AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

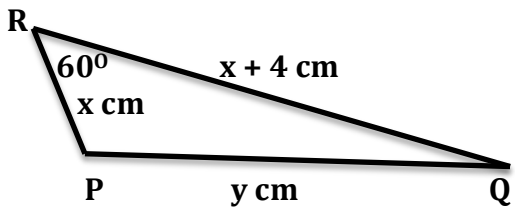
SOLVE THE TRIANGLE

The area of triangle PQR is $3\sqrt{3}$ square centimetres. Angle PRQ is 60^o and RQ is 4 centimetres longer than PR.

Find the length of PQ.

Draw ΔPQR accurately.

Calculate all the angles in the triangle.



SOLUTION

Area = ½x(x + 4)sin 60^0

= ½x(x + 4)×√3/2

So √3/4 x(x + 4) = √27 = 3√3

$x^2 + 4x - 12 = 0$

(x + 6)(x – 2) = 0

So x = 2 as it must be positive.

By the cosine rule : $y^2 = 4 + 36 - 24\cos 60^0 = 28$

$y = 2\sqrt{7}$ cm (or 5.29 cm to 2 decimal places).

When this triangle is constructed accurately using the 3 lengths we notice that ∠RPQ is obtuse.

By the sine rule : sin ∠RPQ/6 = sin 60^o/y

sin ∠RPQ = 6 (√3/2)/ (2√7) = 0.9820 so ∠RPQ = 180 - 79.1 = 101^o (to nearest degree)

Check ∠RQP = $\sin^{-1}$ 2(√3/2)/ (2√7) =19.1 and the angles of the triangle add up to 180^o.

Alternatively, using the cosine rule

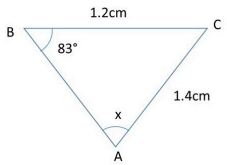
cos ∠RPQ = (4 + 28 – 36)/ 8√7= - 0.1890 showing that this angle is obtuse.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

What is the correct first step in solving the following to find x?



A $\cos x = \frac{1.4 \times \cos 83}{1.2}$ B $\sin x = \frac{1.2 \times \sin 83}{1.4}$

C $\sin x = \frac{1.4 \times \sin 83}{12}$ D $\cos x = \frac{1.2 \times \cos 83}{1.4}$

C. is the correct answer.

Common Misconceptions

A., B and **D** Learners who give these answers do not know the sine rule or how to use it.

https://diagnosticquestions.com

Why do this activity?

This non routine activity combines the use of algebra and solving a quadratic equation with the use of the area formula and the cosine and sine rules. Drawing a sketch of the triangle will help learners to think about its possible shape but they may not deduce at the start that the triangle is obtuse. The activity develops mathematical thinking about what information is given and how it can be used and applied.

By doing an accurate construction from the values learners find for x and x + 4 cm, and the given angle of 60^o, they will discover that angle P is obtuse and this will be evident if the cosine rule is used to find angle P.

This is an excellent example for using Geogebra or other graphing software.

Learners have to understand that, if they choose to use the sine rule they will need another property of the triangle to determine whether angle P is acute or obtuse, although this is unambiguous if they use the cosine rule.

Intended learning outcomes

Practice in using the area formula and the sine and cosine rules to find lengths and angles of a triangle.

Suggestions for teaching

Resources needed: compasses, protractors, rulers, pencils and calculators.
Start with the diagnostic question.
Then write the question on the board and tell the learners to work on it on their own.

After 5 to 10 minutes, when most of the learners are drawing the triangle accurately, they can work with a partner and check their answers.

You might like to suggest that one of the pair calculate the angles from the cosine rule while the other one uses the sine rule and then they compare answers. That way they should be able to understand why angle RPQ is 101^o and not 79^o.

If Geogebra is available, the variable length PR=x can be constructed with the given angle PRQ = 60^o and then the point Q such that RQ = x + 4. The software will calculate the area of the triangle. Vary the diagram with these conditions and note the change in angle RPQ as the area is increased.

Learners who finish quickly can be given the Extension Question.

Finish the lesson by asking a pair of learners to explain not just how they found their answers, but how they decided that angle P is obtuse.

Key questions

What information is given? How can you use it.
Have you used all the information given?
Can you find x?
When you know x, can you draw the triangle using the given information before finding y?
When you have calculated the value of x, how do you find y?
How many methods do you know that you could use here?
How do you know which answer to pick?

Possible extension

Now suppose the angle PRQ is still 60^o and RQ is still 4 centimetres longer than PR but the area changes. What area would make angle RPQ a right angle? Suppose that the same conditions apply but the area increase still further is the angle at P acute or obtuse?

When RP = 4 cm and RQ = 8 cm then the angle at P is 90^o and the area is $8\sqrt{3}$ cm^2.

Increasing the area makes angle P an acute angle and as the area increases so angle P decreases.

Possible support

The question asks learners to calculate PQ and after that to draw the triangle. They may find it helpful. when they have found x to be 2 cm, to use this information to draw the triangle accurately and to find an approximation to PQ by measurement.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**