AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

QUADRATIC MATCHING 2

The graphs and some properties of seven quadratic functions are given here with their equations in the forms:

$y = ax^2 + bx + c$,
$y = (x + p)(x + q)$ and
$y = a(x + r)^2 + s$

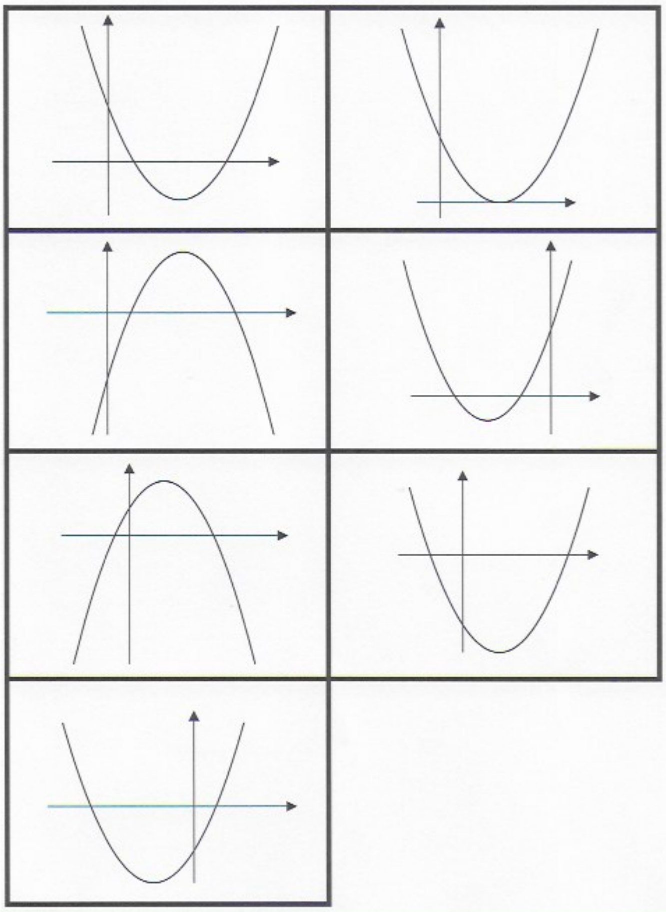
Match them up and put them into 7 sets.

Make a poster showing the graph of each function with the matching equations and properties.

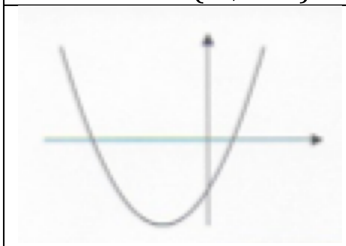
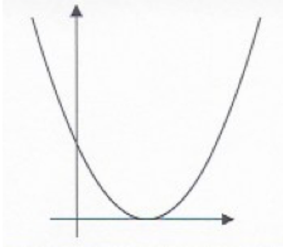
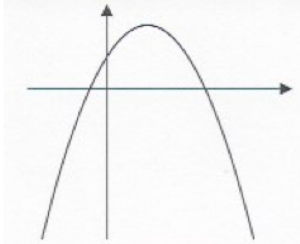
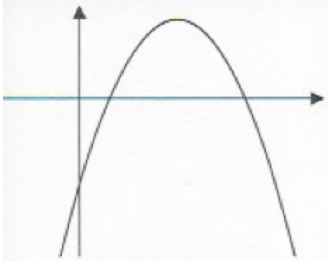
In all these examples a = +1 or -1. Choose your own quadratic function where a is not equal to +1 or -1 and complete your poster your own 8th set with its graph, equations and properties.

