N O W T Y T N E W T G P</p> <p>D E O W T D N T O R H F</p> <p>T Z E F W T K H K U T N</p> <p>U W O T Y W N G Z X E W</p> <p>Y U E F R L B I J E I O</p> <p>R P O N U U V E T N X S</p> <p>L U G K T Z O X J I U I</p> <p>R Y W S G Y I F M R I N</p> <p>Q N X P B S T W E L V E</p> <p>K B C C B C H Y S V B X</p>	<p>T W E L V E N D F J L N K T N</p> <p>U F G K Q K I E N V P M H W E</p> <p>S T H Z G F N E E B L I Z E E</p> <p>Y E A H M M E X I T R J C N T</p> <p>D O V H S T T K U T R R P T F</p> <p>B D Z E X U E J E V J U E Y I</p> <p>L E H I N J E E N X M I O Y F</p> <p>U O S M Q T N B C G G G M F F</p> <p>W N L E S E E K X H A C D J X</p> <p>Z E E H E E B E T M B X Z M T</p> <p>W I G N Q L J W N E V E S U R</p> <p>J N N Z I Q E V H M B G N U P</p> <p>L E K U X N U V U N H K F X Z</p> <p>T N E E T H G I E X R C L I H</p> <p>H H Y U E W V O U N D U U S X</p>
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Changes in my classroom practice

“Mathematics education begins and proceeds in a language, it advances and stumbles because of language and its outcomes are often assessed in language.”

Durkin and Shire (1991)

Teaching strategies for language and communication development.

Language and communication are essential in teaching and learning mathematics. Language is used as the main tool in the transmission of mathematical knowledge, values and beliefs, as well as cultural practices. It enables teachers and learners to communicate with each other. In the transition from mother-tongue to the stage when learners embrace this new tool and can use it comfortably, the English language itself, as the Language of Instruction (LOI), has to play an increasing role alongside mother-tongue communication. As students progress through school, competence in the LOI becomes a prerequisite for success on the learning journey.

The Language & Learning Challenge

Mathematics has its special vocabulary, technical term, phrases, conventions for using symbols to represent the words, and methods of arguing within a given situation. Learners must be introduced and get to know all of these through the use of natural language.

Several strategies can be adopted by the teachers to minimise confusion about meanings that arise from the different ways that language is used in a mathematics classroom. These strategies include:

- monitoring the ambiguity that arises when the same word has different meanings;
- exploring meanings and making a class reference system, for example as a wall display;
- enriching contextual cues to remind learners of mathematical meanings (people maths and illustrations);
- confronting ambiguity and exploiting it to the learners’ advantage, for example by encouraging learners’ creative work like writing a poem or making jokes.

Monitoring lexical ambiguity

In preparing materials for use in mathematics lessons, the teacher must consider whether any words might have a different meaning for the learner from that intended or assumed in the specialist context. Sensitivity to the learners’ needs is increased if the teacher is aware in advance of potential misconceptions.

The teaching strategy introduced in the first Workshop Activity gives teachers a method for helping learners to understand the new words that they meet in mathematics and, at the same time, to help them to develop fluency in English by seeing examples of the word used in different ways in everyday talk.

Teachers are generally concerned with 3 parallel learning objectives. Code switching, that is teaching in the LOI but sometimes using the mother tongue to explain the mathematics, is often used as a means of bridging gaps between them.

1. to understand a new concept and what they hear the teacher say about it
2. to talk about the idea in their mind and to communicate accurately in the LOI
3. to write about the idea correctly using words in the LOI and written symbols

Exploring meanings: Reference systems and wall displays

Dictionaries can be used very effectively when teachers adopt a policy that learners should take responsibility for their own learning and should develop competence in learning independently. In a disadvantaged community where the main language is Xhosa, the author observed a Year 6 class working in groups and making 3D models of polyhedra from scrap card. When learners did not know the meaning of a word they went to the bookshelf and brought a dictionary back to their table and found the meaning. The class had several different dictionaries for reference purposes. From time to time the teacher asked about a particular word and wrote it on the chalkboard. Several learners fetched dictionaries and read definitions of that word and the class discussed the varied definitions and how they should understand and use the word.

Wall displays are also important, can be made by teachers and learners and don’t need to cost much. Better displays with less work can be set up if teachers collaborate to compile the definitions and illustrations. Teachers can search the internet for ideas and inspiration for making their own displays.

