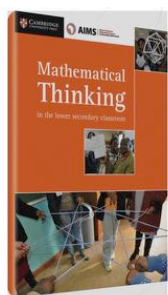


## MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



### Mathematical Thinking in the lower secondary classroom

Edited by Christine Hopkins, Ingrid Mostert and Julia Anghileri

978-1-316-50362-1

These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from [Amazon](http://www.amazon.com) or from <http://www.cambridge.org/za/education> Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to <http://www.cup.co.za>

For reviews and curriculum map see <https://aiminghigh.aimssec.ac.za/mathematical-thinking/>

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**EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:**






**PAGE 1**

**TITLE PAGE**

- Teaching strategy focus. *Each guide focuses on and exemplifies a teaching methodology that is widely used.*
- Curriculum content and learning outcomes.
- Summary of mathematical topic (FACT BOX.)
- Resources needed

**PAGES 2 & 3**

**WORKSHOP ACTIVITIES FOR TEACHERS**

Two pages for you to work through with your colleagues. These activities are to be shared and discussed. For each activity there is a list of resources needed , how to organise the activity (e.g. individual, pairs, whole class) , how long the activity will take , when to pause, think and try the activity , and when to record your work .

**PAGES 4 & 5**

**CLASSROOM ACTIVITIES FOR LEARNERS**

Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

**PAGES 6 - 10**

**CHANGES IN MY CLASSROOM PRACTICE**

Pages on using the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates. For follow-up activities you will find lots more lesson activities on the AIMING HIGH Teacher Network

<https://aiminghigh.aimssec.ac.za/category/lesson-activities/>

## Number systems and equations Making connections

By Liezel du Toit



Liezel taught mathematics at secondary schools in South Africa for more than 20 years.

She now works as a freelancer and enjoys the different opportunities to be involved in mathematics and the teaching of mathematics.

She often teaches at AIMSSEC courses, and she trains and mentors mathematics teachers, tutors groups of secondary school learners, teaches artisan students and writes textbooks.

# Number systems and solving equations

## Teaching strategy: Making connections within and beyond mathematics

### Curriculum content:

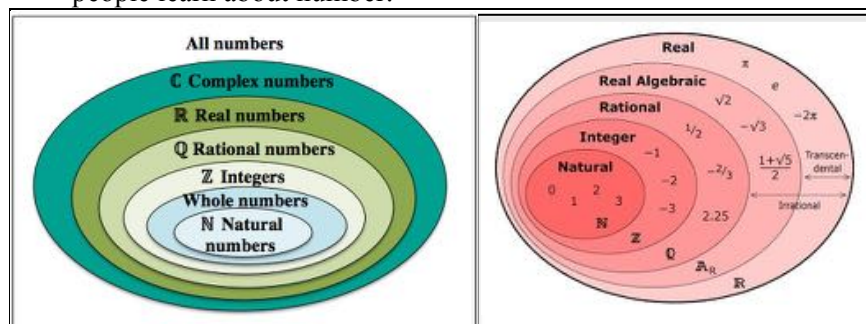
- Number systems. Review of number knowledge.
- Solving equations and expanding number concepts:
  - Understand that real numbers can be rational or irrational; apply that to understanding solutions of quadratic equations;
  - Relate real numbers to decimal representations, and round numbers appropriately to the context;
  - Considering solutions of quadratic equations and note that there exist numbers other than those on the real number line, the complex numbers.

### Prior knowledge needed:

- How to solve linear equations.
- How to solve quadratic equations by using factors or the quadratic formula.

### Intended Learning Outcomes At the end of this activity teachers and learners will:

- Know how to interpret solutions to linear and quadratic equations, and relate them to both graphs and sets of number.
- Understand the connections between the graphs of functions, solutions of equations and sets of numbers.
- Be able to round real numbers to an appropriate degree of accuracy and be able to place irrational numbers between the appropriate two integers on the number line.
- Appreciate the connections between the nature of solution(s) of equations and the sets of numbers that make up the number system and how this relates to the historical development of number and the way people learn about number.



### Fact box

**Natural Numbers  $\mathbb{N}$**  are the numbers 1, 2, 3 ...

**Integers  $\mathbb{Z}$**  are the numbers ...-3, -2, -1, 0, 1, 2, 3...

**Rational Numbers  $\mathbb{Q}$**  are numbers of the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers and  $b$  is not 0.

**Irrational numbers** (e.g.  $\sqrt{2}$  or  $\pi$ ) cannot be written in this form.

**Real numbers  $\mathbb{R}$**  are either rational or irrational. Each real number corresponds to a unique point on the one dimensional number line and every point on the number line corresponds to a unique real number.

