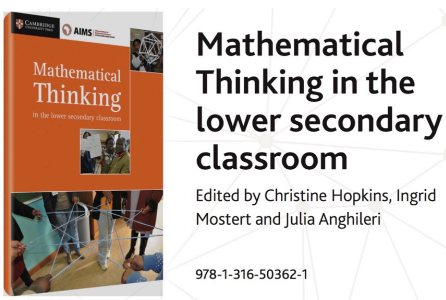
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from Amazon or from http://www.cambridge.org/za/education Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to http://www.cup.co.za

For reviews and curriculum map see https://aiminghigh.aimssec.ac.za/mathematical-thinking/

The AIMSSEC App can be downloaded from the internet onto any android smart phone, laptop or tablet in 3 or 4 minutes and it is free.

Go to Google Play and search for **aimssec** and follow instructions. **Please register on the AIMSSEC App so that,** when you are connected to the internet, you can add comments, ask questions and join in professional discussions on the AIMING HIGH Teacher Network website.

After downloading the AIMSSEC App, everything will be available on your own phone or other device so that you will be able to use all the AIMING HIGH resources, lesson activities and professional development workshop guides offline, that is WITHOUT USING THE INTERNET.

From time to time when you connect to the internet you can update your AIMSSEC App content with recently added resources.

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 TITLE PAGE

- Teaching strategy focus. *Each guide focuses on and exemplifies a teaching* Curriculum content and learning outcomes.
- Summary of mathematical topic (FACT BOX.)
- Resources needed

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS

Two pages for you to work through with your colleagues. These activities are to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to pause, think and try the activity, and when to record your work.

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS

Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 - 10 CHANGES IN MY CLASSROOM PRACTICE

Pages on using the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates. For follow-up activities you will find lots more lesson activities on the AIMING HIGH Teacher Network https://aiminghigh.aimssec.ac.za/category/lesson-activities/

CHILDREN IN FAMILIES
Mathematical modelling and making real life connections.
By Sue Pope

Sue Pope is Head of Service: Science, Mathematics and Core Skills at SQA – Scottish Qualifications Authority. She is a mathematics educator who has worked in schools, universities, local authorities and national policy. She believes everyone can learn mathematics and is entitled to the best possible learning experiences. Building positive attitudes towards mathematics is as important as ensuring successful learning recognised through academic achievement – attitudes last a lifetime. Sue has worked with mathematics educators from around the world to develop curriculum policy and teaching materials. Sue is a long-standing active member of the Association of Teachers of Mathematics.

Children in families

Teaching strategy: Mathematical modelling and making real life connections

Curriculum content: statistics, probability, simulation

Prior knowledge: simple probability, tally charts, bar charts, two-way (contingency) tables

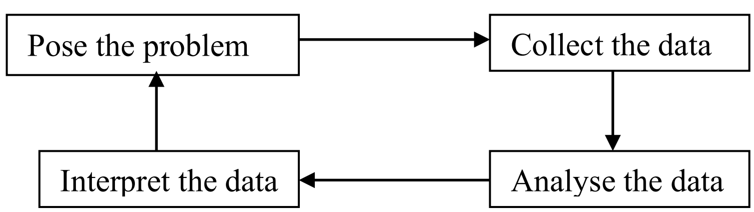
Intended Learning Outcomes: At the end of this activity teachers and learners will

- Know how to use simulation to generate data to solve a problem
- Understand that the outcomes of random processes can be surprisingly predictable
- Be able to design an experiment, collect, represent and interpret data
- Appreciate that *simulation* (modelling) can be used to explore hypothetical situations, and how probability models can be used to evaluate statistical data
- Have experienced the statistical problem-solving cycle

Fact box

Hypothesis: a statement of what we think might be true.
Simulation: a simulation imitates the working of a real world situation, for example we make a *mathematical model* of a situation, and then *simulate* how it works using a random generator (such as a coin) to generate data for a hypothetical situation.

The statistical problem-solving cycle



Tree Diagrams and Contingency Tables are powerful ways of representing the outcomes of successive events.

In a **tree diagram** outcomes are represented at the end of each branch and the branching shows the order in which the events occur. Probabilities are written on the branches. Every possible outcome of the successive events can be traced through the tree.

