WORKHOP GUIDES FOR TEACHERS TO LEARN TOGETHER

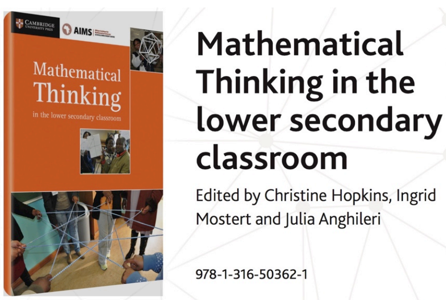
UPPER SECONDARY NA5 GROWTH AND DECAY

Guide for your own self-help professional development workshop and resources for inquiry based lessons.

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from Amazon or from http://www.cambridge.org/za/education Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to http://www.cup.co.za

For reviews and curriculum map see https://aiminghigh.aimssec.ac.za/mathematical-thinking/

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From time to time when you connect to the internet you can update your AIMSSEC App content with recently added resources.

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 **TITLE PAGE**

- Teaching strategy focus. *Each guide focuses on and exemplifies a frequently used teaching method.*
- Curriculum content and learning outcomes.
- Summary of mathematical topic (FACT BOX.)
- Resources needed

PAGES 2 & 3 **WORKSHOP ACTIVITIES FOR TEACHERS**

Two pages for you to work through with your colleagues. These activities are to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to pause, think and try the activity, and when to record your work.

PAGES 4 & 5 **CLASSROOM ACTIVITIES FOR LEARNERS**

Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 - 10 **CHANGES IN MY CLASSROOM PRACTICE**

Pages on using the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates. For follow-up activities you will find lots more lesson activites on the AIMING HIGH Teacher Network https://aiminghigh.aimssec.ac.za/category/lesson-activities/

UPPER SECONDARY GROWTH AND DECAY

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Growth and Decay

Teaching strategy: Learners presenting their ideas to an audience

Curriculum content: Growth (linear and exponential) and decay and applications to finance. Investment & loan options, depreciation, and other applications.

Prior knowledge needed: Arithmetic and geometric sequences. Learners do not need to have memorised specific formulae for the nth term or sum of the first n terms of an arithmetic or geometric sequence but should have worked on such problems.

Intended Learning Outcomes At the end of this activity teachers and learners will:

- Appreciate that money and goods can change value due to a range of factors (e.g. interest, inflation, depreciation, demand, supply)
- Know that geometric series, $S = a + ar^2 + ar^3 + \dots$, with a common ratio r less than 1 in magnitude $(-1 < r < 1)$ converge and other geometric series diverge
- Know and apply formulae for the sum of a geometric series (finite and convergent infinite)
- Be able to model change in value using multiplicative factors and determine interest rates for different periods of compounding
- Be able to represent linear (e.g. simple interest and 'straight line depreciation') and exponential (compound interest and 'depreciation on a reducing balance') models of financial change algebraically and pictorially (using a timeline), and solve related problems
- Appreciate that compounding can lead to exponential growth or decay and be able to solve related loan, investment and other problems.
- Appreciate that financial problems do not simply involve calculations but also risk, need, ethics, etc. Be able to evaluate 'too good to be true' or unfair financial schemes
- Be able to monitor and evaluate calculations using estimates and make appropriate use of technology (use of calculators and spreadsheets, not rounding too early in calculations).

Fact box

Sum of geometric series: For a geometric series $S = a + ar + ar^2 + ar^3 + \cdots$

$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \frac{a\,(1-r^n)}{(1-r)}$ or equally, $S_n = \frac{a\,(r^n-1)}{(r-1)}$.

When $-1 < r < 1$ the infinite sum converges and $S = \frac{a}{1-r}$.

Multiplicative factors: Increasing an amount by *i* % can be done by multiplying by $(1 + \frac{i}{100})$

Similarly, decreasing an amount by *i* % can be done by multiplying by $(1 - \frac{i}{100})$.