**Complex numbers  $\mathbb{C}$**  correspond to points in the 2 dimensional plane including the real axis (the  $x$ -axis) but might have an imaginary part that involves  $i = \sqrt{-1}$ , e.g.  $3 + 4i$  which is the point (3 ; 4).

To solve an equation in algebra, we can often use a graph.

There are **numbers in higher dimensions**: none in 3 dimensions, quaternions in 4-dimensions ...

A **zero** of a function  $f(x)$  is the same as a **root** or **solution of the equation**  $f(x)=0$ .


For example, to solve the quadratic equation  $3x^2 - 7 = 0$ , we might draw the parabola  $y = 3x^2 - 7$  and see where it meets the  $x$ -axis (the line  $y = 0$ ). The points, called the  **$x$ -intercepts** of the graph, are the **roots** or **solutions** of the equation  $3x^2 - 7 = 0$ , that is the real numbers  $\pm\sqrt{7/3}$ .


The points of intersection of the parabola  $y = ax^2 + bx + c$  and the  $x$ -axis (the ' $x$ -intercepts') give solutions of the equation  $ax^2 + bx + c = 0$ , namely  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . If  $b^2 < 4ac$  the parabola does not intersect the  $x$ -axis as, for example, in the equation  $x^2 - 4x + 5 = 0$  whose solutions are the complex numbers  $4 \pm 2i$ .

*Resources needed for the activities: Pen, paper, calculator, copy of worksheets on pages 10 & 11.*



## Workshop Activities for Teachers

### Activity 1: Number systems

 Divide up the group if there are more than five teachers.

 30 minutes

Discuss the following questions. (Pause  think and try this)


1. We use numbers every day. Where do the numbers come from? Do you think numbers were invented or discovered? WHY did people start to use numbers? *Some parts of our number system took thousands of years to develop, for instance the use of place value and the use of zero as both a place holder and a number. Why are these ideas important?*
2. Give examples of the numbers that you will use if:
  - (a) you want to know how many learners are in the classroom now;
  - (b) your friend borrowed money from you and has not paid it back yet;
  - (c) you want to divide two loaves of bread between ten people;
  - (d) you want to find the length of the diagonal of a square with side 7 cm. 
3. We can write each rational number  $\frac{a}{b}$  in decimal form. Convert  $\frac{5}{8}$  into its decimal form. Now convert  $\frac{3}{11}$  into its decimal form. What about  $\frac{1}{3}$  and  $\frac{4}{7}$ ? Explain how you can convert any given rational number into its decimal form. 
4. Describe some different patterns that decimals can have. What do you notice about the patterns in the decimal expansions of rational numbers? Are these always **repeating** patterns. How do you know?
5. Every real number can be written in decimal form. Use your own words to describe the difference between a rational and an irrational number. Can you give some examples of irrational numbers?
6. How would you explain to a primary school learner what the real numbers are?
7. How would you explain to a primary school learner how to order numbers – with and without a calculator?
8. The set of real numbers is made up of all the points on the number line which is the same as the  $x$ -axis. The set of complex numbers is made up of all the points in the Cartesian plane and they are widely used in higher mathematics and science. All real numbers are complex numbers but only some of the complex numbers are real numbers. The complex numbers that are not real are sometimes called non-real numbers. For example  $2 + 5i$  and  $3 - 4i$  (the points (2 ; 5) and (3 ; - 4)) are complex numbers but not real numbers. What number do you think  $(2 + 5i) + (3 - 4i)$  is? Could you subtract instead of adding these numbers? Try multiplying these numbers.
9. What is the difference between a complex number and an undefined number? What undefined numbers can you think of? If you have a calculator, what does it ‘think’?
10. Why is it important to know about the number system?

*As teachers we can and should learn from each other! How can you use or adapt these questions in your classroom to teach the number system?*

### Activity 2: Find the link


 The worksheet on page 10.

 Pairs.


 60 minutes

When we solve equations our solutions can be numbers of different types.  
There is a link between solving equations and interpreting graphs.


#### A. Linear equations

- There are three possible cases if we solve linear equations: the equation might have no solutions, a unique (one) solution, or an infinite number of solutions.
- If we think of the straight-line graph of  $y=f(x)$  where  $f(x)$  is linear, that graph might be parallel with the  $x$ -axis (no solutions to  $y=0$ ), on top of it (an infinite number of solutions to  $y=0$ ) or cross it at one point (a unique solution).
- Work in pairs and complete number 1 of the worksheet on page 10. The equations given are a little more tricky since they don't start in the form  $f(x)=0$ .
- What concepts do you want your learners to develop while doing number 1? 


