

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

QUADRATIC MATCHING	1
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	y=(x-8)(x+2)
y = (x+4)(x+2)	y = (x+2)(4-x)
y=(x-4)(2-x)	y=(x-8)(x-2)
y = (x-4)(x-4)	y=(x+8)(x-2)

Graphs and equations of 7 quadratic functions are given here.

Match them and put them into 7 sets. Write down the coordinates of the intercepts with the axes.

Make a poster showing the graph of each function with the matching equations.



In all these examples the coefficient of the quadratic term is +1 or -1. Choose your own quadratic function where this coefficient is not equal to +1 or -1 and complete your poster your own 8th set with its graph, equations and properties.

HELP

First use the cards in set C and match the equations of the quadratic functions with the factorised forms.

Then match the graphs given in set B to the equations to make up the 7 sets.

Then use the cards in set E and match this information about the intercepts of the graph with the axes to the 7 sets.

NEXT

Match the remaining cards in sets A1 and A2 with the other cards.

Resources: Cards sets A1, A2, B, C, D and E.

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	y=(x-8)(x+2)
y = (x+4)(x+2)	y = (x+2)(4-x)
y = (x-4)(2-x)	y=(x-8)(x-2)
y=(x-4)(x-4)	y=(x+8)(x-2)
$y = \left(x+3\right)^2 - 25$	$y = (x-4)^2$
$y = \left(x - 5\right)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x-1)^2 + 9$	$y = \left(x+3\right)^2 - 1$
$y=\left(x-3\right)^2-25$	Minimum at (3, –25)
Minimum at (–3, –1)	Maximum at (1, 9)

CARD SET A1 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties. The quadratic functions are written in the forms: $y = ax^2 + bx + c$; y = (x + p)(x + q); and $y = a(x + r)^2 + s$

Maximum at (3, 1)	Minimum at (5, –9)
Minimum at (4, 0)	Minimum at (-3, -25)
x = 0, y = -16	x = 0, y = 16
x = 0, y = 16	x=0,y=-8
x = 0, y = 8	x = 0, y = 8
x = 0, y = -16	y = 0, x = 8 or -2
y = 0, x = -4 or -2	y = 0, x = -2 or 4
y = 0, x = 4 or 2	y = 0, x = 8 or 2
y = 0, x = 4	y = 0, x = -8 or 2

CARD SET A2 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties The quadratic functions are written in the forms: $y = ax^2 + bx + c$; y = (x + p)(x + q); and $y = a(x + r)^2 + s$

CARD SET B

Match the graphs to the corresponding cards showing the equations and properties of the functions.



CARD SET C

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	y = (x - 8)(x + 2)
y = (x+4)(x+2)	y = (x+2)(4-x)
y = (x-4)(2-x)	y = (x-8)(x-2)
y=(x-4)(x-4)	y = (x+8)(x-2)

CARD SET D

$y = \left(x+3\right)^2 - 25$	$y = \left(x - 4\right)^2$
$y=\left(x-5\right)^2-9$	$y = -(x-3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y=\left(x+3\right)^2-1$
$y = \left(x - 3\right)^2 - 25$	Minimum at (3, –25)
Minimum at (-3, -1)	Maximum at (1, 9)

CARD SET E Intercepts with the axes

x = 0, y = -16	x = 0, y = 16
x = 0, y = 16	x=0, y=-8
x = 0, y = 8	x = 0, y = 8
x = 0, y = -16	y = 0, x = 8 or -2
y = 0, x = -4 or -2	y = 0, x = -2 or 4
y = 0, x = 4 or 2	y = 0, x = 8 or 2
y = 0, x = 4	y = 0, x = -8 or 2

Adapted from the STANDARDS UNIT professional development materials produced by the UK Department for Education and Skills. Author Malcolm Swan.