A **contingency table** can be used for two successive events and can record frequencies or probabilities. The frequencies can be scaled to make the numbers easier to work with, for example imagine 100 or 1000 total outcomes.

***Resources needed**: coins and if possible dice or cards numbered 1 to 6, squared paper.*

Workshop Activities for Teachers

Activity 1: Children in families

Squared paper for collecting and representing data *Whole group* *e.g. 20 minutes*

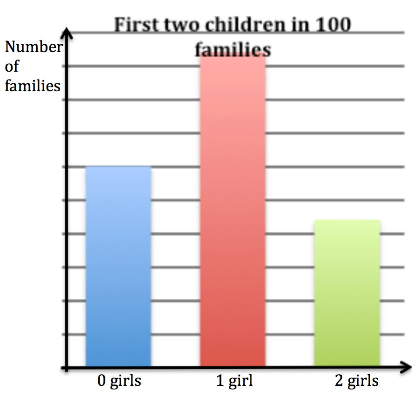
Problem: what is the most likely combination of boys and girls in the first two children in any family?
Imagine the first two children in families you know and record whether they are boys or girls in order of birth.
Collect the information for everyone in the group.
What combination of boys and girls is most common?
Assuming boys and girls are equally likely, calculate the probability of different birth sequences of boys and girls.

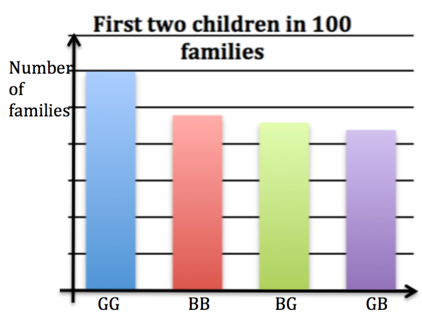
How does your data compare with the calculated probabilities?

Before you read on stop and try this for yourself.

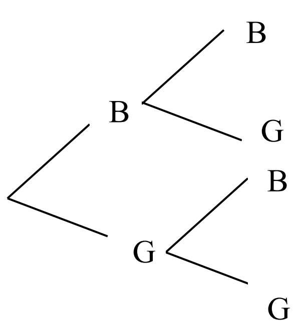
This problem can be extended to exploring different numbers of children.

Notes to help you do Activity 1
Did you expect the four possible outcomes: BB; BG; GB and GG; to occur equally often? This investigation will help you to understand that they are equally likely and that data you collect from a sample or from an experiment can vary from what you expect.
The collected data is best represented in a bar chart (the bars should be separate because it is discrete data). Two charts are possible, the number of girls (or boys) in the family (0, 1, 2) and each of the four ordered combinations. They might look like this:





A tree diagram is helpful when looking at outcomes in this situation



$p(BB) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$p(BG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(GB) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

You might expect to get equal frequencies in your collected data, but this is a common misconception.

Activity 2: A hypothetical society

Coin, squared paper to record outcomes *Work in pairs then collect data from everyone.*

Imagine living in a society where families can have as many children as they like but stop after the birth of the first boy. What do you think such a society would be like? Talk about this with your partner.

Assuming that a boy or a girl is equally likely, you can use a coin to *simulate* this situation for yourself. Decide which side of the coin represents a girl, then the other side represents a boy. Toss the coin, keep going until you get a boy and record the size of 20 families.

What is the average family size?

Just one in each family is a boy, so what is the ratio of girls to boys?

Collect the results of the simulation experiments from your whole group. What is the average family size? What is the ratio of boys to girls? Is this what you expected?

Notes

A possible result for 20 families:

B, B, B, GB, GGB, GGGGGGB, B, GGB, B, B, B, GB, GGB, B, GGB, B, B, GGB, B, GGGB

No. girls	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	**11**	**2**	**5**	**1**			**1**

In the 20 families there are 20 boys and 21 girls, so the average family size is 2.05.

Extending the simulation to 100 families gave the following results:

No. girls	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Frequency	**58**	**13**	**17**	**7**	**3**		**1**				**1**

In the 100 families there are 100 boys and 96 girls, so the average family size is 1.96

These activities show the **statistical problem-solving cycle**: you *start with a problem or context* that is likely to capture learners' interest and curiosity. In the case of Activity 1 you collect data from the group of learners you are working with and they make up a sample taken from the whole population of families. In Activity 2 you generate experimental data using a coin to simulate the equally likely outcomes of boys and girls. Then you *analyse the data*: in Activity 1 by drawing a bar chart and comparing it to the tree diagram that gives theoretical probabilities for the scenario, and in Activity 2 you calculate the average number of children in a family. This allows a *conclusion* to be drawn about the original problem that may lead to *further investigations*.