#### B. Zero product property

- The "Zero Product Property" says that:  
if  $p \times q = 0$  then  $p = 0$  **or**  $q = 0$  (or both  $p = 0$  **and**  $q = 0$ )
- Work in pairs and complete number 2 of the worksheet on page 10.
- What misconceptions can learners have while answering number 2? 

#### C. Quadratic equations

- Work in pairs and complete number 3 of the worksheet on page 10.
- What is similar about the three questions?
- What is different about the three questions? 

#### D. Quadratic equations and parabolas

- There is a connection between the solutions for a quadratic equation and the graph of the parabola.
- What does it mean to say that  $f(x)$  is a quadratic expression? The graph of the parabola  $y = f(x) = 0$  cuts the  $x$ -axis at either two points, one point or no points. The equation  $f(x)=0$  then has two, one (sometimes, we say 'one repeated' since it almost cuts twice) or no real solutions. Of course, as we showed in question 3, it can (and will) then have complex roots.
- Work in pairs and complete number 4 of the worksheet on page 10.
- What questions can you ask your learners when they work through this question? 

#### E. In conclusion


When will you use the different questions on the worksheet in your classroom? What will you have to change so that your learners can make good use of them?



## Classroom Activities for Learners

### Activity 1: Number systems

 Groups of 3

 30 minutes


Discuss the following questions.

1. We use numbers every day. Where do the numbers come from? Do you think numbers were invented or discovered? WHY did people start to use numbers? *Some parts of our number system took thousands of years to develop, for instance the use of place value and the use of zero as both a place holder and a number. Why are these ideas important?*
2. Give examples of the type of numbers that you will use if:
  - (a) you want to know how many learners are in the classroom now;
  - (b) your friend borrowed money from you and has not paid it back yet;
  - (c) you want to divide two loaves of bread between ten people;
  - (d) you want to find the length of the diagonal of a square with side 7 cm.
3. We can write each rational number  $\frac{a}{b}$  in decimal form. You can convert any rational number into its decimal form by dividing  $a$  by  $b$ . Divide 5 by 8 to convert  $\frac{5}{8}$  into its decimal form. Now convert  $\frac{3}{11}$  into its decimal form. What about  $\frac{1}{3}$  and  $\frac{4}{7}$ ?
4. Using your answers to question 3, describe some different patterns that decimals can have. What do you notice about the patterns in these decimal expansions of rational numbers? Will there always be **repeating** patterns? How do you know?
5. Every real number can be written in decimal form. Use your own words to describe the difference between a rational and an irrational number. Can you give some examples of irrational numbers?
6. How would you explain to a primary school learner what the real numbers are?
7. How would you explain to a primary school learner how to compare the size of real numbers (that is how to order numbers) – with and without a calculator?


#### Ideas for teaching Activity 1:

- Use the questions as an investigation if learners have access to the internet or other sources.
- Use number 3 as a quick competition between pairs of learners. Set a time limit and perhaps have a small prize available for the winners.
- Make a poster of the number system for the classroom and refer to it during maths lessons.


### Activity 2: Find the link

 Use the worksheet on page 10 to do this activity.


 Pairs

 60 minutes

### Activity 3: Where do I belong?

 Use the Venn diagram on page 11.

 Pairs then whole class

 30 minutes

## SUGGESTED ANSWERS TO ACTIVITY 1

1. There are many sources on the internet for background information, for example the NRICH article [What are Numbers](#) and articles on <http://www.storyofmathematics.com/index.html>. The number system developed as society and civilization needed tools to count their livestock, to trade, to navigate and to describe the stars and the world. We think Indian mathematicians were first to use a symbol for "nothing", but Chinese mathematicians were also early users of such a symbol. Find out more on the internet, for example this article on [a brief history of numbers and counting](#). Many different cultures developed counting in different bases, not always base ten though we have ten fingers. For example, the Babylonians used base 60 which is the origin of our time and angle measurement. Why is 60 a good base to use?
2. (a) For example, 35 or 50 or 120 learners. These numbers are *natural numbers* in the set  $\mathbb{N} = \{1; 2; 3; \dots\}$ . There might be nobody in the classroom in which case you would use the number 0 (zero). Zero is a whole number in the set  $\mathbb{N}_0 = \{0; 1; 2; 3; \dots\}$   
 (b) For money that is owing we might use for example -R50 or -£20 or -€30 or -\$10. These negative whole numbers are integers in the set:  $\mathbb{Z} = \{.. -3; -4; -1; 0; 1; 2; 3 \dots\}$ . If the money owing includes cents, for example -R10.50, then this is a rational number.  
 (c) For dividing 2 loaves between 10 people we use the fraction or rational number  $\frac{2}{10} = \frac{1}{5}$ . These fractions are sometimes called *common* fractions or *simple* or *vulgar* fractions. Fractions are rational numbers.  
 (d) The diagonal of a square with side 7 has length  $7\sqrt{2}$  cm. This is an example of an irrational number. Why? There's an interesting and elegant [proof that  \$\sqrt{2}\$  is irrational](#) suitable for maths teachers and for gifted students on NRICH. With a light change the argument works for square roots of any prime number. Also see [homeschoolmaths.net](http://homeschoolmaths.net)