Present / Future value: There are a number of formulae for Present / Future value for loans, investments, annuities, hire-purchase etc. As these rely on whether the instalments are paid at the start or end of periods and any initial deposit or discount, we have not included any particular formula and recommend learners 'construct' a formula using a timeline for a given problem.

Resources needed: Calculators, worksheets for learners (see pages 12 and 13), graphing software (eg Geobebra). Examples of advertisements from loan companies from newspapers or magazines.

Workshop Activities for Teachers

Pairs Every 10 minutes, or when done with an activity or sub-activity, discuss your findings with the whole group. A board or flip chart for sharing ideas.

Activity 1 Infinite geometric sum

20-30 minutes

For the questions below, how would you convince other educators or learners about your answer? For any of the series that converge, how would you find the sum?

1. Does the series $H = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$ converge?

2. Does the series $G = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots$ converge?

3. More generally, for which values of r does the series $S = 1 + r + r^2 + r^3 + \cdots$ converge?

Before you read on, stop and try this for yourself and discuss with your colleagues.

Notes to help with the activity

1. The (harmonic) series H does not converge. We do have $\frac{1}{n} \to 0$ as $n \to \infty$ but that is not enough to make the series converge. We can group the terms such that each group adds up to more than $\frac{1}{2}$ and, as there are an infinite number of such groups, the series diverges.

$$\frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \cdots + \frac{1}{16}\right) + \cdots > \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + 8\left(\frac{1}{16}\right) \ldots$$

2. The series G converges to 2. One could use the formula for the sum of a geometric series but could also see the sum as 'travelling' from 0 to 2, where at each stage one covers half the remaining distance. One first covers 1 unit, then $\frac{1}{2}$, then $\frac{1}{4}$, and so on. In a lesson, learners could be asked to 'act' this out in the classroom: you could use 1 desk width then ½ of the next, then ¼ of it, then….and think about how much of the second desk is left each time.

3. The geometric series $S = 1 + r + r^2 + r^3 + \cdots$ converges when $-1 < r < 1$. Consider the partial sum $S_n = 1 + r + r^2 + r^3 + \cdots + r^{n-1}$. On multiplying both sides by r, we get $rS_n = r + r^2 + r^3 + r^4 \ldots + r^n$. Subtracting the second row from the first,

$$(1 - r)S_n = 1 - r^n \text{ or, if } r \neq 1, \quad S_n = \frac{(1 - r^n)}{(1 - r)}.$$

Now if $r^n \to 0$ as $n \to \infty$, which happens when $-1 < r < 1$, the infinite sum $S = \frac{1}{1-r}$.
Notice that the more general case in the 'Fact box' follows by multiplying through by a.
Also see https://aiminghigh.aimssec.ac.za/grade-12-geometric-series/

4. Another example worth 'joining up' for learners is that of recurring decimals.

$$\frac{1}{3} = 0.333\ldots = 0.3 + 0.03 + 0.003 + \cdots = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \cdots$$

Learners should be able to represent recurring decimals as geometric series and then see how the formula for the infinite sum gives a fractional form for the recurring decimal.

Activity 2 The power of compounding

Pen, paper, calculator, graphing software or GeoGebra applet 30-45 minutes

For the questions below, how would you convince other educators or learners about your answer?

1. Your employer suggests (Deal 1) that they could keep your salary as is or (Deal 2) they could first increase your salary by 80% and then only drop it by 50%. Which is the better deal?

2. A shop is offering a 30% discount on all goods. You also have to pay 20% sales tax. Is it better to get the discount and then pay the tax or add the tax first and then get the discount?

3. How long will it take for an investment to double if you get 9% interest per year (i) simple interest or (ii) compounded annually? What about time for quadrupling?

4. How long will it take for an investment to double if you get *i* % interest per year (i) simple interest or (ii) compounded annually? What about time for quadrupling?