NOTES FOR TEACHERS

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$	$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
y = (x + 8)(x - 2)	y = (x - 4)(x - 4)	y = (x + 2)(4 - x)	y = (x - 2)(4 - x)
x = 0, y = -16	x = 0, y = 16	x = 0, y = 8	x = 0, y = -8
y = 0, x = -8 or 2	y = 0, x = 4	y = 0, x = -2 or 4	y = 0, x = 2 or 4

			EXAMPLE
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$	$y = x^2 - 6x - 16$	$y = 11x - 5 - 2x^2$
y = (x - 2)(x - 8)	y = (x + 2)(x + 4)	y = (x + 2)(x - 8)	y = (2x - 1)(5 - x)
x = 0, y = 16	x = 0, y = 8	x = 0, y = -16	x = 0, y = - 5
y = 0, x = 2 or 8	y = 0, x = -2 or - 4	y = 0, x = -2 or 8	y = 0, x = 5 or ½

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes. Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1.Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: D

Common Misconceptions

A. Most common mistake. Learner might know the turning point is (1,-4) and see +1 and -4 in the equation.

B. "I think this because the 4 in the equation has to be positive and the 1 has to be negative as the coordinates are opposite."

C. Probably has no idea and just guessed. <u>https://diagnosticquestions.com</u>

Why do this activity?

This activity provides a good review of quadratic functions. It can be done with sets of cards or from the worksheet on page 1.

The activity can be extended to work on the completed square form of quadratic equations and to finding maxima and minima by symmetry and inspection. See Quadratic Matching 2:

https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-2/

This leads to work on transformations of graphs and finding turning points by differentiation.

Learning objectives

In doing this activity students will have an opportunity to:

- identify different forms and properties of quadratic functions;
- connect quadratic functions with their graphs and properties including intersections with axes.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations;
- think flexibly, be creative and innovative and apply knowledge and skills;
- develop the **skill of visualizing**, interpreting and/or creating visual images to represent concepts and situations.

Suggestions for teaching

Explain to the class that they have to sort the information into 7 sets of information with each set made up of a quadratic function written in 2 different forms and a graph. The learners should work in pairs to create posters showing the 7 sets and one additional set of their own that they can create once they have done the other seven.

Encourage learners to explain their reasoning both to you as you move around the room and to each other.

To review and extend learning in a plenary:

Ask 7 pairs of learners in turn to come to the board and explain how they decided that a set of cards belong together.

Ask learners to answer questions using show boards (mini-whiteboards) for example:



Show them one of these graphs and ask:

Give me a possible equation for this graph.

Can you give me that equation in a different form?

Can you give me a completely different equation?

Or ask any of the Key Questions below varying the coefficients.

Key questions

- (1) What are the x and y intercepts of y = (x 8)(x + 2)? How can you tell?
- (2) What are the x and y intercepts of $y = (x 4)^2$? How can you tell?
- (3) Show me the equation of a quadratic that intercepts the y axis at -16. Now show me the same equation in a different form.
- (4) Show me the equation of a quadratic that intersects the x axis at -4 and -2. Now show me the same equation in a different form.

Follow up

Quadratic Matching 2: <u>https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-2/</u>



Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: <u>http://aiminghigh.aimssec.ac.za</u> Subscribe to the **MATHS TOYS YouTube Channel** <u>https://www.youtube.com/c/mathstoys</u> Download the whole AIMSSEC collection of resources to use offline with

the **AIMSSEC App** see <u>https://aimssec.app</u> or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12 Lower Primary **Upper Primary** Lower Secondary Upper Secondary Approx. Age 5 to 8 Age 8 to 11 Age 11 to 15 Age 15+ South Africa Grades R and 1 to 3 Grades 4 to 6 Grades 7 to 9 Grades 10 to 12 East Africa Nursery and Primary 1 to Primary 4 to 6 Secondary 1 to 3 Secondary 4 to 6 USA Kindergarten and G1 to 3 Grades 4 to 6 Grades 7 to 9 Grades 10 to 12 UK Reception and Years 1 to 3 Years 7 to 9 Years 10 to 13 Years 4 to 6