3.  $\frac{5}{8} = 0.625$ ,  $\frac{3}{11} = 0.2727 \dots = 0.\overline{27}$  that is 0.27 recurring.  $\frac{1}{3} = 0.\overline{3}$  and  $\frac{4}{7} = 0.\overline{571428}$ .

You can convert any rational number  $\frac{a}{b}$  into its decimal form by dividing  $a$  by  $b$ . The decimal forms will always have repeating patterns, either repeating zeros at the end or a repeating pattern of digits (recurring decimals). All rational numbers can be written as either terminating decimals or as repeating decimals of length at most  $b$ , because  $a \div b$  has no more than  $b$  different remainders as you work through the division.

5. Notice the definition: A rational number is any number **that can be written** in the form  $\frac{a}{b}$  where  $a$  and  $b$  are both integers and  $b \neq 0$ . Any other real number is irrational. A repeating decimal (rational number) can, in theory, be changed into a non-repeating decimal (irrational number) by randomly choosing one of the digits in each repeated section and replacing it by a randomly chosen digit so breaking the pattern. Numbers of the form  $\sqrt{p}$  where  $p$  is a prime number are all irrational and the number  $\pi$  is irrational. Note that the rational number  $\frac{22}{7}$  is not equal to  $\pi$ , it is a close approximation to  $\pi$ .

Real numbers  $\mathbb{R}$  are ALL the numbers that we can find on a number line. A real number, whether rational or irrational, has an exact position on the number line: it can represent a length. For example,  $\sqrt{2}$  is the length of the diagonal of a 1 by 1 square;  $\pi$  is the circumference of a circle of diameter 1. Note that  $\pi$  is an irrational number. So if we wrote it in decimal form we could carry on indefinitely, down the road, to the nearest town, to the coast, across the ocean, out into space ....

7. Numbers can be ordered by comparing their positions on the number line. Numbers to the left are smaller than numbers to the right of them.

**For teachers** 8. Complex numbers may or may not be on the real number line ( $x$ -axis) but we can

calculate with them. This is not required for the South African National School Certificate.

We write  $\sqrt{-1} = i$ , then  $\sqrt{-4} + \sqrt{-9} = \sqrt{(-1)(4)} + \sqrt{(-1)(9)} = 2i + 3i = 5i$ .

**Addition & Subtraction**  $(2 + 5i) + (3 - 4i) = (5 + i)$  and  $(2 + 5i) - (3 - 4i) = (-1 + 9i)$ .

**Multiplication**  $(2 + 5i)(3 - 4i) = 6 - 8i + 15i - 20i^2 = 6 + 7i - 20(-1) = 26 + 7i$

**Division**  $\frac{(2 + 5i)}{(3 - 4i)} = \frac{(2 + 5i)(3 + 4i)}{(3 - 4i)(3 + 4i)} = \frac{(-14 + 23i)}{25} = \frac{-14}{25} + \frac{23i}{25}$ .

*Note: We divide surds in a similar way*  $\frac{(2 + \sqrt{5})}{(3 - \sqrt{2})} = \frac{(2 + \sqrt{5})(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{6}{7} + \frac{2\sqrt{2} + 3\sqrt{5} + \sqrt{10}}{7}$

9. A **complex number** is any number corresponding to a point in the Cartesian plane and it may or may not be a real number. An **undefined number** is not part of the number system, for example:  $\frac{3}{0}$  or  $0^0$ . A calculator will give an error message if we key in  $3 \div 0$  or  $\sqrt{-1}$  but more powerful software can calculate with complex numbers.

## Solutions for Activity 2 Find the Link and Ideas for Discussion of Worksheet 1

### Question 1

(a) Left hand side:  $y = 2x + 6$  (Straight line with positive gradient and y-intercept at 6.)

Right hand side:  $y = -3x + 1$  (Straight line with negative gradient and y-intercept at 1.)