Before you read on, stop and try this for yourself and discuss with your colleagues.

Notes:

1. One way to tackle problems such as the first two, is to use a helpful amount to work out what happens. For example, 100 increases to 180 given an 80% increase. A 50% decrease reduces the amount to 90. So, an 80% increase followed by a 50% decrease is actually a 10% decrease! We need to ask ***'% of what?'***

2. The main reason for sharing the first two problems is to discuss representing percentage change **multiplicatively**. Not only can this aid understanding, but it is critical when considering compound interest. Working additively, an increase by 80% can be seen as 100% +80% = 180%. So one has 180% of the original amount. But 180% is the same as multiplying by 1.8 (note that 100% +80% is the same as 1 + 0.8 = 1.8). Similarly, a decrease by 50% is the same as multiplying by (1 – 0.5) = 0.5. Carrying out (compounding) both changes can therefore be represented as multiplying by $1.8 \times 0.5 = 0.9 = (1-0.1)$, a 10% decrease.

Notice that it does not make a difference to the final change if we were to do the 50% decrease first and then the 80% increase. This gives a solution to the second problem – it makes no difference whether we do discount first and then tax, or tax first and then discount. Both result in the (multiplicative) change $(1-0.3)\times(1+0.2) = 0.7\times 1.2 = 0.84 = (1-0.16)$, a 16% decrease in the original price.

3. Simple interest at *i* % per period, after *n* periods, is given by the multiplier $(1+n(\frac{i}{100}))$.

Compound interest on the other hand results in the multiplier $(1+\frac{i}{100})^n$.

Though one may not need a timeline, one could introduce a timeline for this problem so as to reinforce how it can help represent a problem. Learners may need to write a few cases before they recognise a pattern (here done at n=3), and then represent/generalise it algebraically. Also it may not be clear whether to represent the initial state as *n*=0 or *n*=1 (or some other value!). It may be helpful to express the situation in clear language. For example, below we consider the amount at the **end of a given year, after interest has been calculated**.

Simple interest:	1	1+0.09	1+ 0.09+0.09	1+3×0.09	$1+n(0.09)$	2
Year:	0	1	2	3	… n …	?
Compound interest:	1	1+0.09	(1.09)(1+ 0.09)	$(1.09)^3$	$(1.09)^n$	2

The simple interest problem results in a linear equation $1+n(0.09) = 2$, whereas the compound interest problem requires logarithms (or trial and improvement with exponents), $(1.09)^n = 2$.

4. An exact formula for doubling time in the simple interest case, for i%, is (**100 ÷ *i***). While the difference in doubling time between simple and compound may not seem significant, it becomes increasingly significant if one considers higher multiples. Thus for 9% interest per year, an amount doubles in approximately 11 years using simple interest and doubles in approximately 8 years using compound interest. But for quadrupling, simple interest will take over 33 years, whereas compound interest 16 years.

5. A similar analysis can be carried out for linear versus exponential decay ('straight line depreciation' versus 'depreciation on a reducing balance'), where one has multiplier (1 – i%) and half-life instead of doubling time.

Activity 3 Compounding and aggregation/accumulation 20-30 minutes

Resources: Pen, paper, calculator or spreadsheet

1. A person can afford to pay back R 1000 per year for 20 years. If the interest rate is 8%, compounded annually, what maximum amount of loan can the person afford to take?

2. A person wishes to receive a pension/payment of R 1000 per year for 20 years. If the interest rate is 8%, compounded annually, what amount, P, does the person need to deposit?

3. A person wishes to buy a machine on hire purchase with no initial payment. The machine costs \$P. If the purchase is to be paid for using equal annual instalments over 20 years, what would be the annual instalment, \$A?

Note: Rather than use particular formulae, a timeline can help determine the needed formulae. All the above problems use the same timeline. If a problem does not clarify when instalments are due, we can state our assumptions at the outset (here payments are made at the end of the year).

