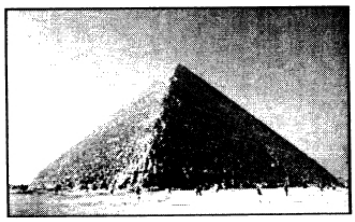
GREAT PYRAMID

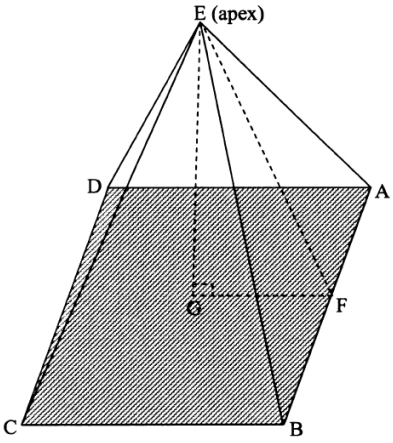


Great Pyramid at Giza in Egypt

The Great Pyramid at Giza in Egypt was built around 2500 BC. The pyramid has a square base ABCD with sides 232.6 metres long. The distance from each corner of the base to the apex E was originally 221.2 metres.

Draw a diagram of the pyramid to show two right angled triangles from which you can write down trigonometric equations to answer the following questions.

You may find it helpful to make a square based pyramid with rolled paper sticks and cut out a paper triangle like triangle EGF to fit inside.



1. If F is a point on AB such that EF is perpendicular to AB and G is vertically below the apex E, what can you say about triangles EFA and EGF?

2. Calculate the size of the angle at the apex of a face of the pyramid (for example angle AEB).
Can you find this by a different method?

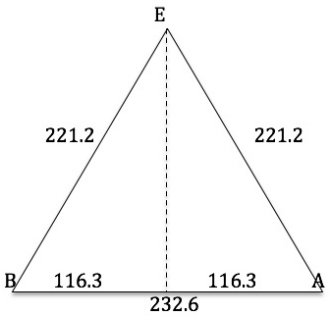
3. Calculate the angle each face makes with the base (for example angle EFG).

4. Calculate the height of the pyramid.

This question is from the South African National School Certificate Examination Mathematics Paper 2 November 2013 Qu. 13

HELP

You will find it useful to draw this diagram so that it is easy to see the right angled triangles.



NEXT

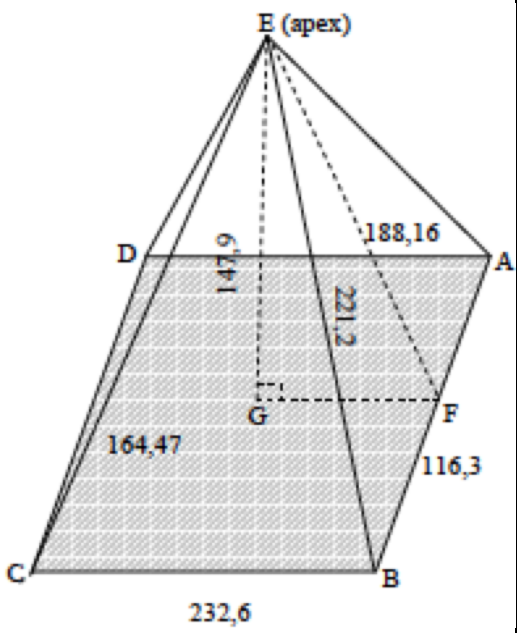
Find as many lengths and angles in the pyramid as possible, for example CG and angle ECG.

NOTES FOR TEACHERS

SOLUTION

1. If F is a point on AB such that EF is perpendicular to AB and G is vertically below the apex E, then triangles EFA and EGF are right angled.

