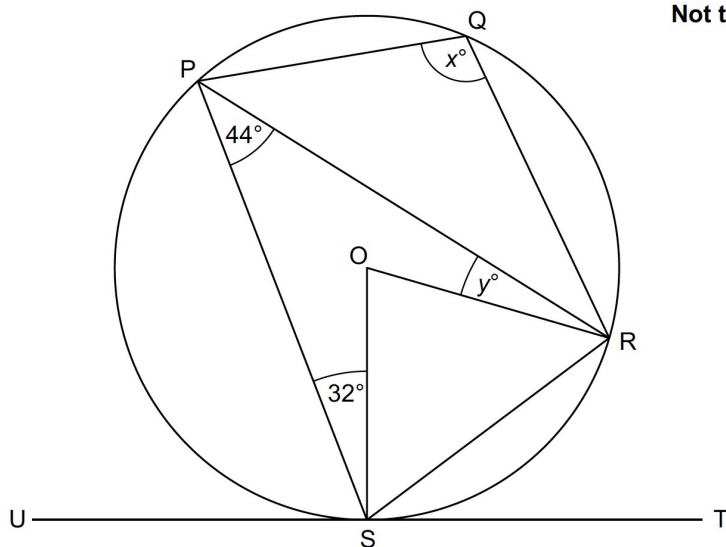


CYCLIC



Not to scale The diagram shows a circle centre O with points P, Q, R and S on the circumference. UST is a tangent to the circle. Angle RPS = 44° and angle PSO = 32° .

Find angles x° and y° .

SOLUTION

Method 1

$\angle SOR = 88^\circ$ the angle at the centre is twice the angle at the circumference

$\angle OSR = \angle ORS = \frac{1}{2}(180^\circ - 88^\circ) = 46^\circ$ isosceles triangle, angles in triangle add to 180°

$\angle PSR = 78^\circ$

$\angle PQR = x^\circ = 180^\circ - 78^\circ = 102^\circ$ opposite angles of cyclic quadrilateral add to 180°

$\angle PRO = y^\circ = 180^\circ - (44^\circ + 32^\circ + 46^\circ + 46^\circ) = 12^\circ$ angles of $\triangle PSR$ add to 180°

Method 2

$\angle SOR = 88^\circ$ $\angle PSU = 90^\circ - 32^\circ = 58^\circ$ OS is perpendicular to tangent UT

$\angle PRS = 58^\circ$ angle in the alternate segment subtended by chord PS

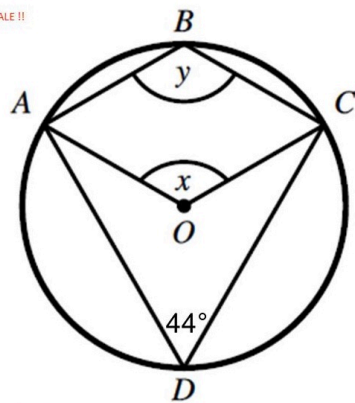
$\angle PRO = y^\circ = 58^\circ - 46^\circ = 12^\circ$

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

- Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

!! NOT TO SCALE !!



Angle ADC = 44°

What is the size of angle x ?



136°



22°



88°



44°

C. is the correct answer. Note that the diagram is not to scale and although angle x looks as if it is bigger than 88° that is the answer because the angle at the centre subtended by arc ABC is double the angle at the circumference.

Common Misconceptions

A. This answer “136 degrees is the only number that is an obtuse angle out of all of the other numbers. Other numbers are acute angles.” shows learner thought the angle must be obtuse and used $180^\circ - 44^\circ$.

B. This learner half remembered the theorem but instead of doubling the angle at the circumference, halved it.

D. Probably just a guess.

Follow up diagnostic question: Which theorem is used to find angle y ?

A. The angle in the alternate segment theorem.	B. The angle at the centre of a circle is double the angle at the circumference.	C. Opposite angles of a cyclic quadrilateral are supplementary.	D. Opposite angles of a cyclic quadrilateral are equal.
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<https://diagnosticquestions.com>

Why do this activity? This activity requires learners to focus their attention on what is relevant to solving the problem. The tangent is a distractor because it is not needed in the solution.

Intended learning outcomes

For learners to get practice in using and applying circle theorems.

Suggestions for teaching

Start the lesson with the diagnostic question and the follow-up to review the circle theorems. Make sure that the learners understand that option D in the follow-up is NOT true, it is not one of the circle theorems.

Notice that it is not necessary to use tangent or the alternate segment theorem in this solution but some learners may do so in order to find angle y° .

Use the 1 – 2 – 4 - more teaching strategy. Give the learners 10 minutes to do the problem **working on their own** as they would do in a test. Then ask the learners to work in pairs, to compare answers and methods and to help each other. Have they used different methods? Then get each pair to work with another pair and compare methods. Finally invite learners to come and use the board and explain their work to the class.

Summarize what has been learned.

Key questions

What do you notice in the diagram?

Can you see any equal lengths? Have you marked them on your diagram?

Can you see any equal angles? Have you marked them on your diagram?
If you can't find angle x or angle y right away what angles can you find?

Possible extension

Why is it impossible to find $\angle QPR$ or $\angle QRP$? Explain.

Answer: Point Q could be anywhere on the arc PR, changing both $\angle QPR$ and $\angle QRP$ while angle $\angle PQR$ would always be 102° and the rest of the angles in the diagram would stay the same.

Possible support

It usually helps to mark on the diagram the size of ALL the angles you can find even if you don't know how to find the angles needed. As you fill up the diagram with the angles you know it may help to you to see how to find other angles.

This is a General Certificate of Secondary Education (GCSE) question from OCR Examination Board taken in the UK by 16 year olds.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.

	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6