Enriching contextual cues

When mathematical definitions need to be distinguished from everyday or scientific meanings, the teacher should encourage the learners to discuss the differences and draw pictures or construct meaningful sentences to contrast the two meanings, for example:

- The *difference* between my two bags is that one is red and the other is blue.
- The *difference* between 12 and 7 is 5 because $12 - 7 = 5$.

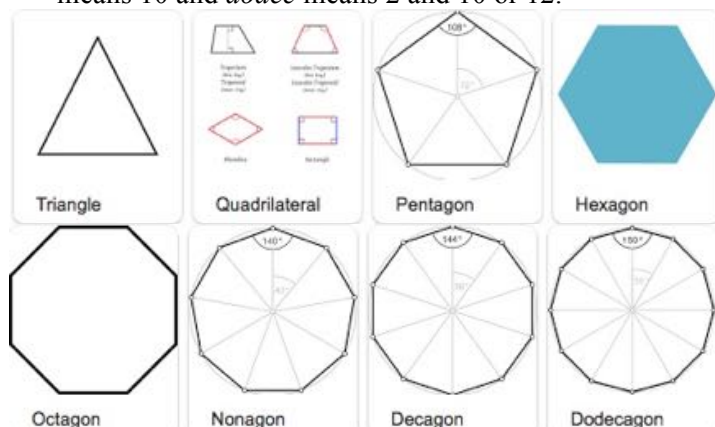
Enriching strategies and practices to adopt in every classroom include: keeping a word wall; posting new words on the wall alongside their definitions and illustrations to make the new words more meaningful; use of dictionaries; journal entries of new words, creative writing and illustrating, jokes, games and word puzzles.

Exploiting ambiguity to advantage

Word origins can be used to clarify distinct meanings of mathematics word pairs that are commonly confused such as (*numerator* / *denominator*). The root *nom* means to name, hence the *denominator* names the fraction, while the *numerator* tells the *number* of parts of interest. Thus $\frac{3}{5}$ means that something is divided into *five* equal parts, and we are referring to *three* of those parts.

Analysing word origin allows teachers to share with learners the ‘words behind the words’ so that learners connect terms that sound foreign to the words they already know.

- *Parallel* comes from ‘alongside’ ... (*para-*) as in a paramedic who works alongside medical professionals.
- *Percent* means *divided by 100* from ‘each hundred’.
- Do the class know the word ‘polygamy’ which means having more than one mate? The word *polygon* means ‘many-angles’. It comes from the Classical Greek *πολύς* (*polús*) "much", "many" and *γωνία* (*gōnía*) "corner" or "angle".
- The word *polyhedron* means ‘many-faces’. It comes from the Classical Greek *πολύεδρον*, that is *poly-* meaning many + *hedron* meaning base or face.
- In the names of polygons and polyhedra *tri* means 3, as in tripod and tricycle, *quad* means 4, as in quadrangle and quadruplets, *pent* means 5, *hex* means 6, *sept* means 7, *oct* means 8, *non* means 9, *dec* means 10 and *dodec* means 2 and 10 or 12.






Confronting ambiguity

Workshop Activity 1, and others like it bring potential conflicts of meaning into focus and are examples of teaching strategies to develop learners’ appreciation of the varieties of meanings in other contexts of the mathematical words they learn.

Young learners enjoy jokes and, as they become more sensitive to linguistic ambiguities, teachers can introduce word play, jokes, puns, and riddles and encourage learners to make up their own. Activities like quizzes, word searches, crossword puzzles and creative writing in mathematics are useful both for practice and for learning vocabulary and spelling.

Some examples:

<p>Why was 6 afraid of 7?</p> <p>Because 7, 8, 9.</p> <p>7</p>	 <p>HELP LEARNERS TO PLAY WITH WORDS</p> <p>I ordered a takeaway from the local Chinese last night. I ordered a 23, a 13, a 31 and a 79.</p> <p>I had to take them back. They tasted odd.</p>	<p>Question: Why won't Ada drink a glass of water with 8 pieces of ice in it?</p>  <p>Answer: It's too cubed.</p>
<p>Question: What does zero say to eight?</p> <p>Answer: Nice belt!</p> <p>0 8</p>		<p>Surgeon: I have so many patients! Who do I work on first?</p>  <p>Nurse: Simple. Use the order of operations.</p> 