2. AF = 116.3 m (half the length of AB).
   EA = 221.2 m (the sloping edge length given)
   sin ∠AEF = 116.3/221.2 = 0.5258
   ∠AEF = 31.7°
   ∠AEB = 63.4°



Alternative method using the cosine rule for ΔAEB:

$2(221.2)^2 \cos \angle AEB = 221.2^2 + 221.2^2 - 232.6^2$

$\cos \angle AEB = 1 - \frac{1}{2}(232.6)^2/(221.2)^2$

$= 0.4471$

$\angle AEB = 63.4^\circ$

3. By Pythagoras Theorem
   $EF^2 = 221.2^2 - 116.3^2 = 35403.75$
   EF = 188.16 m
   cos ∠EFG = 116.3/118.16 = 0.6181
   ∠EFG = 51.8°

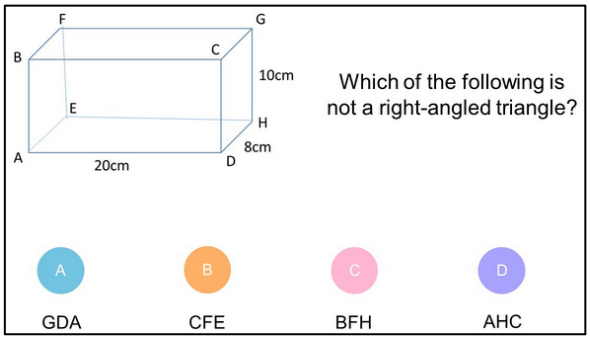
4. EG = 116.3tan ∠EFG = 147.9m

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

D is the correct answer.



Common Misconceptions

A) AD⊥CDHG so AD⊥DG (line in plane CDHG) so GDA is a rt triangle. Learners may think the lines should go to opposite corners of the shape in order to be at right angles

B) EF⊥BCGF so EF⊥FC (line in plane BCGF) so CFE is a rt triangle.
Learners may think CFE is not a right-angled triangle because it does not look like one in the diagram.

C) BF⊥ EFGH so BF⊥FH (line in plane EFGH) so BFH is a rt triangle.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners an opportunity to develop their ability to visualise 3D structures and to apply the trigonometry they have learned to a real world problem. It is important for learners to be able to use and apply the mathematics that they learn.

Learning objectives

In doing this activity students will have an opportunity to:

- develop visualisation of 3D structures;
- practise solving problems using right angled triangles and trigonometry.

Generic competences

In doing this activity students will have an opportunity to **develop visualization** and apply their knowledge to **solve problems** in a real life context.

Suggestions for teaching

You may find it helpful to make a square based pyramid with paper sticks and to cut out a paper triangle like triangle EGF to fit inside.

Keep a supply of paper sticks so that you can quickly make demonstration models. Just roll the paper tightly around a piece of string and secure with selotape. Untie the string and dismantle the model if you want ease of storage for your paper sticks.

You can decide whether to use the Diagnostic Quiz at the start of the lesson or at the end.

To give learners practice in reading comprehension you could give them the first paragraph and tell them you will give them 5 minutes to draw a diagram of the pyramid. Ask them to find two right angled triangles from which they can write down trigonometric expressions to find angles and lengths in the diagram.

Ideally the learners should draw LARGE diagrams on show-boards and raise their showboards when you say 'NOW' so you can see quite quickly what they have managed to do. If you don't have showboards then learners can all draw their diagrams large scale, in their notebooks, or on paper, and hold them up to show the teacher. *The image shows learners raising showboards but for a different activity.*

Then ask learners to come and draw the diagram on the board. Label the vertices of the pyramid and draw two right angled triangles EFA and EGF.

Use the **1 - 2 - 4 - More** teaching strategy. Start with the students working on their own. Then tell them to work in pairs and suggest that they each find answers and check with each other if they have the same answers. Then tell pairs to work with another pair in a four-some and again to check with the other pair if they have the same answers. After that engage the whole class in a plenary session when students present and explain their methods of solution to the class.

Finally reflect on and summarize what has been learnt.

Key questions

- Can you draw (or show me) any right-angled triangles in the pyramid?
- What can you say about triangles EFA and EFB?
- What can you say about triangles EFG?
- What lengths do you know in that triangle?
- If you know two lengths in a right-angled triangle how can you find the angles?

Follow up

In a soccer stadium https://aiminghigh.aimssec.ac.za/year-12-in-a-soccer-stadium/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6