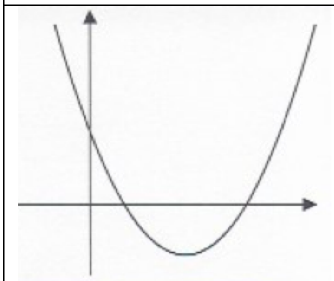
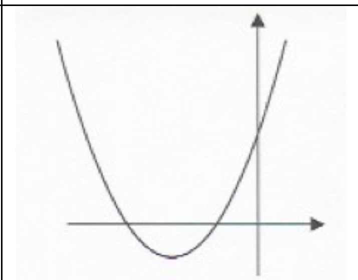
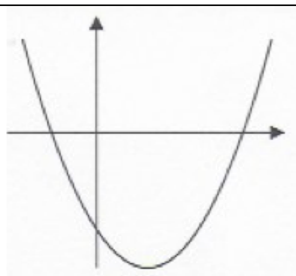
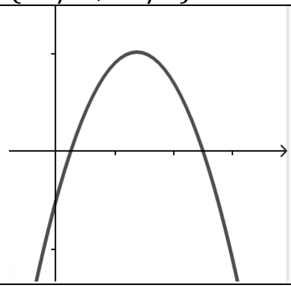
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$
$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	Minimum at (3, –25)
Minimum at (–3, –1)	Maximum at (1, 9)

Maximum at (3, 1)	Minimum at (5, –9)
Minimum at (4, 0)	Minimum at (–3, –25)
$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2



SOLUTION

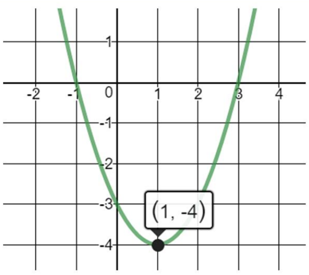
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$	$y = 8 – x^2 + 2x$	$y = 6x – x^2 - 8$
$y = (x + 8)(x - 2)$	$y = (x – 4)(x – 4)$	$y = (x + 2)(4 – x)$	$y = (x – 2)(4 – x)$
$y = (x + 3)^2 – 25$	$y = (x – 4)^2$	$y = -(x – 1)^2 + 9$	$y = -(x – 3)^2 + 1$
x = 0, y = -16	x = 0, y = 16	x = 0, y = 8	x = 0, y = -8
y = 0, x = -8 or 2	y = 0, x = 4	y = 0, x = -2 or 4	y = 0, x = 2 or 4
Minimum at (-3, - 25)	Minimum at (4 , 0)	Maximum at (1, 9)	Maximum at (3, 1)
			

			EXAMPLE
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$	$y = x^2 - 6x - 16$	$y = 11x - 5 – 2x^2$
$y = (x - 2)(x - 8)$	$y = (x + 2)(x + 4)$	$y = (x + 2)(x - 8)$	$y = (2x – 1)(5 – x)$
$y = (x - 5)^2 – 9$	$y = (x + 3)^2 - 1$	$y = (x – 3)^2 -25$	$y = -2(x – 11/4)^2 + 81/8$
x = 0, y = 16	x = 0, y = 8	x = 0, y = -16	x = 0, y = - 5
y = 0, x = 2 or 8	y = 0, x = -2 or - 4	y = 0, x = -2 or 8	y = 0, x = 5 or ½
Minimum at (5, - 9)	Minimum at (-3 , -1)	Minimum at (3, -25)	Maximum at (11/4 , 81/8)
			

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.



Which of the following could be the equation of this curve?

A $y = (x + 1)^2 - 4$

B $y = (x - 1)^2 + 4$

C $y = (x + 1)^2 + 4$

D $y = (x - 1)^2 - 4$

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

D. is the correct answer.

Common Misconceptions

A. Most common mistake. Learner might know the turning point is (1,-4) and see +1 and -4 in the equation.

B. "I think this because the 4 in the equation has to be positive and the 1 has to be negative as the coordinates are opposite."

C. Probably has no idea and just guessed.

https://diagnosticquestions.com

Why do this activity?

