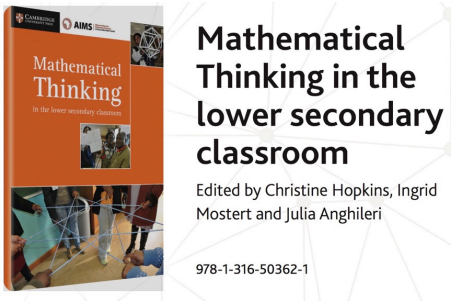
AIMSSEC

WORKSHOP GUIDES FOR TEACHERS TO LEARN TOGETHER
UPPER SECONDARY
NA3 STRAIGHT LINE GRAPHS & SIMULTANEOUS EQUATIONS
Guide for your own self-help professional development workshop

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from Amazon or from http://www.cambridge.org/za/education Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to http://www.cup.co.za

For reviews and curriculum map see https://aiminghigh.aimssec.ac.za/mathematical-thinking/

The AIMSSEC App can be downloaded from the internet onto any android smart phone, laptop or tablet in 3 or 4 minutes and it is free.

Go to Google Play and search for **aimssec** and follow instructions. **Please register on the AIMSSEC App so that,** when you are connected to the internet, you can add comments, ask questions and join in professional discussions on the AIMING HIGH Teacher Network website.

After downloading the AIMSSEC App, everything will be available on your own phone or other device so that you will be able to use all the AIMING HIGH resources, lesson activities and professional development workshop guides offline, that is WITHOUT USING THE INTERNET.

From time to time when you connect to the internet you can update your AIMSSEC App content with recently added resources.

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 TITLE PAGE

- Teaching strategy focus. *Each guide focuses on and exemplifies a teaching* Curriculum content and learning outcomes.
- Summary of mathematical topic (FACT BOX.)
- Resources needed

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS

Two pages for you to work through with your colleagues. These activities are to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to pause, think and try the activity, and when to record your work.

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS

Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 - 10 CHANGES IN MY CLASSROOM PRACTICE

Pages on using the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates. For follow-up activities you will find lots more lesson activities on the AIMING HIGH Teacher Network
https://aiminghigh.aimssec.ac.za/category/lesson-activities/

Straight Line Graphs and Simultaneous Equations

Representing and connecting different mathematical ideas.
By CHRISTINE HOPKINS

Christine Hopkins studied mathematics at Cambridge University and has extensive experience of school teaching and teacher training internationally. She has worked as a teacher trainer in England at Roehampton University, and in Cambodia and Africa, trying to identify the skills and approaches that we can use and share as teachers to improve the understanding and enjoyment of mathematics by those we teach. Christine was Assistant Dean at Roehampton University. Her writing and editing includes professional development material translated into Khmer from the BETT Cambodian Project and her role as chief editor of the AIMSSEC Mathematical Thinking in Lower Secondary Classrooms. She is fascinated by the teaching of mathematics and has worked as a teacher with children from 9 – 18 years of age and found at all ages the same spark of excitement at solving a problem or understanding something that seemed too difficult.

Straight Line Graphs and Simultaneous Equations

Teaching strategy: Mathematical connections and multiple representations.

Curriculum content: Linear simultaneous equations in two variables, representation of pairs of linear equations algebraically and graphically.

Prior knowledge: How to solve linear equations. How to calculate with fractions and negative numbers.

Intended Learning Outcomes At the end of this activity teachers and learners will:

- ✓ Know how to solve simultaneous linear equations
- ✓ Understand that each equation gives information about two variables and that two distinct equations give a single solution namely the values of the two variables.
- ✓ Be able to construct an equation and solve simultaneous equations by trial and error and by algebraic manipulation
- ✓ Appreciate why adding each side of two equations gives a valid statement
- ✓ Have experienced a concrete representation of an abstract idea
- ✓ Be able to make links between these different representations, choosing the best for a given situation.

Fact box

Linear equation in one unknown:

General form: $ax + b = c$ Example: $2x - 5 = 7$

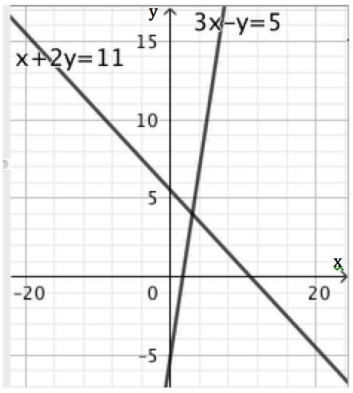
Simultaneous equations in two unknowns:

(equations that are both true at the same time)

General form: $ax + by = c$ Example: $x + 2y = 11$ (1)

$dx + ey = f$ $3x - y = 5$ (2)

The second example can be solved by looking at the coefficients (numbers) of x and y and choosing how to eliminate either x or y, or by rearranging and substituting.