Value of loan:	P	$(1.08)\,P$	$(1.08)^2 P$	$(1.08)^{20}P$
End of year:	0	1	2	… 20
Value of payments:	0	A	$A + (1.08)\,A$	$A + (1.08)\,A + \ldots + (1.08)^{19}A$

Given P (or A), we wish to solve for the other variable (or given three of $P, A, i, n,$ solve for …) in

$A + (1.08)\,A + \ldots + (1.08)^{19}A = (1.08)^{20}P$. We can now use the sum for a geometric series.

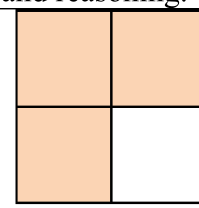
Classroom Activities for Learners

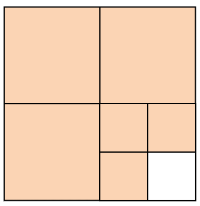
Activity 1: Square pizza! Representing visual fractions as a geometric series

Worksheet (see page 12) Diagrams for demonstration on board *Pairs* *20 – 40 minutes*

This task can be used to motivate the formula for the sum of a geometric series. At each stage (starting at 1b), a pair of learners can be asked to come up and share their work and reasoning.

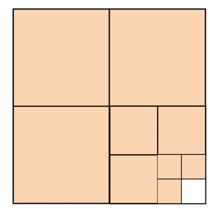
1a. Four learners decide to meet and share a square pizza equally. But on the day, one of them, Tara, is late to arrive. The other three eat their shares (shaded) and leave one share. What fraction has each learner eaten thus far?





1b. As Tara has still not arrived, they cut the remaining square into four equal squares, eat one share each (shaded portion) and leave one portion.
How much has any one learner eaten in this second slicing? How much has one learner eaten thus far? Write your answer as the sum of two terms.

1c. The three learners continue doing this as Tara is still to arrive! What fraction of the pizza has each learner eaten at the third slicing?

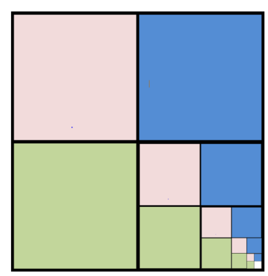


1d. At each stage, what fraction of the pizza has any one of the three learners eaten? Represent this mathematically as a series. What can you say about the sum of this series?

1e. If Tara doesn't show up at all, for all practical purposes, how much of the pizza will each of the other learners have eaten? Justify your answer!

1f. Calculate the total length of the internal edges (knife cuts!) drawn up to any stage. Determining the sum in this case may not be straightforward but learners should be able to write out the terms of the series.

Ideas for Learning Activity 1



1. This activity provides a good setting for introducing examples 2 and 3 from Teacher Activity1 (formula for sum of geometric series).

2. Giving a length for the original sidelength of the pizza, say 20cm, can motivate the role of the initial term, *a*, in a geometric series.
3. Colouring the three different shares with different colours may help learners 'see' the fraction.

Activity 2: My Prize

Worksheet (see page 12), calculators or spreadsheets
Pairs, followed by presentation to whole group *60- 90 minutes*

You have won a prize and have to choose one of the following five options (where you get money over a period of 20 years). Estimate the total amount for each option. Based on your estimates, which option would you choose and why? Check your estimates using suitable calculations and comment on your initial choice of best option. You may be asked to present your choice and reasoning to the class.

1. R60 000 will be given at the end of each year for 20 years.

2. R8 000 will be given at the end of year 1. R16 000 at the end of year 2, R24 000 at the end of year 3, and so on for 20 years.

3. R10 000 is given at the end of year 1. At the end of year 2, R12 000 is given. Each year the money given is 20% more than the money given in the previous year.

4. R500 000 is placed in a bank account that earns 7% interest per annum, compounded annually. You get the final sum at the end of 20 years.

