SUM ON

The sum of the first 48 terms of an arithmetic series is 4 times the sum of the first 36 terms.

What is the sum of the first 30 terms?

$S_{48} = 4S_{36}$

$S_{30} = ?$

SOLUTION

Let the first term be a and the common difference be d. The information given tells us that:

$24(2a + 47d) = 4 \times 18(2a + 35d)$

Simplifying this equation by dividing by 24:

2a + 47d = 3(2a + 35d)

Then collecting like terms:

2a + 47d = 6a + 105d

4a + 58d = 0

2a + 29d = 0 .

Notice that there are many solutions to this equation and we have no information that tells us which one to choose.

The sum of the first 30 terms is *15(2a + 29d)* but we have just found *2a + 29d* = 0 so

the sum of the first 30 terms is zero.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
    "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5.   If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Find the next term

13, 7, 1, -5 ......

A) -10   B) 1

C) -12   D) -11

D. is the correct answer.

Common Misconceptions

A. B. C. Learners who gave these answers may not understand sequences or they may be confused by negative numbers and unable to add and subtract integers.

https://diagnosticquestions.com

Why do this activity?

This is a non standard question and learners have to think for themselves and not just follow a procedure or algorithm that they have met before. The activity helps to develop mathematical and creative thinking.

Intended learning outcomes

Learners will use and apply knowledge of how to find the sum of an arithmetic series.

Suggestions for teaching

Start with the diagnostic question to review arithmetic sequences and to remind learners that the common difference can be negative.

Then use the 1 – 2 – 4 – more teaching strategy. Learners should work on their own for about 10 minutes, then discuss their working with a partner so that they can compare methods and see if they have come to the same conclusion. If they have, how would they explain their reasoning to other people? Then pairs of learners should compare methods and see if they have come to the same conclusion and again talk about the best way to explain their reasoning to other people?

Finally have a whole class discussion and ask learners to come to the board and explain to the class how they found the answer.

Key questions

What information are you given?
Can you use the information given to write down an expression or an equation?
What formula can you use?
What is the formula for the sum of the first 30 terms?
Did the answer surprise you?

Possible extension

How many solutions does the equation $2a + 29d = 0$ have if a and d are integers? ?
Can you find a sequence that fits the information given?
Write down the first 5 terms of your sequence and the 26th, 27th, 28th, 29th and 30th term. Explain why this sequence has the sum you have found.

Possible support

Note that open questions are much more conducive to learning and the development of understanding than closed or leading questions. The following leading questions are just a recipe, or a set of instructions for solving the problem. Learners often follow instructions like this without understanding why and without developing their own creative thinking and problem solving skills.

Ask the leading questions one by one:
1) Can you write down the formula for the sum S_{48} of the first 48 terms of an arithmetic sequence?
2) Can you do the same for the formula for the sum S_{36} of the first 36 terms?
3) Can you use the given information and your formulae from 1) and 2) to write down an equation and then simplify this equation?
4) Look at your equation in 3). What does this tell you about the sum of 30 terms?