The graph of each equation in the second example is a straight line.

Every point on the line satisfies (fits) the equation.

Two distinct straight lines usually cross at a single point. The point where the two lines cross gives the solution of BOTH equations SIMULTANEOUSLY.

***Resources**: Two differently coloured dice* or *2 different sets of cards numbered 1 to 6. Showboards. Graph paper.*

Workshop Activities for Teachers

Activity 1: Guessing the scores on the dice

Two dice for each pair, showboards Whole group then pairs 40 minutes

A. Trying the learner activity without using algebraic techniques

For this activity there is one rule: NO ALGEBRAIC TECHNIQUES can be used which learners do not know at the very start of their work on simultaneous equations.
Read though the first classroom activity for learners. In this activity the teacher throws two dice and hides the scores. The teacher gives the learners two clues so that they can work out the scores on the dice by trial and improvement (e.g. 'the numbers on the two dice add to 10; the number on the blue die is two more than the number on the red die').

The purpose of this activity is to give learners an understanding of how simultaneous equations work BEFORE introducing the algebraic techniques.

Many of you might be tempted to start using the formal techniques you know for solving simultaneous equations. Try the activity with one member of your group acting as the teacher. The others must solve the problem by simple methods like listing possibilities.

Repeat the activity working in pairs. One teacher throws the dice and makes up the clues. The second teacher writes down all the possible scores for the first clue and then chooses the pair which fits the second clue as well.

B. Demonstrating algebraic techniques

Choose two teachers to demonstrate to the group how they teach the formal methods of solving simultaneous equations. The teacher demonstrating throws the dice but this time the clues are recorded as equations: $r + b = 7$

$2r + b = 10$ *(What do these mean, for a red and a blue die?)*

There are many solutions for each equation but you are going to find the one solution that works *simultaneously* for both.

There will probably be at least two different methods. Discuss the small differences in how you talk about the process and what you ask the learners to record. Can you agree on what the learners find difficult and what you can do to help them to understand?

An important point in discussing formal methods is WHY it is OK to add and subtract equations – this is not something that has been done before. Teachers should discuss in pairs how they would justify this. Think of different ways of recording and discuss the different choices.

Notes to help you do Activity 1

1. List all possibilities for the first clue; for $r + b = 7$ the possibilities could be (1, 6) or (2, 5) etc
2. Should we include (6, 1) as well as (1, 6)? Trying the next part will help you to decide. If the first number is red and the score on the red plus three times the score on the blue is 14 (2, 4) works but not (4, 2) so the order matters.
3. To make the activity easier, make the first clue the sum of the red and blue scores $r + b =$ the first few times. The second clue could be $3r + b =$ or $2r – b =$ etc
4. Adding the same to both sides of an equation produces a true statement. By adding two equations you are adding the same to both sides.

Activity 2 Representing simultaneous equations as graphs

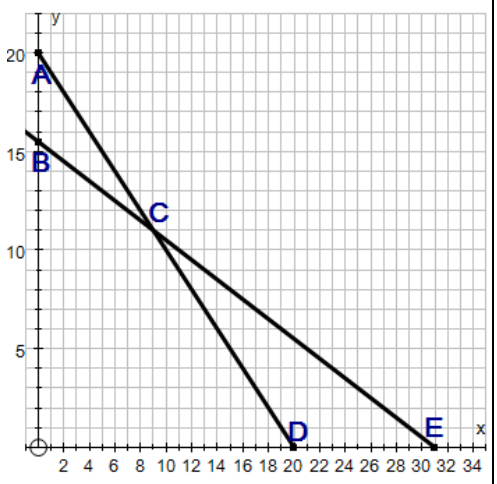
Graph paper Pairs then whole group discussion 20 minutes

In the second classroom activity, take x for the age of the girl and y for the age of the boy (in years).
For the clue x + y = 20 plot all the guesses, for example (1,19) (5,15)… The points lie on a straight line.
Discuss whether all the points on the line make sense in the problem (5⅓, 14⅔) is OK but perhaps not (20,0).

We can't know which point gives the right answer because there are so many points on the line.

Now plot the points for the other clue: x + 2y = 31.
This gives another line. The point C where the lines cross fits both clues so the co-ordinates of C give the solution.

