FUNCTION FLOW

Function Machine 1:

Input ——— × 3 ⟶ – 5 ⟶——— Output
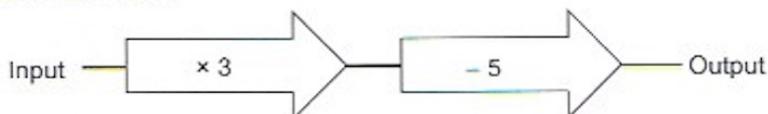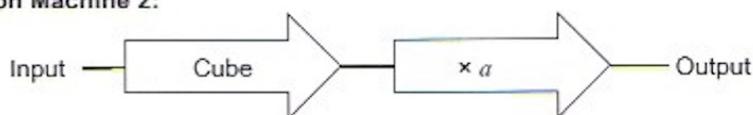
The input to this function machine is **7.** Write down an expression for the output.

If the output is x what was the input?

This flow diagram represents another function where **a** is a constant.
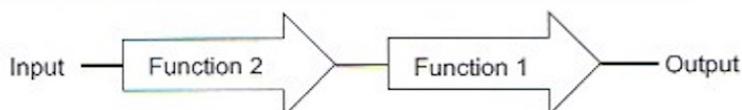
Function Machine 2:

Input ——— Cube ⟶ × a ⟶——— Output

If the input is 4 what is the output?

The functions from Function Machines 1 & 2 are combined to give Function Machine 3 where the output of Function Machine 2 is used as the input for Function Machine 1.

Function Machine 3:

Input ——— Function 2 ⟶ Function 1 ⟶——— Output

The number 2 is put into Function Machine 3 and the output is 3. Find **a.**

Make up some questions of your own using these function machines.

HELP

You might start by completing this table of values for Function Machine 1.

Input x	-2	-1	0	1	2	3	4
$3x$	-6				6		12
Output $3x$ - 5		-8				8	

NEXT

Make up and answer your own questions based on these 3 function machines, exchange your questions with other learners and answer each other's questions.

Draw graphs of the functions given by these machines.
$f_1(x) = 3x – 5$
$f_2(x) = ax^3$
$f_3(x) = 3ax^3 – 5$

NOTES FOR TEACHERS

SOLUTION

If the output to Function Machine 1 is 7 then the input was 4.

If the input to Function Machine 1 is x then the output is 3x – 5. We can write this as $f_1(x) = 3x – 5.$

If the input to Function Machine 2 is 4 then the output was $64a.$ We can write this as $f_2(4) = 64a.$

If the number 2 is put into Function Machine 3 then the output is $f_3(2) = 3 × (8a) – 5$ and if this is 3 then $24a – 5 = 3$ and so $a = ⅓.$

Why do this activity?

The activity gives practice in writing algebraic expressions, in finding inverse functions and also in combining functions. It can be used to introduce function notation.

Diagnostic Assessment 1 This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

Diagnostic Questions

Given an input of x, which expression is the output of this function machine?

INPUT
x

↓

Cube it

↓

Subtract 3

↓
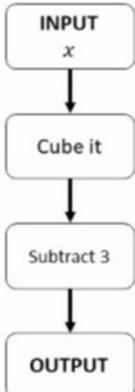
OUTPUT

A $x^3 - 3$

B $3x - 3$

C $(x - 3)^3$

D Not enough information

https://diagnosticquestions.com

A. is the correct answer.

Common Misconceptions
B. The learner is confused about how to write down the cube of a number.
C. The learner has got the operations in the wrong order.
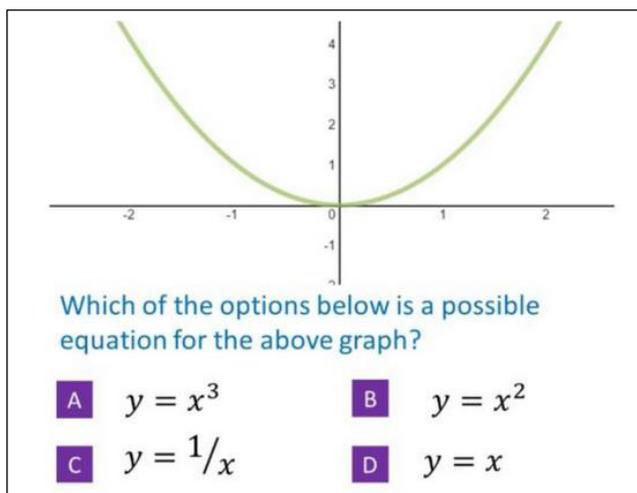**D**. The learner does not understand the information given.

DIAGNOSTIC ASSESSMENT 2 This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

Which of the options below is a possible equation for the above graph?

A $y = x^3$

B $y = x^2$

C $y = 1/x$

D $y = x$

The correct answer is: B

Possible misconceptions:
- Learners don't recognise what happens to a graph when x is replaced by -x.
- They are unable to distinguish between the graphs of linear, quadratic and cubic functions.
- The have not learned to visualise the general shapes of graphs.

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Learning objectives

Learners will:
- review what they know about functions;
- develop a better understanding of how functions combine a series of operations;
- gain fluency in using function notation e.g. learn how a series of operations is written down as an algebraic expression;
- learn the terms input (independent variable) and output) (dependent variable).

Generic competences

In doing this activity learners will develop communication and visualisation skills

Suggestions for teaching

Start the lesson with Diagnostic Assessment 1 and, if necessary, some discussion about the correct way to write down this sequence of operations.

Write the question on the board or make copies of the question from page 1 to give to the learners. Encourage learners who finish the rest of the task to make up their own questions and to exchange with another learner so they answer each other's questions.

Leave sufficient time if possible for learners to come to the board and explain their solutions to the class.

Finish the lesson with a brief summary of what has been learned.

If your class have met graphs of quadratic and cubic expressions they could do the Diagnostic Assessment 2 at the end of the lesson to make the connection between equations of functions and graphs.

Key questions
- What is the inverse operation for ×3 (that undoes or reverses multiplying by 3)?
- Can you tell me what the inverse operation for -5 is (that undoes or reverses subtracting 5)?
- What is the output of that function machine if the input is 6?
- Can you write down an algebraic expression for the output of that function machine if the input is the variable *x*?
- How are you going to answer that question?

Follow up
Primary and Lower Secondary

Mind Reader https://aiminghigh.aimssec.ac.za/mind-reader/

Shifting Times Tables https://aiminghigh.aimssec.ac.za/shifting-times-tables/

Secondary

Steps https://aiminghigh.aimssec.ac.za/steps/

Function Game: https://aiminghigh.aimssec.ac.za/function-game/

Building Functions https://aiminghigh.aimssec.ac.za/function-flow/

Swop https://aiminghigh.aimssec.ac.za/swop/

Undoing https://aiminghigh.aimssec.ac.za/undoing/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
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