The two lines have one point of intersection. If we solve the equation, we find  $x = -1$ . This means that the x-coordinate of the point of intersection is equal to  $-1$ .

*Matching graph: Diagram 2*

(b) Left hand side:  $y = -4x + 2$  (Straight line with negative gradient and y-intercept at 2.)

Right hand side:  $y = -4x - 6$  (Straight line with negative gradient and y-intercept at -6.)

The two lines have equal gradients ( $m = -4$ ), so they are parallel. The two lines have NO points of intersection. There is no solution.

*Matching graph: Diagram 3*

(c) The left-hand side is equal to the right-hand side:  $y = 3x + 6$  (Straight line with positive gradient and y-intercept at 6.)

The two lines have ALL their points in common. We can substitute any real number into the equation and the left-hand side will be equal to the right-hand side so all real numbers are solutions, that is there are infinitely many solutions.

*Matching graph: Diagram 1*

### Question 2

Consider the  $x$  in front of the given brackets (factors) as being in a bracket of its own:

$$(x)(2x + 5)(3x + 9)(x - \sqrt{-4}) = 0$$

If  $x$  can be any type of number the solutions are:  $x = 0$  or  $x = -\frac{5}{2}$  or  $x = -3$  or  $x = \sqrt{-4}$

(a) If  $x \in \mathbb{N}$ ,  $x = -3$  (Remember: 0 is NOT a natural number.)

(b) If  $x \in \mathbb{Z}$ ,  $x = 0$  or  $x = -3$

(c) If  $x \in \mathbb{Q}$ ,  $x = 0$  or  $x = -\frac{5}{2}$  or  $x = -3$  (Remember:  $0 = \frac{0}{1}$  and  $-3 = \frac{-3}{1}$ )

(d) If  $x \notin \mathbb{R}$ ,  $x = \sqrt{-4} = 2i$  (Complex numbers are of the form  $a + bi$  where  $a, b$  are real numbers.)

### Question 3

(a)  $x = 3$  or  $x = -3$

Rational numbers

(b)  $x = \sqrt{7}$  or  $x = -\sqrt{7}$

Irrational numbers

(c)  $x = \sqrt{-9} = 3i$  or  $x = -\sqrt{-9} = -3i$

Complex numbers

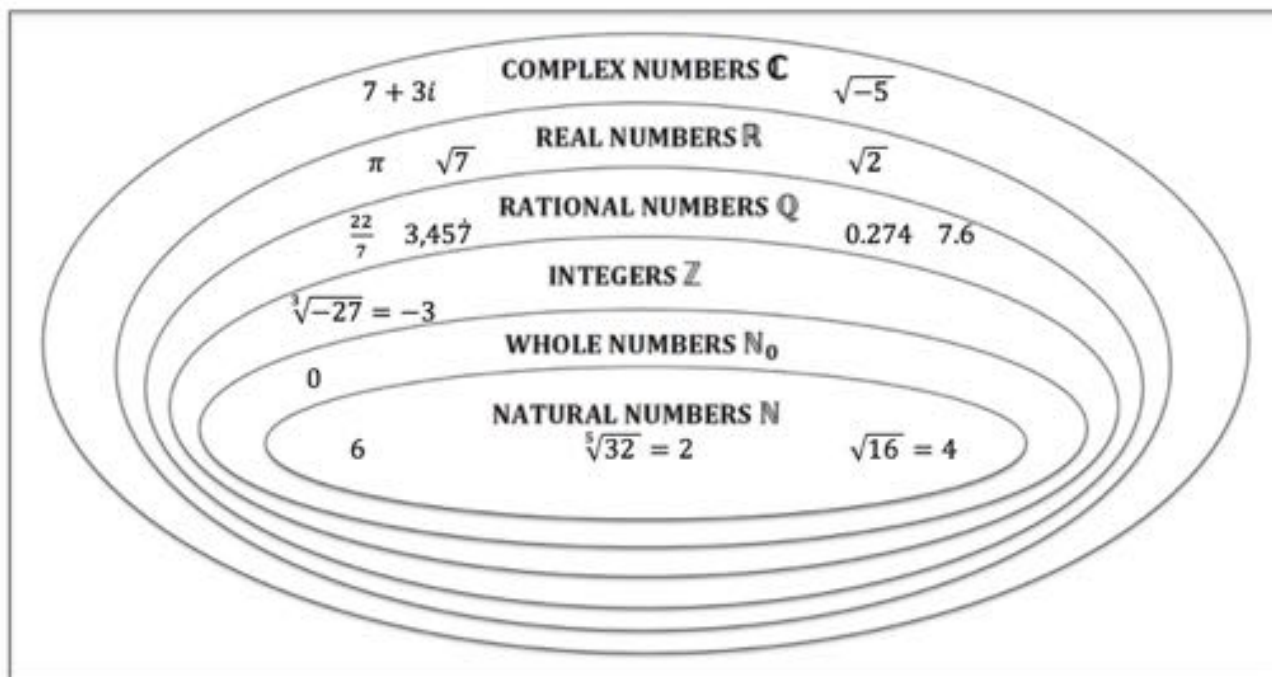
## DISCUSSION OF WORKSHEET 1 (CONTINUED) AND KEY QUESTIONS TO ASK