5. You get R2 at the end of year 1, R4 at the end of year 2, and the amount is doubled each year for 20 years.

How would your best choice change if

(a) you took into account factors such as uncertainty in what happens over the long term and other risks?

(b) the investment period was different from 20 years. Pick any two options and compare them over a number of years? Is one option always better than the other? If not, at what point in time does this change? How could you use a graph to represent this?

Ideas for Learning Activity 2

i. Depending on the time set aside for this activity, learners may start by simply comparing two options and presenting their work to the rest of the class. Presentations by pairs of learners who have worked on different options, could help the class to see the wider comparisons.

ii. On the face of it, each option gives an increasing amount of money over the preceding option. But other than Option 4, none of the options take interest into account. So one could increase their value by re-investing their annual payouts. This provides the opportunity to discuss that the value of money or objects is not static. Demand, supply, confidence, inflation, interest rates – many factors can cause differences in the present value and future value of an amount. Learners can choose two options and consider them using realistic interest, inflation rates and other risks.

iii. Solutions for the given problems:

1. R60 000 will be given at the end of each year for 20 years.
R1 200 000 [60 000 × 20]

2. R 8 000 will be given at the end of year 1. R 16 000 at the end of year 2, R 24 000 at the end of year 3, and so on for 20 years.
R1 680 000 [8 000 × (1+2+3+…+19+20) = 8 000 × 20(21)/2 = 8 000 × 210]

3. R 10 000 is given at the end of year 1. At the end of year 2, R 12 000 is given. Each year the money given is 20% more than the money given in the previous year.
R1 866 879.99 [10 000 $(1 + 1.2 + 1.2^2 + 1.2^3 + \ldots + 1.2^{19})$ = 10 000 × 186.6879996]

4. R500 000 is placed in a bank account that earns 7% interest per annum, compounded annually. You get the final sum at the end of 20 years.
R1 934 842.23 [500 000 $(1.07)^{20}$ = 500 000 × 3.86968446]

5. You get R2 at the end of year 1, R4 at the end of year 2, and the amount is doubled each year for 20 years.
R2 097 150 [$2+2^2+2^3+\ldots+2^{19}+2^{20} = 2(2^{20}-1)/(2-1)$]

Activity 3: Payday / short-term loans

Worksheet (see page 13), calculators or spreadsheets, examples from actual advertisements
Pairs, followed by presentation to whole group *40 – 60 minutes*

Below are three loan offers taken from a British loan comparison website.
3a. Which looks the best value?

Loan A: Borrow £100, pay back £107.21 in a week's time.
Loan B: Borrow £100, pay back £114 in a week's time.
Loan C: Borrow £100, pay back £105 in a week's time.

3b. Now look at the 'small print' on each (the actual small print is harder to work out!)

Loan A: Interest rate 1% per day, compounded daily.
Loan B: 2% simple interest per day.
Loan C: 5% interest per week, compounded weekly.

3c. What would happen if you couldn't pay off the loan in a week? This is why a number of governments now require that loan companies provide APR (Annual Percentage Rate) so that borrowers can make better decisions. Calculate the APR for each of these loans.

3d. Next re-rank the options, based on the APR or amount due in a month.

3e. Look up payday loan offers in your country. Report back on the range of interest rates. What information are money lenders required to provide? What regulations are in place?

Ideas for Learning Activity 3

1. For a (nominal) annual interest rate of *i* %, compounded *n* times a year, the compounding multiplier is $(1+\frac{i/100}{n})^n$, and the APR is then given by $((1+\frac{i/100}{n})^n - 1)\times 100\%$.
Before working on the above problems, using a timeline, say for monthly compounding and a nominal rate of 24%, learners should be introduced to how the above formula arises.

2. To the nearest whole number, the APR for the three loan companies are:
Loan A: 3678% Loan B: 730% Loan C: 1164%
Learners should recognise that any APR in excess of 20% could be considered extortionate / unjust. One may suggest accounting for inflation, especially if inflation is high, but that only deepens the hardship for the borrower if wage inflation does not keep up with price inflation.