Activity 3 Discussion of statistical applications in the real world *20 minutes*

Share ideas about the use of surveys or experiments to make predictions and how you might obtain publicity material or newspaper articles that you could use in class, for example about opinion polls to predict election results, police records to predict crime, population samples to predict the total population, consumer surveys about a new product being marketed, clinical trials to test the effectiveness of new drugs or medical treatments, data used to assess insurance risk etc.

Discuss the *Medical Statistics* example on page 9 and why doctors use several tests as well as the patient's symptoms in making a diagnosis.

Classroom Activity for Learners

Activity 1: Children in families

Squared paper, coins *Whole class* *1 hour*

What the teacher does	What the learners do
Invite learners to tell you by a show of hands about the eldest two children in their own family. You will need to decide whether step brothers and sisters are included.	Write down combinations of BB, BG, GB and GG for their own families.
What combination of children do you think is most likely?	Make a hypothesis.
Collect the group's results. Invite children to represent the data.	Share their results and record the class results. Make a bar chart.
Are these results what you might expect? Introduce tree diagrams.	Complete the tree diagram and calculate the probabilities.
What frequencies might you expect given these probabilities?	Consider the relationship between experimental and expected frequencies.
Gather ideas and ask learners to give reasons for their answers.	
Explain how a coin can be used to simulate an experiment. Tells learners to work in pairs to simulate the first two children in 10 families. Collect the class results and compare simulated data with the expected frequencies.	Toss a coin twice to simulate the first two children in a family. Repeat to get results for 10 families. Notice that the simulated data is like experimental data and doesn't match the expected frequencies exactly.
Introduce the hypothetical society in which people stop having children once they've had a boy. What would the society be like?	Hypothesise what the society might be like.
Invite the children to use the coin to simulate the hypothetical society where families can have as many children as they like but always stop after the birth of the first boy.	Use a coin to simulate ten families in the hypothetical society. Find the average family size and number of boys and girls.
Collect the class results. Encourage reflection on the outcomes.	Share their results and record the class results. Find the average family size and number of boys and girls – how does this compare with their hypothesis?

Ideas for teaching Activity 1

A discussion about how many boys and girls there are in families, and whether or not there are any patterns in the number of boys and girls would be a good start. What sort of questions could we ask? What assumptions do we need to make?

It is important to agree that boys and girls are equally likely. There is the potential for students to ask their own questions (e.g. what happens for the first three or four children in families?)

Activity 2: How good is the prediction from a survey or simulation?

Squared paper, coins, newspaper article *Whole class* *1 hour*

What the teacher does	What the learners do
How accurate do the class think such predictions are? Point out that in many real world situations surveys or experiments are the **only** way to predict frequencies or probabilities of future events. Tell the class they will do a simulation and calculate the probabilities to check how accurate it is.	Reflect and respond to the question.
What combinations of boys and girls can there be in a family of 4 children?	Write down permutations: BBBB, BBBG, BBGB, BGBB, GBBB, BBGG, BGBG, BGGB, GBBG, GBGB, GGBB, BGGG, GBGG, GGBG, GGGB, GGGG.
How likely is a family of 4 children to be all boys? How can we do a simulation to test this?	Hypothesise
Explain how a coin can be used to simulate families.	Work in pairs. Toss a coin to simulate families of 4 children, record results for 16 experiments.
Collate the group's results and ask learners to represent the data.	Share their results and record all the class results Make a bar chart for the number of boys in the families.
Are these results what you might expect?	
Introduce tree diagram and assist learners to calculate probabilities.	Complete the tree diagram and calculate the probabilities
What frequencies might you expect given these probabilities?	Consider the relationship between experimental and expected frequencies
Compare the simulated data with the expected frequencies. Encourage reflection on the outcomes.	Notice that the simulated data is like experimental data and doesn't match the expected frequencies exactly.
Discuss the difference between a simulation, an experiment and a survey.	Notice that a larger number of simulations usually gives a better prediction.