<p><b>4. Solutions</b>  (a) <math>x^2 + 6x = 0</math></p> <p>Think "parabola":  Parabola: <math>y = x^2 + 6x</math>  For <math>x</math>-intercept(s), let <math>y = 0</math>  <math>x^2 + 6x = 0</math>  <math>x(x + 6) = 0</math>  <math>\therefore x = 0</math> or <math>x = -6</math>  <i>Matching graph: Diagram 6</i></p>	<p><b>Questions to ask</b>  (1) Is it a problem that <math>c = 0</math>?  (2) Why is the following calculation incorrect?  <math>x^2 + 6x = 0</math>  <math>\div x] \quad x + 6 = 0</math>  <math>\therefore x = -6</math>  (3) Are the <math>x</math>-intercepts real numbers? Why?</p>	<p><b>Suggested answers</b>  (1) No. As long as <math>a \neq 0</math> we still have a quadratic equation.  (2) The graph shows that we lose one solution if we divide by <math>x</math>.  (3) Real numbers. We can plot them on the number line (<math>x</math>-axis).</p>
<p>(b) <math>2x^2 + 5x - 8 = 0</math></p> <p>Think "parabola":  Parabola: <math>y = 2x^2 + 5x - 8</math>  For <math>x</math>-intercept(s), let <math>y = 0</math>  <math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math>  <math>= \frac{-(5) \pm \sqrt{25 + 64}}{2(2)}</math>  <math>= -3.61</math> or <math>1.11</math>  (correct to 2 decimal places)  <i>Matching graph: Diagram 4</i></p>	<p>(1) Take note: If <math>c = -8</math>, then <math>-4ac = -4(2)(-8) = 64</math>  (2) Take note:  <math>\sqrt{25 + 64} \neq 5 + 8</math>  (3) Are the solutions real numbers? Why?  (4) Are the solutions rational or irrational numbers? Why?</p>	<p>(1) Take care in calculations with negative numbers. It can help to use brackets in calculations.  (2) Avoid this misconception. In algebra: <math>\sqrt{a^2 + b^2} \neq a + b</math>  (3) Real numbers: they are on the number line.  (4) Irrational numbers involving square roots.  (5) -3.61 or 1.11 is the most common way of writing -3,61 or 1,11</p>
<p>(c) <math>\frac{1}{4}x^2 + x + 1 = 0</math></p> <p>Think "parabola":  Parabola: <math>y = \frac{1}{4}x^2 + x + 1</math>  For <math>x</math>-intercept(s), let <math>y = 0</math>  <math>\times 4] \quad x^2 + 4x + 4 = 0</math>  <math>(x + 2)(x + 2) = 0</math>  <math>\therefore x = -2</math> or <math>x = -2</math>  <i>Matching graph: Diagram 7</i></p>	<p>(1) Are the roots the same or different?  (2) Are the roots real? Are the roots rational or irrational?  (3) How can we shift the graph so that the parabola has two different (unequal) <math>x</math>-intercepts?</p>	<p>(1) The same. We have two equal roots.  (2) Real numbers (they are on the number line). Rational numbers, in fact they are integers. We can easily pinpoint them exactly on the number line.  (3) Shift (translate) the graph downwards.</p>
<p>(d) <math>5x^2 + 3 = 0</math></p> <p>Think "parabola":  Parabola: <math>y = 5x^2 + 3</math>  For <math>x</math>-intercept(s), let <math>y = 0</math>  <math>5x^2 + 3 = 0</math>  <math>5x^2 = -3</math>  <math>x^2 = -\frac{3}{5}</math>  <math>x^2 = \pm i\sqrt{\frac{3}{5}}</math>  No real solutions <math>\therefore x \notin \mathbb{R}</math>  <i>Matching graph: Diagram 5</i></p>	<p>(1) Why does your calculator give "ERROR" for <math>\sqrt{-0.6}</math>?  (2) Why do the arms of the parabola point upwards?  (3) What will happen if we shift the graph exactly 3 units downwards?  (4) What will happen if we shift the graph more than 3 units downwards?</p>	<p>(1) Our calculators only work with real numbers. <math>\sqrt{-0.6}</math> is a complex number.  (2) <math>a = 5</math> so <math>a &gt; 0</math>. The parabola has a minimum turning point.  (3) Turning point on <math>x</math>-axis: one point of intersection between parabola and <math>x</math>-axis.  (4) Turning point below the <math>x</math>-axis: two unequal roots.</p>