This activity provides a good review of quadratic functions and leads to work on finding maxima and minima through differentiation. It can be done with sets of cards (see pages 5 to 9).
There are many ways in which teachers can use this activity. It can be adapted in the following ways:

(1) Split into 2 lessons by providing card sets B and C in the first lesson, see Quadratic Matching 1, and card sets A2 and D in the second lesson.
(2) Cater for all abilities by giving some learners all the cards in sets A1, A2 and B and giving the weaker learners just card sets B and C (as in Quadratic Matching 1) and then later giving the learners who have sorted sets B and C the additional card sets A2 and D.
(3) If some learners are likely to find the activity easy you might like to give them a reduced set of cards and ask them to create the missing ones.
(4) Using cards makes the activity more manageable for some learners but if your school has not got resources to make the cards then you can avoid using cards by simply giving learners a copy of the worksheet on page 1.
(5) Or avoid using cards by copying the question on the board.
(6) If you have computers available and free graphing software such as Geogebra, then learners can experiment with additional quadratic functions in the various forms and their graphs.

Intended learning outcomes

To enable learners to:

- identify different forms and properties of quadratic functions
- connect quadratic functions with their graphs and properties including intersection with axes and maxima and minima.

Suggestions for teaching

According to the class you are teaching and to the resources available in your school, you can choose any of options 1 to 6 given above.

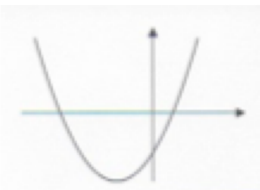
Explain to the class that they have to sort the information into 7 sets of information with each set made up of a quadratic function written in different forms, some associated properties and a graph. The learners should work in pairs to create posters showing the 7 sets and one additional set of their own that they can create once they have done the other seven.

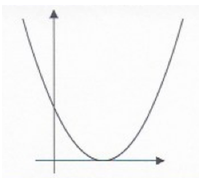
Encourage learners to explain their reasoning both to you as you move around the room and to each other.

To review and extend learning in a plenary:

Ask 7 pairs of learners in turn to come to the board and explain how they decided that a set of cards belong together.

Ask learners to answer questions using show boards (mini-whiteboards) for example:





Show them one of these graphs and ask:
Give me a possible equation for this graph.
Can you give me that equation in a different form?
Can you give me a completely different equation?

Or ask any of the Key Questions below varying the coefficients.
Or ask learners to produce generalisations of these results:

- Show me the equation of a quadratic with a minimum at (r , s). Now show me the same equation in a different form.
- Show me the equation of a quadratic with x intercepts at p and q. Now show me the same equation in a different form.

Key questions

(1) What are the x and y intercepts of $y = (x - 8)(x + 2)$? How can you tell?
(2) What are the x and y intercepts of $y = (x - 4)^2$? How can you tell?
(3) Show me the equation of a quadratic that intercepts the y axis at -16. Now show me the same equation in a different form.
(4) Show me the equation of a quadratic that intersects the x axis at -4 and -2. Now show me the same equation in a different form.
(5) Where are the intercepts of the function $y = (x + 3)^2 - 1$? Where is the minimum of the function? how can you tell?
(6) Show me the equation of a quadratic function with a minimum at (5, -9). Now show me the same equation in a different form.

Possible extension

1. Link the completed square form to transformations of graphs.
2. Learners could be asked to sketch graphs for given quadratic functions with intercepts and stationary points marked.

Possible support

Give learners just card set C and ask them to match the equation of the quadratic functions with the factorised forms.
When they have done this give them card set E and ask them to match the intercepts with the axes.
Then give them the graphs to match to the 7 sets.
If they can achieve all this then give them the completed square equations and coordinates for turning points to match up.

Adapted from the STANDARDS UNIT professional development materials produced by the UK Department for Education and Skills. Author Malcolm Swan.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6

CARD SET A1 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties.
The quadratic functions are written in the forms:
$y = ax^2 + bx + c$
$y = (x + p)(x + q)$
$y = a(x + r)^2 + s$