Ideas for teaching Activity 2

If possible bring to the lesson a newspaper or magazine article about a real-world situation that reports expected frequencies or probabilities as the result of a survey or experiment, or use the example on Medical Statistics (page 9). Ask the learners to comment on how accurate they think this prediction is. Ask if they can think of other examples where surveys or experiments are used to make predictions (for example clinical trials to test the effectiveness of new drugs or medical treatments, opinion polls to predict election results, consumer surveys on a new product being marketed, samples from populations to predict the total population, for instance 'how many fish in a lake?', police records to predict crime, use of data for assessing insurance risk etc.).

In this lesson learners are encouraged to discuss real world applications of probability and to make hypotheses. The objective is to provide motivation for thinking mathematically and relating their schoolwork to applications in the real world. The problem of assessing the accuracy of predictions from polls and experiments leads to practice in finding all the different elements of a sample space, and to practice in drawing graphs, tree diagrams and contingency tables and calculating relative frequencies and probabilities.

Changes in my classroom practice

It is important to capture learners' interest at the start of a lesson and here this is done by exploring the number of children in their own families. The other activities follow the statistical problem-solving cycle. To collect data we tossed a coin to simulate a sequence of events that have two equally likely outcomes. In this way, we made a mathematical model of a real-life situation, recording the frequencies of the different combinations of children in families. This led to work on representing data, on probability, and on the use of tree diagrams. The results may be surprising and not what the learners expect. Random number generators such as different polyhedral dice, spinners, spreadsheets or computers can be used to simulate events. Collecting data from surveys or simulations, or using published statistics, gives plenty of scope for learners to ask their own questions and to develop the investigations further.

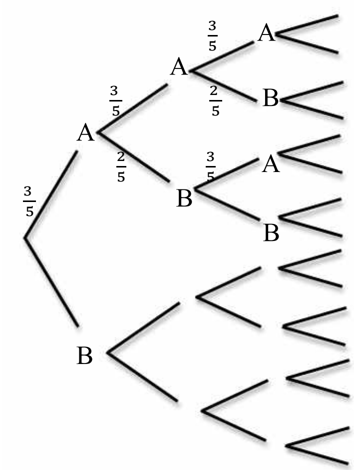
Errors and misconceptions

Experimental results rarely match theoretical probabilities exactly, but it is not uncommon for learners to expect an exact match or to think that experimental results actually give probabilities. For example, when throwing dice they might expect to get the numbers one to six within six throws of the dice. They may be surprised that the same outcome occurs again, and again. This clustering of outcomes is completely normal in random events, and well worth investigating.

There are two common fallacies associated with random events: negative recency – *I've just had a cluster of heads so next time I will get tails* and positive recency – *I've just had a cluster of heads so next time I'll get a head*. The coin does not have a memory – whatever has happened before, the chances of the head or tail remain the same, and equal assuming the coin is unbiased.

Key Questions to check on knowledge and understanding

1. What assumptions have we made? (this is important for thinking about possible differences between the real world and our model, e.g. are boys and girls actually equally likely?)
2. What possible sources of bias might there be in our experimental data?
3. Explain how information is shown on a tree diagram using as an example a tree diagram for the combinations of boys and girls in a family of four children.



4. This tree diagram shows some of the probabilities of two players, Ali and Bev, winning points in a game. Ali is a better player and consistently has a probability of $\frac{3}{5}$ of winning each point. This could be a gambling game that Ali has set up so as to win more than half the time. Bev has a probability of $\frac{2}{5}$ of winning each point. Fill in on the diagram all the missing probabilities and labels of A for Ali winning a point and B for Bev winning a point.

The probability of Ali winning the first 3 points and Bev winning the 4th point is

$$\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{2}{5} = \frac{54}{625} = 0.0864$$

Calculate the probability of:

(i) Ali winning 3 out of the first 4 point, (ii) Bev winning the first 4 points

(ii) Ali and Bev both winning 2 points.

