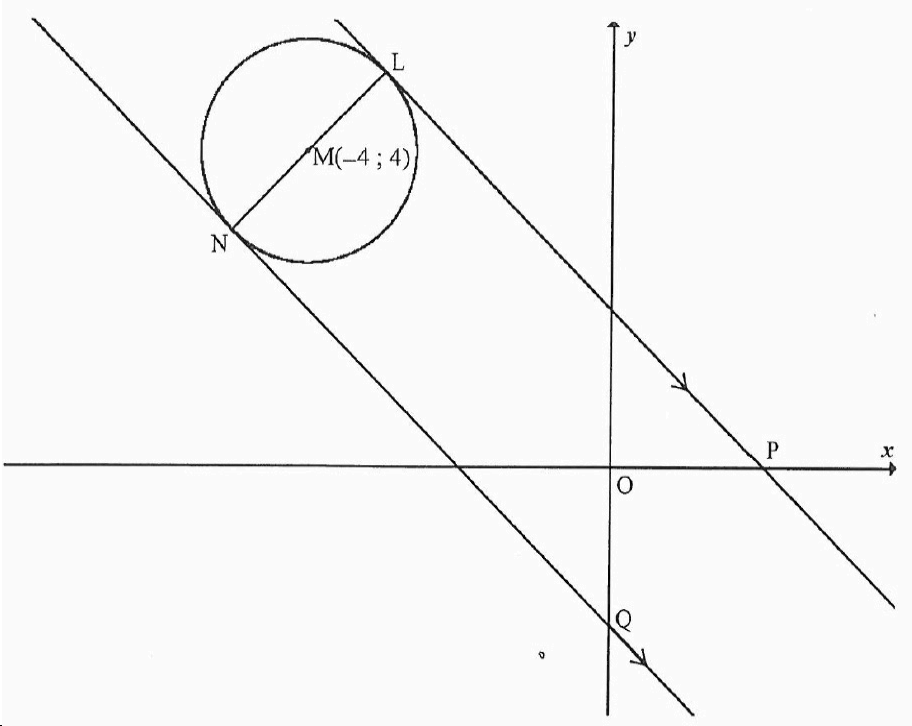
COIN IN SLOT



The line LP with equation $y + x - 2 = 0$ is a tangent at L to the circle with centre M(-4 , 4).
LN is a diameter of the circle. LP is parallel to NQ where P is on the x-axis and Q is on the y-axis.
Find:
a) the equation of the line LN;
b) the coordinates of L;
c) the equation of the circle;
d) the coordinates of N;
e) the equation of NQ.
f) The length of the diameter is doubled and the circle is translated 6 units to the right. Find the equation of the new circle?

Help

It may help you to realise that each part of the answer is a stepping stone to the next part.

Extension

Reflect the diagram in the x-axis. Then answer the corresponding questions for your new diagram.

NOTES FOR TEACHERS

SOLUTION

a) Line LP has equation y = -x + 2 and gradient -1
As LN and LP are perpendicular, line LN has gradient +1 and equation y = x + c where 4 = -4 + c so c = 8.
The equation of LN is y = x + 8

b) At the point L: –x + 2 = x + 8 so x = - 3. So the coordinates of L are (-3, +5)

c) ML = $\sqrt{2}$ as it is the hypotenuse of a 45° isosceles right angled triangle with edge 1 unit.
The equation of the circle is $(x + 4)^2 + (y – 4)^2 = 2$.

d) The coordinates of N are (-5, 3) .

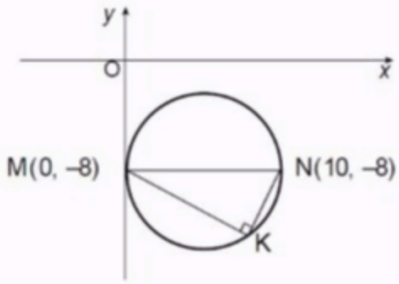
e) The equation of NQ is y = -x + c where 3 = -(-5) + c so c = -2. The equation of NQ is y = -x – 2
This can be deduced from the symmetry of the diagram about the line y = -x .

f) The new circle has centre (2, 4) and radius $2\sqrt{2}$ so it has equation
$(x – 2)^2 + (y – 4)^2 = 8$

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The diagram shows a circle with the *y*-axis as a tangent.
M and N have coordinates (0, −8) and (10, −8) and angle MKN equals 90°.



What is the equation of the circle passing through M, K and N ?

A $(x+5)^2+(y-8)^2=100$

B $(x-10)^2+(y+8)^2=100$

C $(x+5)^2+(y-8)^2=5$

D $(x-5)^2+(y+8)^2=25$

D. is the correct answer.

Common Misconceptions

A. "A because x value is -5 and y value is +8 and the radius is 10 making it = to 100" shows confusion about the radius.

B. Just using the numbers in the diagram without understanding.

C. Here the learner is substituting in the coordinates of the centre and the radius without understanding that the equation arises from Pythagoras Theorem giving the squares of the distances from (x , y) to the centre and the square of the radius.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners practice in using the equations of lines (including perpendicular lines) and also equations of circles. The last part requires an understanding of the derivation of the equation of the circle.

Intended learning outcomes
Learners to practice using and applying equations of lines and circles.

Generic competences
In doing this activity students will have an opportunity to:
- reason logically and to apply knowledge and skills;
- develop the skill of interpreting visual images to represent concepts and situations;
- communicate in writing and speaking:
    - communicate, exchange ideas, criticise, and present information and ideas to others
    - have empathy with others, listen to different points of view
    - analyze, reason and record ideas effectively.

Suggestions for teaching
After doing the diagnostic question and reviewing equations of circles give learners a worksheet copied from the top part of page 1 or write the question on the board.

Use the 1 - 2 – 4 – more teaching strategy. Ask learners to work on their own as if in a test. When a few learners have finished ask the learners to work in pairs to check their methods and answers. If some learners did not get very far because they did not understand what to do, then the other learner should explain the steps in the solution. If they get different answers then they should sort out for themselves where the mistake is. If they have used different methods they should explain their methods to each other. The objective is to understand the concepts involved.

Then pairs can join with another pair and check methods and answers in a similar way.

Finally have different pairs come to the board to explain each of the five parts.

This question comes from the South African Matric Paper 2 November 2010.

Key questions
Can you find the gradient of that line?
If the equation of a straight line is y = mx + c then what do you know about m and c?
What do you know about the gradients of a radius OT and a tangent at T to a circle?
What do you know about the gradients of perpendicular lines?
What has Pythagoras Theorem got to do with the equation of a circle?

Follow up
The activity 'Lines' https://aiminghigh.aimssec.ac.za/years-10-12-lines/ could be used as an introduction to this task.
Line Match https://aiminghigh.aimssec.ac.za/years-10-12-line-match/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundatio Phase	**Upper Primary**	**Lower Secondary**	**Upper Secondary**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**