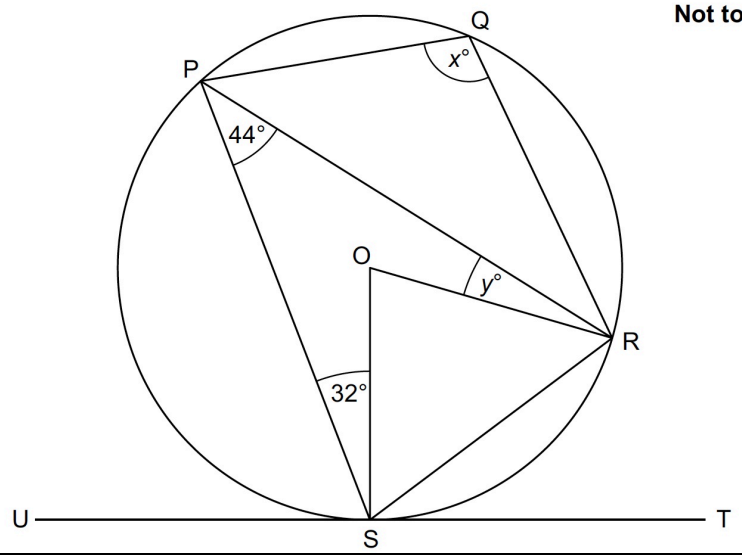
CYCLIC



Not to scale

The diagram shows a circle centre O with points P, Q, R and S on the circumference. UST is a tangent to the circle. Angle RPS = 44° and angle PSO=32°.

Find angles x° and y°.

Help

It usually helps to mark on the diagram the size of ALL the angles you can find even if you don't know how to find the angles needed. As you fill up the diagram with the angles you know it may help to you to see how to find other angles.

Extension

Why is it impossible to find ∠QPR or ∠QRP? Explain.

NOTES FOR TEACHERS

SOLUTION

Method 1

$\angle SOR = 88^o$ the angle at the centre is twice the angle at the circumference

$\angle OSR = \angle ORS = \frac{1}{2}(180^o - 88^o) = 46^o$ isosceles triangle, angles in triangle add to 180^o

$\angle PSR = 78^o$

$\angle PQR = x^o = 180^o - 78^o = 102^o$ opposite angles of cyclic quadrilateral add to 180^o

$\angle PRO = y^o = 180^o - (44^o + 32^o + 46^o + 46^o) = 12^o$ angles of ΔPSR add to 180^o

Method 2

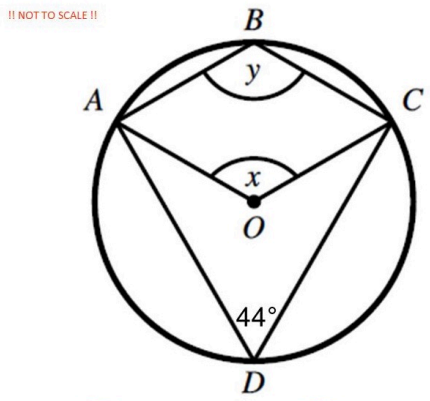
$\angle SOR = 88^o$ $PSU = 90^o - 32^o = 58^o$ OS is perpendicular to tangent UT

$\angle PRS = 58^o$ angle in the alternate segment subtended by chord PS

$\angle PR0 = y^o = 58^o - 46^o = 12^o$

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



Angle ADC = 44°

What is the size of angle x?

A 136° B 22° C 88° D 44°

C. is the correct answer. Note that the diagram is not to scale and although angle x looks as if it is bigger than 88^o that is the answer because the angle at the centre subtended by arc ABC is double the angle at the circumference.

Common Misconceptions

A. This answer "136 degrees is the only number that is an obtuse angle out of all of the other numbers. Other numbers are acute angles." shows learner thought the angle must be obtuse and used $180^o - 44^o$.

B. This learner half remembered the theorem but instead of doubling the angle at the circumference, halved it.

D. Probably just a guess.

Follow up diagnostic question: Which theorem is used to find angle y?

A. The angle in the alternate segment theorem.	**B**. The angle at the centre of a circle is double the angle at the circumference.	**C**. Opposite angles of a cyclic quadrilateral are supplementary.	**D**. Opposite angles of a cyclic quadrilateral are equal.

https://diagnosticquestions.com

Why do this activity? This activity requires learners to focus their attention on what is relevant to solving the problem. The tangent is a distractor because it is not needed in the solution.

Intended learning objectives

For learners to get practice in using and applying circle theorems.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations and proofs;
- visualize - develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for teaching

Start the lesson with the diagnostic question and the follow-up to review the circle theorems. Make sure that the learners understand that option D in the follow-up is NOT true, it is not one of the circle theorems.

Notice that it is not necessary to use tangent or the alternate segment theorem in this solution but some learners may do so in order to find angle y^o.

Use the 1 – 2 – 4 - more teaching strategy. Give the learners 10 minutes to do the problem **working on their own** as they would do in a test. Then ask the learners to work in pairs, to compare answers and methods and to help each other. Have they used different methods? Then get each pair to work with another pair and compare methods. Finally invite learners to come and use the board and explain their work to the class.

If time you might ask "Why is it impossible to find ∠QPR or ∠QRP?" and ask learners to explain their answers. The answer is that point Q could be anywhere on the arc PR, changing both ∠QPR and ∠QRP while angle ∠PQR would always be 102^o and the rest of the angles in the diagram would stay the same.

To end the lesson, summarize what has been learned.

This is a General Certificate of Secondary Education (GCSE) question from OCR Examination Board taken in the UK by 16 year olds.

Key questions

- What do you notice in the diagram?
- Can you see any equal lengths? Have you marked them on your diagram?
- Can you see any equal angles? Have you marked them on your diagram?
- If you can't find angle x or angle y right away what angles can you find?

Follow-up ideas

Investigating Circle Theorems https://aiminghigh.aimssec.ac.za/years-10-11-investigating-circle-theorems/
Quad with Inscribed Circle: https://aiminghigh.aimssec.ac.za/years-10-12-circle-inscribed-in-quadrilateral/
Salinon https://aiminghigh.aimssec.ac.za/years-9-11-salinon/
Article about Cyclic Quadrilaterals https://nrich.maths.org/1310

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	**Upper Primary**	**Lower Secondary**	**Upper Secondary**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**