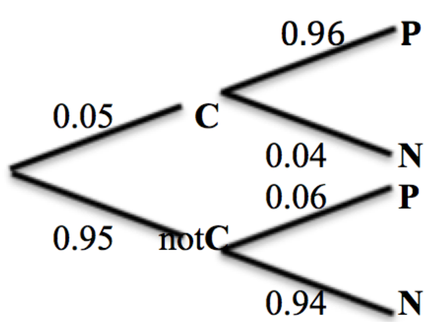
Key Questions to develop understanding

1. There are three possible types of two children family. Why are they not all equally likely?
2. Why use a simulation rather than a survey?
3. Work out (a + b), $(a + b)^2$, $(a + b)^3$ and $(a + b)^4$.
4. How do the distributions of the numbers of boys and girls in one child, two children, three children and 4 children families relate to the coefficients in the expansions of $(a + b)^n$?
5. Have any of the families you generated surprised you?
6. How many times would you need to toss a coin to decide whether it is biased?

Examples of real life connections

Medical statistics

The chances of getting a particular type of cancer is 5%.
A screening test is available that gives a positive result for 96% of those who have the disease but fails to show the disease in 4% of cases. The test is positive for 6% of the population who do not have the disease (a false positive) and correctly shows that the other 94% do not have it.



The probability of testing positive when you don't have cancer is $0.95 \times 0.06 = 0.057$
The probability of testing positive when you do have cancer $0.05 \times 0.96 = 0.048$
This means that of all the positive test results the percentage for those who do not have cancer is $\frac{0.057}{0.057+0.048} = 54\%$

What does this mean? As there are just two successive events a **contingency table** can be used.

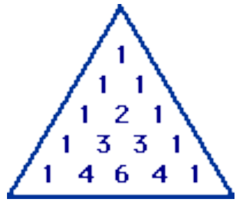
	Cancer	No cancer	
Test positive	0.048	0.057	0.105
Test negative	0.002	0.893	0.895
	0.05	0.95	1
	Cancer	No cancer	
Test positive	48	57	105
Test negative	2	893	895
	50	950	1000

Scaling the probabilities up to whole numbers makes the data easier to understand. So we imagine a population of 1000 people with the contingency table showing the number from 1000 people who fall into each of the four categories.

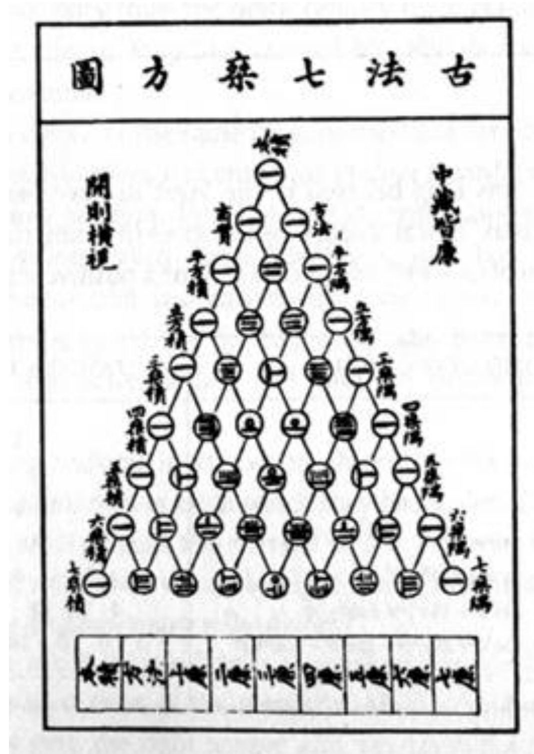
Doctors do not make a diagnosis on the basis of one test alone; they do several different tests. The probability of two tests giving the wrong result is less than the probability of one giving the wrong result and the probability of three tests giving the wrong result is even smaller.

Follow up Activity 1

There are many ways in which these contexts can be explored further.



Children in families is a classic binomial situation because there are two possible variables: the number of girls and the number of boys. So it is possible to make links with what is commonly known as Pascal's triangle although the triangle was known much earlier in China and India.



One idea to follow up is the link between Pascal's triangle and the coefficients of the terms in the expansions of $(a + b)^0$, $(a + b)$, $(a + b)^2$, $(a + b)^3$, … which leads to the Binomial Theorem.