## SOLUTION FOR VENN DIAGRAM WORKSHEET

VENN DIAGRAM OF SETS OF NUMBERS



### Teaching strategy: Making connections within and beyond mathematics

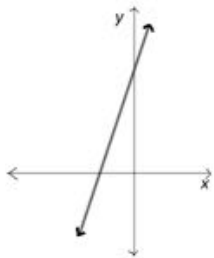
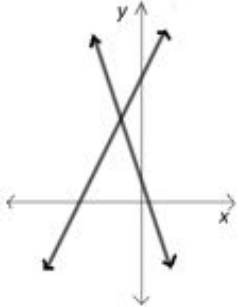
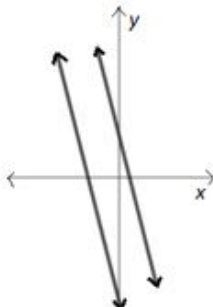
The Venn diagram is not complete: learners will be able to name subsets of the natural numbers, for example, even numbers, or multiples of 7, or prime numbers, or....

Young children start by counting with small whole numbers, and learners with younger brothers or sisters will know 0 has little meaning for them, so that children's learning parallels historical developments of numbers. Gradually children meet fractions through sharing in home or school situations: these developments from whole numbers only developed historically as societies needed them. It is important that learners gradually come to recognize a need for a greater range of numbers in their work, but for many, the need for e.g. complex numbers will not be obvious – until they find out that the development of e.g. the whole of modern electronics, and so cell phones and computers, depend on using complex numbers!

Learners may be fascinated by the historical development of number and of mathematics: whatever our culture, these are part of our cultural roots. *What uses did your learners' ancestors have for number, and how did they record those? Ask learners what they know. What uses for number did their grandparents have? How and why have those needs changed?*

## WORKSHEET 1

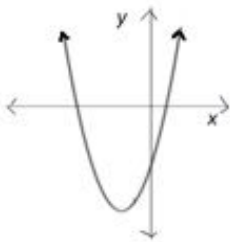
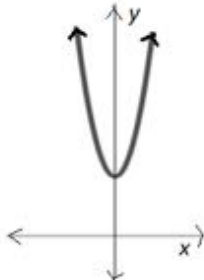
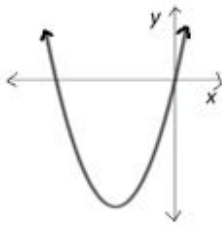
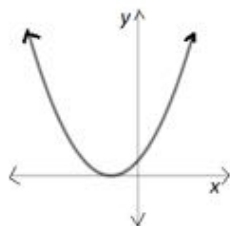
1. Simplify each side of the given equations and then solve for  $x$ .  
 Match the equations to the graphs. Explain your reasoning.

(a) $2(x + 3) = -3x + 1$	(b) $2 - 4x = -2(2x + 3)$	(c) $3x + 6 = 3(x + 4) - 6$
Diagram 1 	Diagram 2 	Diagram 3 

2. Given:  $x(2x + 5)(3x + 9)(x - \sqrt{-4}) = 0$   
 Solve for  $x$  if  
 (a)  $x \in \mathbb{N}$                       (b)  $x \in \mathbb{Z}$                       (c)  $x \in \mathbb{Q}$                       (d)  $x \notin \mathbb{R}$

- 3.1 Solve for  $x$  in the following three quadratic equations:  
 (a)  $x^2 - 9 = 0$                       (b)  $x^2 - 7 = 0$                       (c)  $x^2 + 9 = 0$
- 3.2 Describe the nature (type of number) of the solutions.

