

DERIVATIVE MATCHING

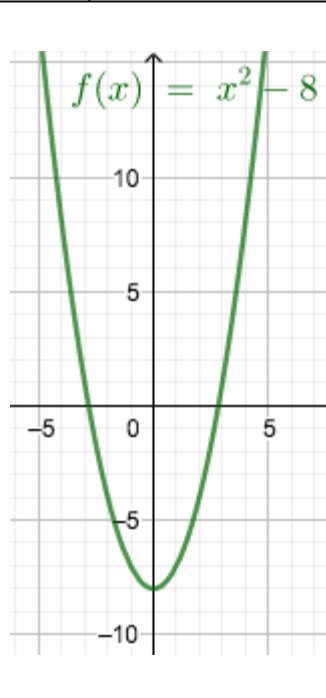
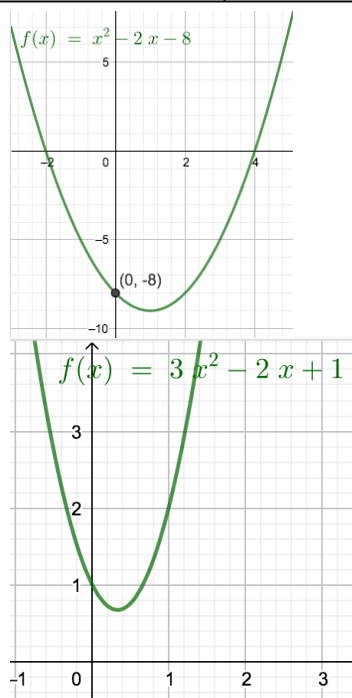
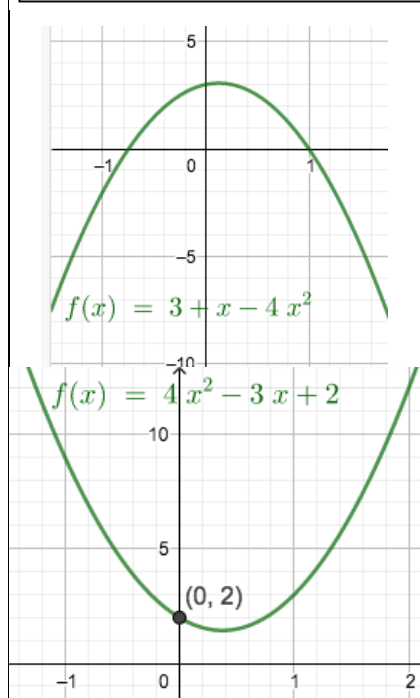
Match the functions with their derivatives and with the values of the functions and derivatives.

$f(x) = 3x^2 - 2x + 1$	$f'(x) = 8x - 3$	$f'(-1) = 9$	$f(-1) = -5$
$f(x) = x^2 - 8$	$f'(x) = 2x - 2$	$f'(2) = 10$	$f(2) = 12$
$f(x) = 3 + x - 4x^2$	$f'(x) = 2x$	$f'(-1) = -4$	$f(0) = 1$
$f(x) = x^2 - 2x - 8$	$f'(x) = 6x - 2$	$f'(1) = 5$	$f(1) = 0$
$f(x) = 4x^2 - 3x + 2$	$f'(x) = 1 - 8x$	$f'(4) = 8$	$f(3) = 1$

What does this information tell you about the graphs of the functions?

SOLUTION

$f(x) = 3x^2 - 2x + 1$	$f'(x) = 6x - 2$	$f'(2) = 10$	$f(0) = 1$
$f(x) = x^2 - 8$	$f'(x) = 2x$	$f'(4) = 8$	$f(3) = 1$
$f(x) = 3 + x - 4x^2$	$f'(x) = 1 - 8x$	$f'(-1) = 9$	$f(1) = 0$
$f(x) = x^2 - 2x - 8$	$f'(x) = 2x - 2$	$f'(-1) = -4$	$f(-1) = -5$
$f(x) = 4x^2 - 3x + 2$	$f'(x) = 8x - 3$	$f'(1) = 5$	$f(2) = 12$



NOTES FOR TEACHERS

These activities are adapted from the STANDARDS UNIT professional development materials produced by the UK Department for Education and Skills. Author Malcolm Swan.

Diagnostic Assessment

This should take about 5–10 minutes.

- Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

What is the derived function of $f(x) = x(x - 4)$	A	$f'(x) = 2x - 4$
	B	$f'(x) = -3x$
	C	$f'(x) = 1$
	D	It is not possible to find the derived function

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A. is the correct answer.

Common Misconceptions

C & D Show no understanding of differentiation

D. This answer shows the learner has no understanding of differentiation or fails to realize that $x(x - 4) = x^2 - 4x$

<https://diagnosticquestions.com>

Why do this activity?

This activity builds on and extends learners' understanding of graphs of quadratic functions. It is useful, soon after they have learned to differentiate powers of x , to provide practice in finding derivatives, substituting values of x into formulas and visualising graphs when they know some of the properties.

Intended learning outcomes

To enable learners to:

- practise differentiating quadratic functions
- practise finding the value of a function and its derivative at specified points
- distinguish between $f(x)$ and $f'(x)$
- relate values of $f(x)$ and $f'(x)$ to the graph of $y = f(x)$

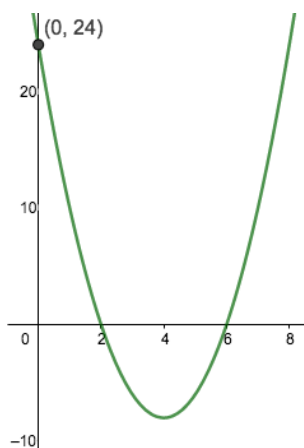
Suggestions for teaching

Learners could work in pairs either using the worksheet copied from page 1 or the set of cards on page 4.

When most pairs have finished sorting the cards into 5 sets, and all pairs have created at least 3 sets of cards, ask the learners to work in pairs and to discuss what each card tells them about the graph of that function.

In a plenary session with the whole class, ask pairs of learners why they matched up their cards to a particular function and what this tells them about the graph.

To review the lesson and summarize what has been learned ask learners to show you their answers to the key questions using showboards (mini-whiteboards).



Either in the same lesson, or in the next lesson, you could sketch this graph (and similar graphs) on the board and ask the learners to tell you everything they can about $f(x)$ and $f'(x)$ at various points on the graph.

Key questions

- If $f(x) = 4x^2 - 3x + 2$, then show me the value of $f(0)$, $f(1)$, $f'(0)$ and $f'(1)$
- Show me a function where $f'(1) = 0$. What can you say about the graph at that point?
- Show me a function where $f'(1) = 1$. What can you say about the graph at that point?
- Show me a function where $f'(2) = -1$. What can you say about the graph at that point?
- Show me two functions where $f(0) = -8$. What can you say about the graphs at that point? What are the differences between the 2 graphs?
- Show me a graph of a function where $f'(0) = 0$
- Sketch a graph of a function where $f'(0) = -5$
- Sketch a graph of a function where $f'(2) = 0$
- Sketch a graph of a function where $f(0) = 10$
- If $f'(x) = 2x - 3$ show me a sketch of a possible function for $f(x)$.

To cater for learners of all abilities:

Possible support

If learners do not understand the difference between $f(2)$ and $f'(2)$ then you could ask other learners to explain it to them.

You could give learners who are likely to struggle a reduced set of cards (say the cards for 3 of the 5 functions). Then if they succeed you could give them the other 2 sets.

Possible extension

You might not give the cards with the derivatives to the most able learners but rather ask them to sort the cards and to work out the derivatives for themselves.

Even more challenging, you might not give the cards with the equations of the functions to the most able learners but rather ask them to sort the cards and to work out the equations for themselves once they know the derivatives and some values.

Learners who finish quickly could be asked to make up more sets of cards in the same style.

Further lesson activities

This idea could be used with other types of functions, for example cubics or functions with fractional or negative indices.

A similar activity could be used to practise substitution of surds or fractions.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6

$f(x) = 3x^2 - 2x + 1$	$f'(x) = 8x - 3$	$f'(-1) = 9$	$f(-1) = -5$
$f(x) = x^2 - 8$	$f'(x) = 2x - 2$	$f'(2) = 10$	$f(2) = 12$
$f(x) = 3 + x - 4x^2$	$f'(x) = 2x$	$f'(-1) = -4$	$f(0) = 1$
$f(x) = x^2 - 2x - 8$	$f'(x) = 6x - 2$	$f'(1) = 5$	$f(1) = 0$
$f(x) = 4x^2 - 3x + 2$	$f'(x) = 1 - 8x$	$f'(4) = 8$	$f(3) = 1$