These coefficients are given, row by row, by the numbers in the triangle. For example

$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ so the coefficients in the expansion of $(a + b)^4$ are 1, 4, 6, 4, 1

Learners can be asked to spot the patterns in the triangle, to extend it to the sixth and seventh rows and to test if these rows give the coefficients in the expansions of $(a + b)^5$ and $(a + b)^6$.

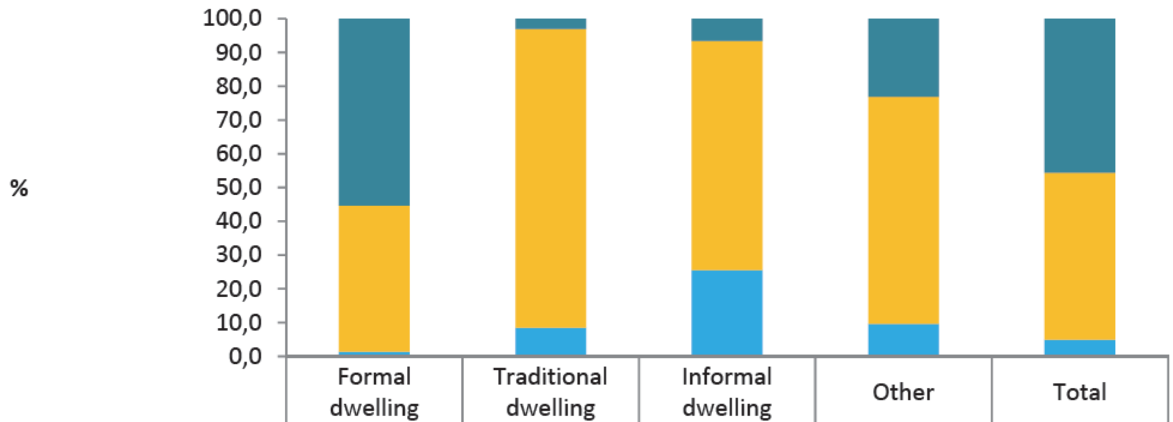
The Chinese image suggests the possibility of learning more about the history of mathematics, or working on different number systems, as well as investigating the properties of the triangle.

Many number sequences and patterns can be found in this triangle including the natural or counting numbers, the square numbers and the triangle numbers. For ideas for investigating Pascal's Triangle see: http://mathforum.org/workshops/usi/pascal/pascallessons.html

Follow up Activity 2: *Access to toilet facilities:*

UN Sustainable Development Goals include access to adequate sanitation. Sub-Saharan Africa has a very high proportion of the population with inadequate sanitation. The 2016 South Africa Community Survey/Census includes the following diagrams (Figure 7.12 shows %s):

Figure 7.12: The distribution of households by location of toilet facilities and type of main dwelling



	Formal dwelling	Traditional dwelling	Informal dwelling	Other	Total
In the dwelling/house	55,4	3,0	6,6	23,1	45,6
In the yard	43,3	88,5	67,8	67,3	49,5
Outside the yard	1,3	8,5	25,6	9,6	4,9

Table 7.15: Distribution of households by location of toilet facilities and type of main dwelling – CS 2016

Toilet location	Formal dwelling		Traditional dwelling		Informal dwelling		Other		Total	
	N	%	N	%	N	%	N	%	N	%
In the dwelling	7 319 207	55,4	32 412	3,0	136 544	6,6	30 818	23,1	**7 518 981**	**45,6**
In the yard	5 724 269	43,3	951 805	88,5	1 400 167	67,8	89 752	67,3	**8 165 993**	**49,5**
Outside the yard	177 250	1,3	91 611	8,5	528 295	25,6	12 866	9,6	**810 022**	**4,9**
Total	**13 220 726**	**100,0**	**1 075 828**	**100,0**	**2 065 007**	**100,0**	**133 436**	**100,0**	**16 494 996**	**100,0**

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They can be used to prepare the contingency table below, where all entries are percentages

	Formal dwelling	Traditional dwelling	Informal dwelling	other	
indoor	44.4	0.2	0.8	0.2	45.6
in yard	34.7	5.8	8.5	0.5	49.5
beyond yard	1.0	0.6	3.2	0.1	5.6
	80.1	6.5	12.5	0.8	100.0

How are these representations related?
What questions can be asked about this data, e.g. access to indoor toilets, type of dwelling etc.?
Which representation is most helpful for each question? Why?