As teachers we may want to invent problems so how do we go about this? To invent another age problem you might use graphing software such as Geogebra, or work backwards, choosing a solution point first, then plotting two lines through it for which you can make up age-clues.



Activity 3: Making connections: sequences, functions and graphs 20 minutes

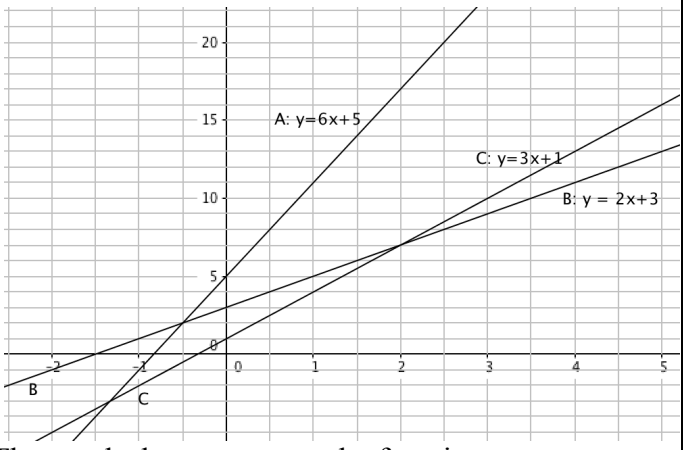
1. Work in pairs. Do the first 2 steps on your own without speaking. Think of a multiplication table, and count in multiples, for example 6, 12, 18, 24, 30,… .
2. Add 5 to every term and write down what you get, for example 11, 17, 23, 29,...
3. Now show the sequences you have written down to each other and work out which multiplication table your partner started with.
4. Discuss how you did this.
5. Number the terms of your sequence and write down ordered pairs, for example: (1, 11) (2, 17), (3, 23), (4, 29), (5, 35) …

Repeat this a few times adding or subtracting a different number each time.

The ordered pairs you write down each time give some inputs and outputs for a linear function.

Plot the corresponding points and join them to form straight lines.

Discuss the connections between the list of multiples, the arithmetic sequences, the (discrete) points you plotted and the continuous lines you have drawn representing related continuous linear functions.



The graphs here represent the functions
A: y = 6x+ 5 (i) from the sequence 11, 17, 23…
B: y = 2x + 3 (ii) from the sequence 5, 7, 9,…
C: y = 3x + 1 (iii) from the sequence 4, 7, 10, …
(*What is true where graphs cross?)*

Classroom Activity for Learners

Activity 1: Guessing the scores on the dice

Two dice or 2 sets of cards numbered 1 to 6. Showboards Whole group 50 minutes

What the teacher is doing	What the learners are doing
1. Teacher chooses a red and blue number either by throwing two dice or by choosing from cards marked 1 to 6. The teacher hides the results ready to be revealed at the end of the activity 2. Teacher gives first clue. The teacher tells the class the number that you get by adding the red number to the blue number. 3. Teacher gives second clue. The teacher tells the class what you get when you add twice the red number to the blue number. 4. Teacher writes RED 1 2 3 4 5 6 on the board and points to each number in turn. Learners must put their hand up when the teacher points to the number they think is the correct red value. Teacher shows the red dice or card Repeat for blue number	Learners work in pairs Learners write down all possible combinations e.g. If the total is 7 they write RED 1 2 etc BLUE 6 5 Each pair tries to work out what the red and blue values must be. They must talk quietly so no one else hears. *If learners have showboards they can write their answer on the board and all show answers together.*

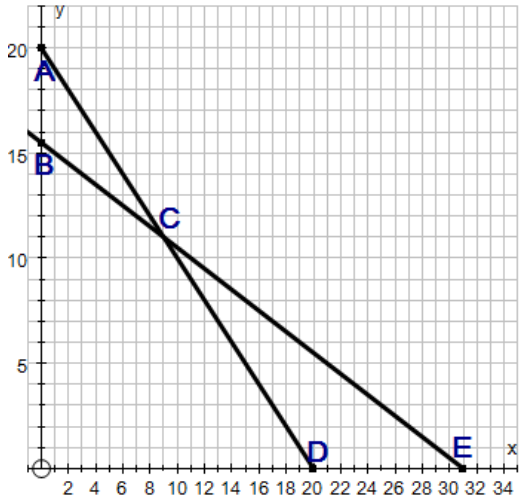
Ideas for Teaching Activity 1

1. Ask some learners to come to the front of the classroom and to choose the numbers and give the clues.
2. Get the learners to work in pairs choosing red and blue numbers and giving clues to their partner. To keep the equations simple the first clue must always be the sum of the red and the blue but the second clue can be more complicated.
3. When the learners have all successfully found the numbers by trial and error work with the whole class.
   - Ask one pair to give you clues.
   - Write as equations on the board
   - Demonstrate the formal method.
   - At the end ask the pair if you have found the correct numbers.