Changes in my classroom practice

Discussion of Teaching Strategy – Learners presenting their ideas

Why? If learners are asked to explain their ideas and reasoning to their peers, they may work hard to develop their presentation and, in the process, may begin to understand the work more deeply. This could start with explaining to the learner sitting next to them, then to a group, then to the whole class. It could be oral or written or, if you have the facilities, electronic. Eventually they might take on a 'teacher' role at the board, developing their presentation in steps, but equally, they might bring a completed presentation to the front and talk the class through it. Any of these forces learners to make their ideas clearer – and is also good preparation for adult life.

How? Depending on how much previous experience learners have with presenting their work to others, you may want to challenge them but also keep their confidence, so start small and build up. It is much harder for learners to write down a clear argument than to explain it orally, and in the early stages you might want to give learners a tight structure to work with. If they are stuck, you might say 'tell me what you're thinking' and after each sentence, ask learners to write down what they have just said. They can learn to support one another in exactly this way.

In addition to the explicit references above to having learners share their ideas and work, there are two approaches worth using as often as you can:

1. Given the significant real-life applications of growth and decay (loans, investment, inflation, buying power, population growth, pollution, radioactive decay) ask learners to research this, in mathematics and in other subjects, and report on it. They can use the internet, textbooks, newspapers or other sources. Even if their initial 'reports' are not very mathematical, one can ask further questions to bring out the mathematics, the context helping to deepen the understanding.

2. Learners can be asked to visualise and communicate some of the dramatic or unexpected results of compounding, growth/decay using simple and/or eye-catching examples.
https://undergroundmathematics.org/exp-and-log/reach-for-the-stars

Key Questions to develop understanding - *use often with your learners*

1. Recognising the interplay between additive and multiplicative use of percentages is crucial. Ask questions such as, how can you represent 20% increase or decrease as a multiplier? What if it were 2.5%? What are the multipliers for 100% or 200% increase, or decrease? What is the effect of an increase by 20% followed by a decrease by 20%? How would you balance a 20% increase? How would you work that out using addition? What are the advantages and disadvantages of each method?

2. Recognising the power of compounding, over long periods of time or due to high interest rates is crucial. Learners can consider extreme cases via 'what would happen if…?' type questions. If you are able to project a spreadsheet, or use calculators, it is very easy to try out different rates and very quickly see the effect.

To check learner's knowledge and understanding

1. When using calculators, learners should not round off early, especially given compounding. *Final* answers should use appropriate rounding, for example, two decimal places for money.

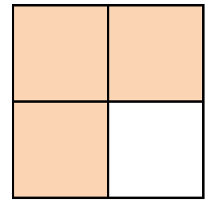
2. Growth and decay are 'two sides of the same coin', though care is needed. 300 is a 100% increase of 150 but 150 is a 50% decrease of 300. Where you can, use graphical display and interactives to help clarify and compare linear and exponential growth/decay.

Further Resources can be found at

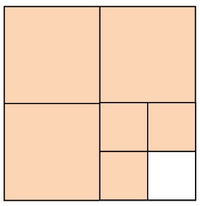
- https://www.everythingmaths.co.za/maths/grade-10/06-finance-and-growth
- Also see the related Grade 11 resources, including on timelines: https://www.everythingmaths.co.za/maths/grade-11/09-finance-growth-and-decay/09-finance-growth-and-decay-03.cnxmlplus
- https://aiminghigh.aimssec.ac.za/grades-9-to-12-dating-made-easier/ **Dating made easier** *(you might then like to begin to make links to carbon dating: talk with science teachers, research it on the Internet, or asks learners to do so):* If a sum invested gains 10% each year how long will it be before it has doubled its value? If an object depreciates in value by 10% each year how long will it take until only half of the original value remains? Why aren't these two answers the same? Is there a rate, used for both gain and depreciation, for which those two answers would actually be the same?
- https://nrich.maths.org/5893 **The legacy** *(very suitable for group work and then for whole-class presentation).*
  Your school has been left R1 000 000 in the will of an ex-pupil ...
  The pupil made some conditions on how the money should be invested and used. These were:
  - The money should have a lifetime of 50 years
  - That the school benefits in some way (spends part of the investment) every year.