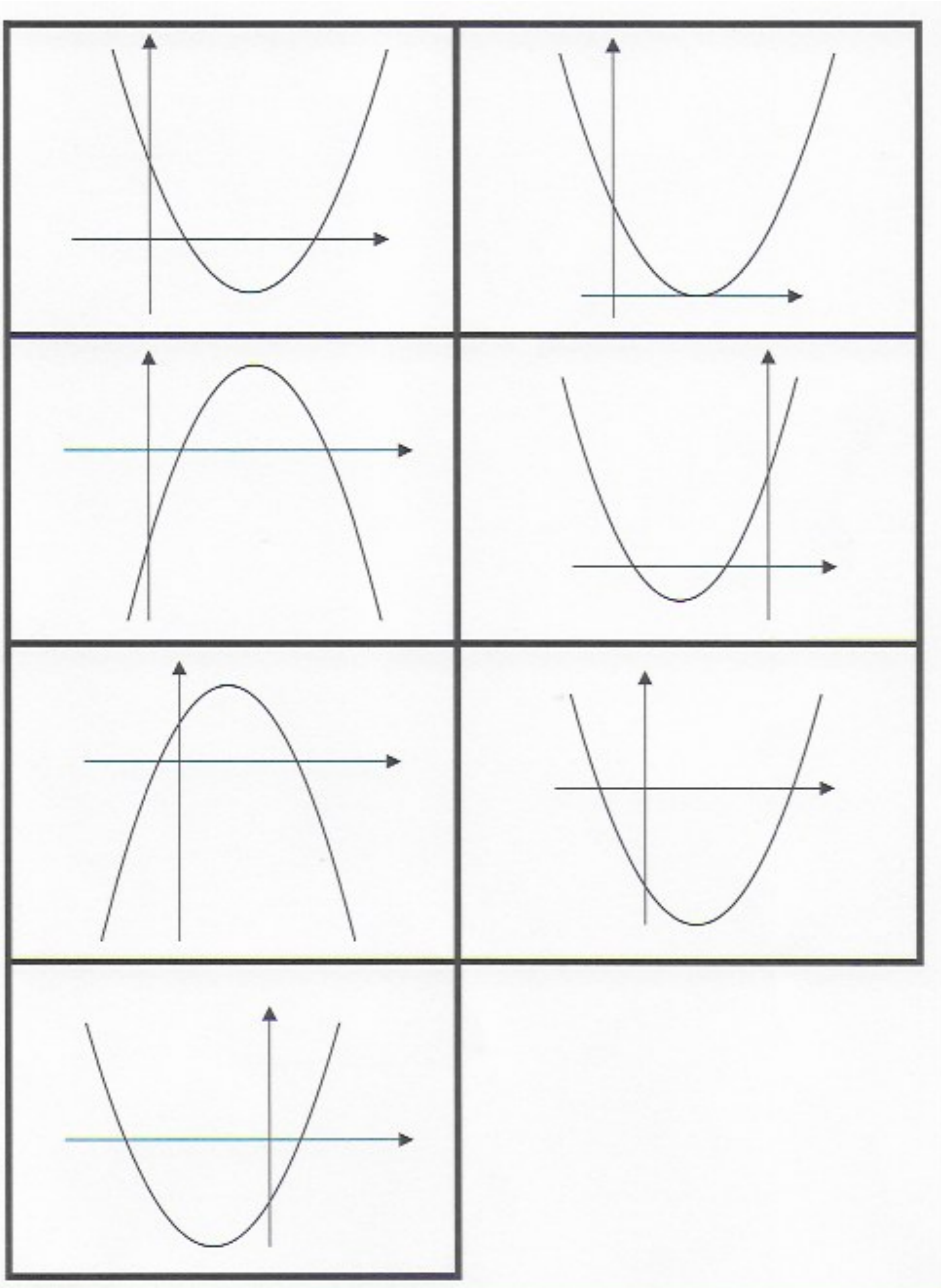
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$
$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	Minimum at (3, –25)
Minimum at (–3, –1)	Maximum at (1, 9)

SET A2 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties.
The quadratic functions are written in the forms:
$y = ax^2 + bx + c$
$y = (x + p)(x + q)$
$y = a(x + r)^2 + s$

Maximum at (3, 1)	Minimum at (5, –9)
Minimum at (4, 0)	Minimum at (–3, –25)
$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2

CARD SET B

Match the graphs to the corresponding cards showing the equations and properties of the functions.



CARD SET C

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$

CARD SET D

$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	Minimum at (3, –25)
Minimum at (–3, –1)	Maximum at (1, 9)

CARD SET E Intercepts with the axes

$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2