Learners now work in pairs to choose red and blue numbers again.
They write the clues as equations and solve formally.

Activity 2: Finding ages in many ways

Showboards. Pairs 50 minutes

What the teacher is doing	What the learners are doing
At the beginning of the next lesson use this variation on the dice activity. 'A woman has two children. The age of the girl added to the age of the boy is 20, twice the girl's age plus the boy's age is 31' The learners can use any method they like to find the ages of the children. Collect lots of possible solutions using just the information that the age of the girl plus the age of the boy is 20, for example 5 and 15, 12 and 8… It looks like x + y = 20 is a straight line. Discuss other points on the line – do they all fit the rule about the ages adding to 20? Repeat for the rule that 'Twice the girl's age plus the boy's age is 31'. Sketch the graph on the board. Ask the learners to: a. Find the coordinates of A and B where the lines cross the y-axis C where the lines intersect D and E where the lines cross the x-axis. b. Explain how these co-ordinates link to the two equations. *Emphasise that we can now understand the problem in words, in algebra and as a graph.*	Discussing the problems in pairs. Trying to first guess a solution and then check with a formal algebraic method. The learners plot the points (5.15) (10,10) (12,8) on a graph and check if other points on the line make sense.  Learners discuss the co-ordinates of A, B, C, D and E Check coordinates of C satisfy the equations and give the solution to the problem.

Ideas for Teaching Activity 2

1. Encourage both trial and improvement and formal methods. Trial and improvement helps the learners to understand the problem. Useful questions: Is the boy's age odd or even? Do you think 1 year and 19 years is very likely?
2. The approach used here is to work really hard on one problem - extracting the general principles. This gives the learners a deep understanding of the problem. They can then proceed to more routine practice with confidence.

Activity 3 Making connections: sequences, functions and graphs

Do Activity 3 from the teachers' workshop activities. If you are short of time, on one day do the part where pairs of learners write down sequences based on multiplication tables and guess which table their partner chose and what they added or subtracted. Discuss this in terms of functions and inverse functions: multiply by 6 add 5 has inverse subtract 5 and divide by 6. Next day graph functions, find intersections and link to simultaneous equations. If time you could follow with the activity STEPS https://aiminghigh.aimssec.ac.za/grades-7-to-9-steps/.

Discussion of Teaching Strategy

These activities make connections between:

- simple problems with equations stated in words (numbers on dice, children's ages)
- pairs of simultaneous linear equations,
- formal algebraic methods of solving the equations,
- multiplications tables,
- arithmetic sequences,
- guessing how the sequence was constructed using inverse operations,
- functions made up of a few ordered pairs giving discrete points on a line,
- continuous linear functions,
- graphs of the functions and
- intersection of pairs of lines.

While one focus of these lessons is to learn to solve simultaneous linear equations, by making these connections learners gain a deeper understanding of the concepts. They also learn or revise several topics from their course at the same time. The objective is for learners to realise that they can find the intersection of two lines more quickly and accurately by formal algebraic techniques but also to understand how this connects with other ideas they have met involving linear relationships. For example, the graphs help to visualise how one equation gives many solutions but two linear equations give just one solution, or no solutions if the lines are parallel. Making these connections helps learners to understand the difference between the function represented by discrete points and the continuous function represented by joining those points.

The purpose of the problem-solving activities is to give learners an intuitive understanding of how simultaneous equations work BEFORE introducing the algebraic techniques. Learners can solve equations easily by trial and error because they are working with restricted sets of numbers (1 to 6 or 1 to 20). For dice, only integer solutions make sense. For the age problems, fractional solutions also make sense so there are many points on the line. As you explain the formal method you can refer back to the trial and error method to help the learners make sense of the problem.

Tables of values are not mentioned. Learners should be growing more confident about plotting a few points and sketching graphs from their knowledge of the properties of the functions without the need for filling in a table of values. Discuss the different ways of plotting a linear graph from the equation. For example, would your learners need to re-arrange $x + y = 20$ as $y = 20 - x$ and make a table of values or could your learners work with the equation $x + y = 20$ and see that when $x=0$ then $y = 20$, and that when $y=0$ then $x=20$, so they quickly have two points on the graph?