  You are asked to produce models of investment and expenditure based on any balance being invested at a fixed interest rate (it is suggested that you could start with a rate of 4%). Your model could also consider different inflation rates.
  What model would you choose to ensure the best return for the school over a period of 50 years?

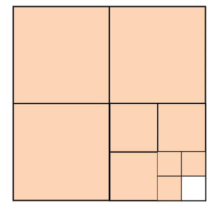
Activity 1 Sharing a square pizza



1a. Four learners decide to meet and share a square pizza equally. But on the day, one of them, Tara, is late to arrive. The other three eat their shares (shaded) and leave one share. What fraction has each learner eaten thus far?



1b. As Tara has still not arrived, they cut the remaining square into four equal squares, eat one share each (shaded portion) and leave one portion.
How much has any one learner eaten in this second slicing?
How much has one learner eaten thus far? Write your answer as the sum of two terms.



1c. The three learners continue doing this as Tara is still to arrive! What fraction of the pizza has each learner eaten at the third slicing?

1d. At each stage, what fraction of the pizza has any one of the three learners eaten? Represent this mathematically as a series. What can you say about the sum of this series?

1e. If Tara doesn't show up at all, for all practical purposes, how much of the pizza will each of the other learners have eaten? Justify your answer!

1f. Calculate the total length of the internal edges (knife cuts!) drawn up to any stage. Determining the sum in this case may not be straightforward but learners should be able to write out the terms of the series.

Activity 2 My prize

You have won a prize and have to choose one of the following five options (where you get money over a period of 20 years). Estimate the total amount for each option. Based on your estimates, which option would you choose and why? Check your estimates using suitable calculations and comment on your initial choice of best option. You may be asked to present your choice and reasoning to the class.

1. R60 000 will be given at the end of each year for 20 years.
2. R8 000 will be given at the end of year 1. R16 000 at the end of year 2, R24 000 at the end of year 3, and so on for 20 years.
3. R10 000 is given at the end of year 1. At the end of year 2, R12 000 is given. Each year the money given is 20% more than the money given in the previous year.
4. R500 000 is placed in a bank account that earns 7% interest per annum, compounded annually. You get the final sum at the end of 20 years.
5. You get R2 at the end of year 1, R4 at the end of year 2, and the amount is doubled each year for 20 years.

How would your best choice change if

(a) you took into account factors such as uncertainty in what happens over the long term and other risks?

(b) the investment period was different from 20 years. Pick any two options and compare them over a number of years? Is one option always better than the other? If not, at what point in time does this change? How could you use a graph to represent this?

Activity 3 Payday / short term loans

Below are three loan offers taken from a British loan comparison website.

3a. Which looks the best value?

Loan A: Borrow £100, pay back £107.21 in a week's time.
Loan B: Borrow £100, pay back £114 in a week's time.
Loan C: Borrow £100, pay back £105 in a week's time.

3b. Now look at the 'small print' on each (the actual small print is harder to work out!)

Loan A: Interest rate 1% per day, compounded daily.
Loan B: 2% simple interest per day.
Loan C: 5% interest per week, compounded weekly.

3c. What would happen if you couldn't pay off the loan in a week? This is why a number of governments now require that loan companies provide APR (Annual Percentage Rate) so that borrowers can make better decisions. Calculate the APR for each of these loans.

3d. Next re-rank the options, based on the APR or amount due in a month.

3e. Look up payday loan offers in your country. Report back on the range of interest rates. What information are money lenders required to provide?