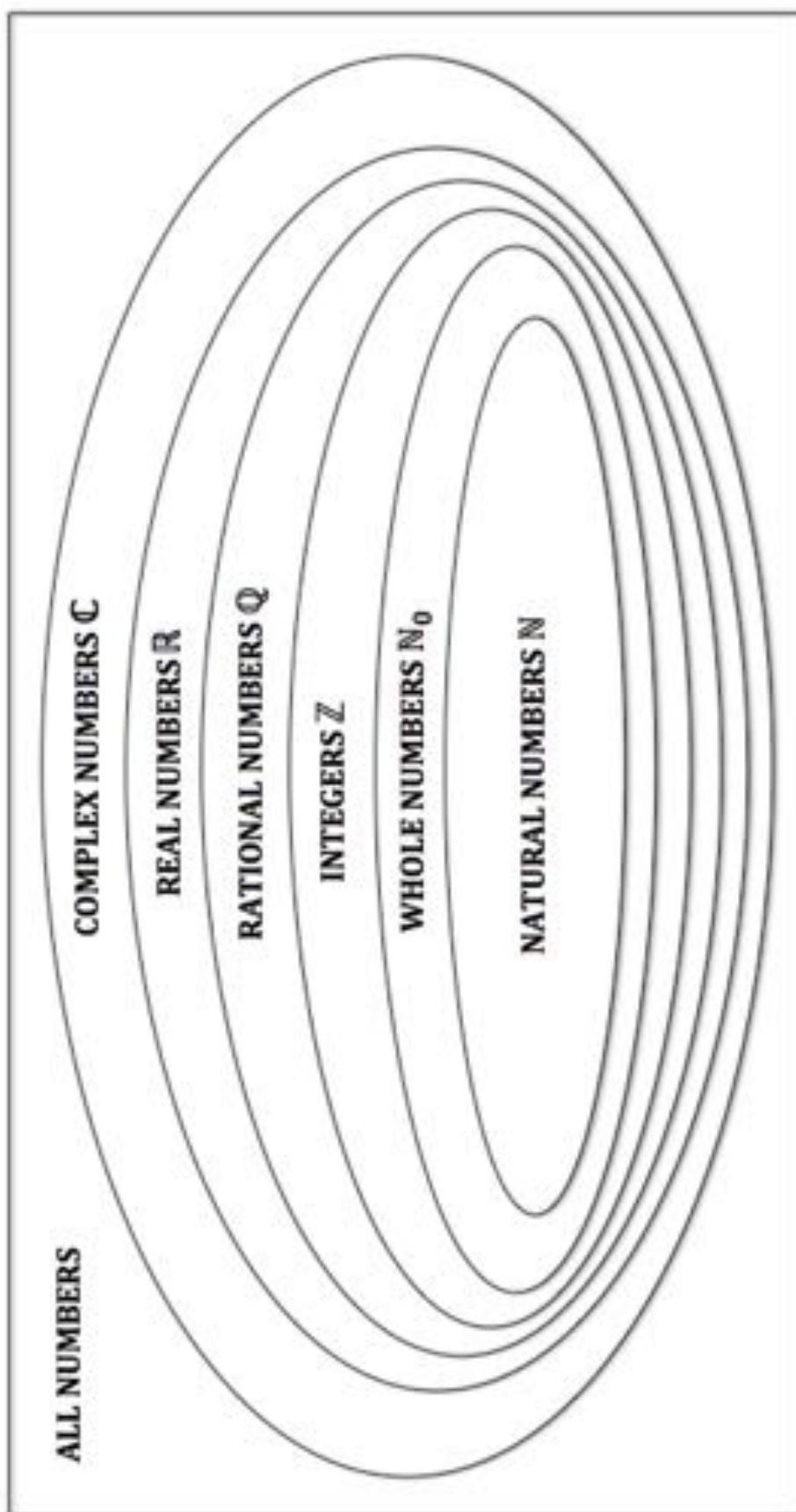
4. Solve for  $x$  in the following quadratic equations.  
 Match the solutions to the graphs. Explain your reasoning.

(a) $x^2 + 6x = 0$	(b) $2x^2 + 5x - 4 = 0$	(c) $\frac{1}{4}x^2 + x + 1 = 0$	(d) $5x^2 + 3 = 0$
Diagram 4 	Diagram 5 	Diagram 6 	Diagram 7 

**VENN DIAGRAM OF SETS OF NUMBERS**

Write the following numbers in the correct part of this Venn diagram. You can write the decimals with decimal commas if you wish.

$\frac{22}{7}$ ;  $\sqrt[3]{-27}$ ; 6; 0; 0.274;  $\pi$ ;  $\sqrt{7}$ ;  $-3$ ;  $\sqrt{-5}$ ;  $\sqrt{2}$ ;  $7.6$ ;  $\sqrt[5]{32}$ ;  $\sqrt{16}$ ;  $7 + 3i$ ; 3,457



Choose some more numbers and write them in the Venn diagram.

You could use this diagram to make a large poster for your classroom.