Key Questions to develop understanding - *use often with your learners*
What do you think the graph will look like?
Where should this graph start?
What does this part of the graph mean?
If you go faster do you go more distance in the same time or less distance?
For numbers that add to 5 should we include (2,3) as well as (3,2)? Why?
If x and y are ages adding to 20 which of these makes sense (3½, 16½) (-1, 21). Explain.
If you plot a graph for 2x + y = 5 what shape do you expect?
How many different ways do you know of plotting the graph of 2x+y = 5?
How do you work out 2y – (-y)?

Questions to check knowledge and understanding
1. The girl is three years older than the boy. If g is the girl's age and b is the boy's age which of these statements are true? Explain the meaning of each statement in words.
b + g = 3 b - g = 3 g - b = 3 b = g+3 g = b+3

2. When working with linear equations we do the same operation to both sides**.**
e.g 3x -7 = 12 (Add 7 to both sides)
3x = 19
To solve the simultaneous equations 2x + y = 17 (1)
x - y = 4 (2) If I add equation (1)+equation (2)
3x = 21
Explain why combining equations like this gives you another equation which is true.

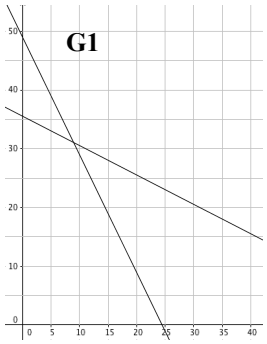
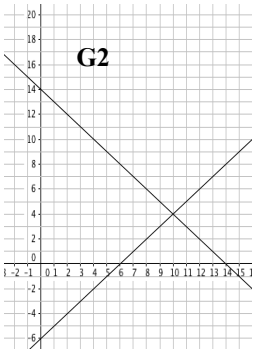
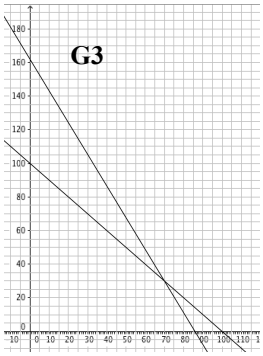
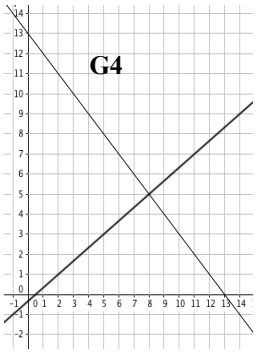
3. What happens when you try to solve these equations?
-x + y = 4 or these -x + y = 4
3x – 3y = -12 3x - 3y = 11
Explain your answer in words and with graphs.

4. Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales the wobble. So I held the baby and stood on the scales while the nurse read off 78kg. Then the nurse held the baby while I read off 69kg. Finally I held the nurse while the baby read off 137kg. What is the combined weight of all three (in kg)?
This problem is taken from the UK Mathematical Challenges

Further Activities:

A. Matchless: https://aiminghigh.aimssec.ac.za/grades-9-to-12-matchless-6/
(There is a value of x, and a value of y, that make all five of these expressions equal in value:
2x + 3y - 20, 5x - 2y + 38, 4x + 5y - 72, x - 4y + 108, 3x – y + 39.
What are these values?
Do you have too much, not enough, or just the right amount of information?)

Multiple representations: Cut out these cards and match them in sets of 4 cards with a word problem, a pair of simultaneous equations, a graph and the coordinates of a point in each set. Teachers may first give out the word problems for learners to solve and later give all the cards for learners to sort and check their solutions.

W1 100 tickets were sold for a concert. Adults paid R85 and children paid R45. The takings were R7300. How many tickets of each sort were sold?	**W2** If twice Sizwe's age is added to the age of his mother, the sum is 49. But if twice her age is added to Siswe's age the sum is 71. Find both ages.	**W3** The sum of two numbers is 13 and twice the first number minus three times the second number is 1. Find the numbers	**W4** A fish can swim at 14 m/s with the current and 6 m/s against it. Find the speed that the fish can swim in still water and the speed of the current.
E1 **x + y = 13** **2x – 3y = 1**	**E2** **x + y = 100** **17x + 9y = 1460**	**E3** **x + y = 14** **x – y = 6**	**E4** **2x + y = 49** **x + 2y = 71**
			
P1 **(10, 4)**	**P2** **(8, 5)**	**P3** **(9, 31)**	**P4** **(70, 30)**